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An Annual Tree Survival and Diameter Growth Model for Loblolly and Slash Pine Plantations in East Texas

Dean W. Coble, Quang V. Cao, and Lewis Jordan

ABSTRACT

An annual growth model that predicts individual tree survival and diameter growth was developed for loblolly pine (*Pinus taeda*) and slash pine (*Pinus elliottii*) trees in East Texas as a function of individual-tree diameter, plantation age, basal area per acre, dominant height, quadratic mean diameter, and presence of fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *fusiforme*). Data from 104,035 loblolly pine and 37,515 slash pine individual tree observations collected on a 3-year cycle from 174 loblolly pine and 80 slash pine permanent plots located in plantations throughout East Texas were used in this study. The survival equation assumes that mortality is constant across the projection length, whereas the diameter growth equation incorporates whole-stand predictions to update stand-level independent variables on an annual basis. Predictions were evaluated in terms of bias and precision, with independent observations for projection lengths from 3 to 24 years. For both survival and diameter growth, bias was lowest and precision highest for 3-year projection lengths. For survival, bias increased and precision decreased as projection length increased through 24 years. For diameter growth, bias was constant (<1 in.) across all projection lengths, whereas precision decreased from <1 in. for the 3-year projection length to <2 in. for the 6–24-year projection lengths. A numerical example is provided that describes how to use the new model to project individual tree survival and diameter growth on an annual basis.

Keywords: growth and yield, maximum likelihood, simultaneous equations, West Gulf Coastal Plain

Annual growth and survival of individual trees is routinely predicted with individual tree growth models (Stage 1973, Mitchell 1975, Hilt 1985, Larsen and Hann 1985, Burkhart et al. 1987, Wensel and Biging 1987, Hilt and Teck 1989, Marquis and Ernst 1992). Traditionally, annual growth is assumed to be constant during the projection period (Cao and Strub 2008). This can be an oversimplification of reality in some cases because growth and survival are nonlinear through time. McDill and Amateis (1993), Cao (2000), Cao et al. (2002), and Nord-Larsen (2006) subsequently introduced interpolation methods to improve on the traditional constant rate method. However, these improvements still required estimates of stand-level variables in the individual tree equations for the years between the beginning and end of the projection period. To address this problem, Cao (2002, 2004) incorporated a whole-stand growth model into his interpolation method to predict stand-level variables on an annual basis. Cao and Strub (2008) examined four methods to estimate the parameters of an annual individual tree survival and diameter system of equations that incorporates annual updates of stand-level variables. They found that the predicting attributes method, which incorporates a whole-stand model to predict stand-level variables on an annual basis, outperformed two other methods that used traditional linear interpolation or assumed that growth and survival was constant for the projection period.

The objective of this study was to develop individual-tree annual survival and diameter growth equations for unmanaged loblolly pine (*Pinus taeda*) and slash pine (*Pinus elliottii*) plantations in East Texas based on the methodologies of Cao and Strub (2008). The prediction equations were validated with observed data to examine the model's performance in terms of bias and precision for projection lengths from 3 to 24 years.

Data Description

This study used 104,035 observations from 174 remeasured permanent plots located in East Texas loblolly pine plantations (Table 1). For slash pine, a total of 37,515 observations from 80 permanent plots were used (Table 1). These plots are part of the East Texas Pine Plantation Research Project (Lenhart et al. 1985), which covers 22 counties across East Texas. Generally, the counties are located within the rectangle from 30° to 35° north latitude and 93° to 96° west longitude. Each plot is approximately 0.25 ac in size (100 × 100 ft). Within a plot, dbh (in inches, measured at 4.5 ft above the groundline), total height (feet), and the survival status (live or dead) were measured three to nine times for each planted tree. Plots were remeasured at fixed 3-year intervals. Data were randomly selected from only one measurement cycle per plot to avoid serial autocorrelation problems among individual tree measurements through time. Dominant height (feet) was determined by averaging the total heights of the 10 tallest trees on a plot (which approximately represents the tallest 40 trees per acre) that were free of damage, forks, and stem fusiform rust.

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Table 1. Observed individual tree and stand characteristics for East Texas loblolly and slash pine plantation pooled data set (for all variables, n = 104,035 observations from 174 remeasured permanent plots for loblolly pine and n = 37,515 observations from 80 remeasured permanent plots for slash pine).

Species and variable	Mean	Standard deviation	Minimum	Maximum
Loblolly pine				
Age (years)	13.8	6.5	2.0	37.0
Dominant height (ft)	44.4	19.0	5.0	95.0
Site index (25-year, ft)	70.4	10.2	30.0	108.0
Trees per acre	497.2	146.4	83.0	1,002.0
Basal area (ft^2) per acre	92.3	52.4	0.5	220.7
Quadratic mean diameter (in.)	5.7	2.3	0.4	13.0
Diameter at breast height (in.)	5.5	2.7	0.1	19.4
Total tree height (ft)	37.8	18.0	5.0	101.0
Slash pine				
Age (years)	12.6	6.1	2.0	35.0
Dominant height (ft)	40.6	18.7	3.0	91.0
Site index (25-year, ft)	80.2	14.1	26.0	138.0
Trees per acre	446.3	181.4	57.0	1,002.0
Basal area (ft^2) per acre	68.6	44.0	0.5	184.6
Quadratic mean diameter (in.)	5.3	2.3	0.5	11.7
Diameter at breast height (in.)	5.1	2.6	0.1	18.3
Total tree height (ft)	34.6	17.7	2.0	97.0

The loblolly and slash pine plot data sets were randomly divided in half to create separate model development and model validation data sets. All possible projection lengths (from 3 to 24 years) in the model validation data sets were used to examine the effects of projection length on the predictions. For example, if a plot was measured at 10, 13, 16, 19, 22, 25, 28, and 31 years, then there will be seven 3-year projection periods, six 6-year periods, five 9-year periods, four 12-year periods, three 15-year periods, two 18-year periods, and one 21-year period. After validation was completed, all observations were pooled to estimate the final regression parameters.

Statistical Methods

This study examined two methodologies from Cao and Strub (2008) to predict annual diameter growth and survival of loblolly and slash pine trees in East Texas: (1) The predicting attributes method, which uses a whole-stand model to predict intermediate values of stand-level variables on an annual basis within the projection period, and (2) the constant rate method, which uses initial values to represent the traditional approach that assumes a constant rate of growth. Although the constant rate method is an oversimplification of reality, we included it in this study to consider the traditional approach. This enabled us to evaluate four combinations of the two methodologies for the East Texas data: (1) predicting attributes for both tree survival and diameter growth, (2) predicting attributes for tree survival and constant rate for tree diameter growth, (3) constant rate for tree survival and predicting attributes for tree diameter growth, and (4) constant rate for both tree survival and diameter growth. The possibility exists that a combination of the two methodologies will work best for the East Texas data.

For the predicting attributes method, we used a whole-stand growth model developed for East Texas (Allen et al. 2010) to predict stand-level variables required in the individual tree equations, which consists of a tree survival equation and a diameter growth equation. In practice, any whole-stand growth model can be used, but the method of Allen et al. (2010) was developed for East Texas pine plantations, so it should work better than other models for this

study. The probability of an individual tree surviving to the next growing season was predicted with the equation,

$$p_{i,t+1} = \left[1 + e^{(\beta_1 + \beta_2 A_t + \beta_3 \hat{H}d_t + \beta_4 \hat{B}_t + \beta_5 (\hat{d}_{i,t}/\hat{D}q_t) + \beta_6 \text{Rust}_{i,t})} \right]^{-1} + \varepsilon_i, \quad (1)$$

and the annual diameter growth of an individual tree was predicted with the equation,

$$d_{i,t+1} = d_{i,t} + e^{(\beta_7 + \beta_8 A_t^{-1} + \beta_9 \hat{H}d_t + \beta_{10} \hat{B}_t + \beta_{11} \ln(\hat{d}_{i,t}))} + \varepsilon_i, \quad (2)$$

where $p_{i,t+1}$ = probability that the ith tree survives at time $t + 1$, given that it was alive at time t; A_t = plantation age (years) at time t; $\hat{H}d_t$ = predicted stand dominant height (feet) at time t; $\hat{B}_t$ = predicted stand basal area (ft^2/ac) at time t; $\hat{d}_{i,t}$ = predicted diameter breast height (dbh, inches) of the ith tree at time t; $d_{i,t+1}$ = dbh of the ith tree at time $t + 1$; $\hat{D}q_t$ = predicted stand quadratic mean diameter (inches) at time t; $\text{Rust}_{i,t}$ = presence (1) or absence (0) of stem fusiform rust for the ith tree at time t; β_k = parameters to be estimated ($k = 1$ to 11); and ε_i = random error.

For the constant rate method, Equations 1 and 2 were modified to use only the dominant height, basal area per acre, quadratic mean diameter, and individual tree diameter observed at the beginning of the projection period, rather than using intermediate values predicted annually by the whole-stand model for these independent variables:

$$p_i = \left[1 + e^{(\beta_{12} + \beta_{13} A_t + \beta_{14} Hd_t + \beta_{15} B_t + \beta_{16} (d_{i,t}/Dq_t) + \beta_{17} \text{Rust}_{i,t})} \right]^{-q} + \varepsilon_i, \quad (3)$$

$$\Delta d = e^{(\beta_{18} + \beta_{19} A_t^{-1} + \beta_{20} Hd_t + \beta_{21} B_t + \beta_{22} \ln(d_{i,t}))} + \varepsilon_i, \quad (4)$$

where p_i is probability that the ith tree survives the growing period; q is the length of the growth period; $\Delta d = (d_{i,t+q} - d_{i,t})/q$, β_k = parameters to be estimated ($k = 12$ to 22); and all other variables are as defined above.

This study adapted the maximum likelihood estimation technique described in Cao and Strub (2008) to simultaneously estimate parameters for Equations 1 and 2. Cao and Strub (2008) also present maximum likelihood methods to estimate parameters for Equations 3 and 4 separately. For all four equations, the maximum likelihood parameter estimates were obtained with the NLIN procedure of SAS (SAS Institute 2004). Like Cao and Strub (2008), autocorrelation was not considered in the maximum likelihood methods used to estimate the parameters for Equations 1 and 2. Autocorrelation is not as important for Equations 3 and 4 because only the initial measurements were used.

Model Validation

The systems of Equations 1 and 2 for predicting attributes and Equations 3 and 4 for constant rate were validated using the validation data sets. Mean difference (MD) was used to evaluate bias in the tree survival and tree diameter growth predictions, whereas mean absolute difference (MAD) was used to evaluate precision in tree survival and tree diameter growth predictions. Standard error of the estimate (SEE) was also included to evaluate precision. Low absolute values for MD, MAD, and SEE represented the best model. MD, MAD, and SEE were defined as

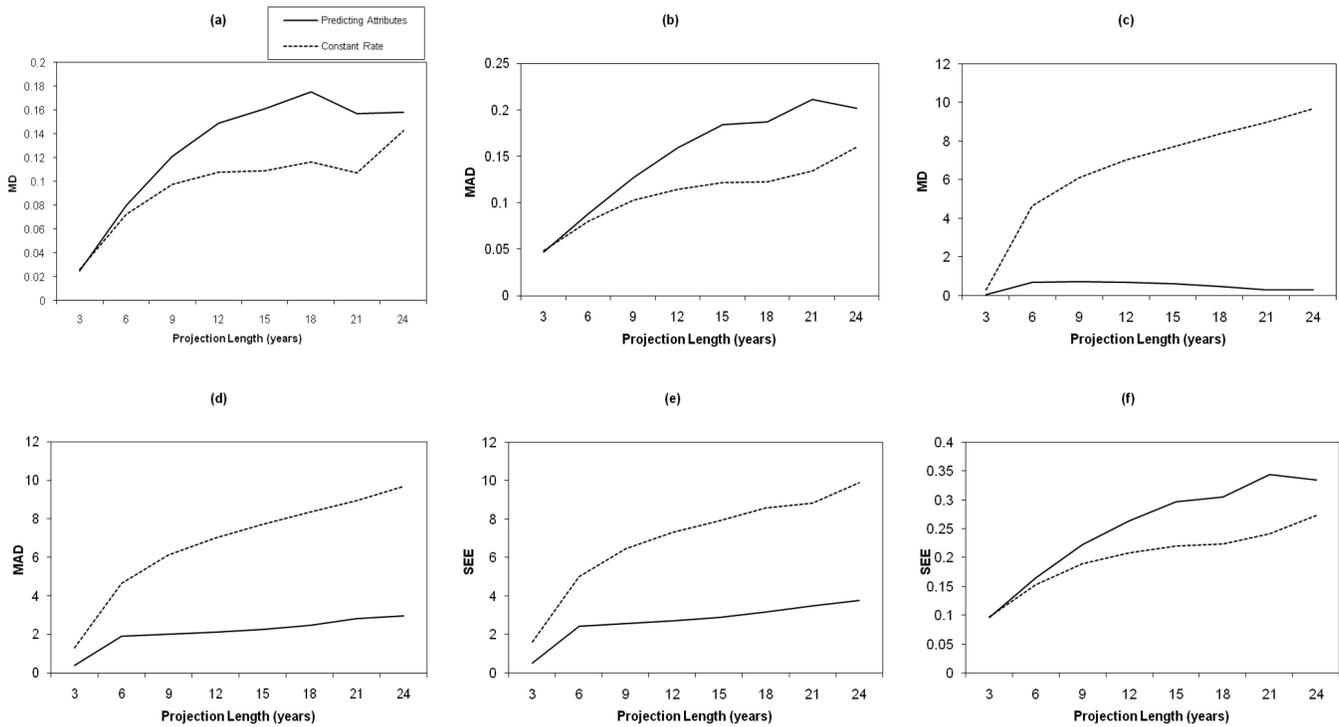$$\text{MD} = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)/n,$$

Figure 1. Loblolly pine evaluation results for the two methods used in this study (predicting attributes and constant rate). a, Mean difference (MD) for tree survival versus projection length; b, mean absolute difference (MAD) for tree survival versus projection length; c, MD for tree diameter versus projection length; d, MAD for tree diameter versus projection length; e, standard error of the estimate (SEE) for tree diameter versus projection length; f, SEE for tree survival versus projection length.

$$MAD = \sum_{i=1}^{n} |Y_i - \hat{Y}_i|/n,$$

$$SEE = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n - k}},$$

where Y_i = observed tree survival or tree diameter value for observation i; $\hat{Y}_i$ = predicted tree survival or tree diameter value for observation i; n = number of observations in the validation data sets; and k = number of estimated parameters in the equation (k = 6 for Equations 1 and 3, whereas k = 5 for Equations 2 and 4).

Results and Discussion

For loblolly pine, tree survival bias (MD) increased similarly for both methods as projection length increased. MD was similar at projection lengths of 3 and 6 years, as well as 24 years, but the predicting attributes method (PA) increased to a higher level of bias than the constant rate method (CR) at all other projection lengths (Figure 1a). Tree survival precision decreased (i.e., MAD and SEE increased) at a greater rate for PA versus CR across all projection lengths (Figure 1b and 1f). Tree diameter bias (MD) was lower and precision was higher (i.e., MAD and SEE lower) for PA versus CR (Figure 1c to 1e). Furthermore, bias and precision for tree diameter estimated with PA were both relatively constant across the range of projection lengths compared with CR (Figure 1c to 1e). Tree diameter was overpredicted by less than 1 in. at all projection lengths (Figure 1c). For precision, MAD was a minimum of 0.5 in. for the 3-year projection length; it then increased to about 2 in. for the 6-year projection length and remained relatively constant through the 24-year projection length (Figure 1d).

The validation results for slash pine were similar to those for loblolly pine. Tree survival bias increased for both methods through the 21-year projection length and then dropped at 24 years (Figure 2a). MD was similar for both methods at projection lengths of 3 and 6 years, but PA began to diverge from CR at the 9-year projection length (Figure 2a). Tree survival precision decreased at a greater rate for PA versus CR across all projection lengths (Figure 2b and 2f). Tree diameter bias was lower and precision was higher for PA versus CR (Figure 2c to 2e). Like loblolly pine, slash pine bias and precision for tree diameter estimated with PA were both relatively constant across the range of projection lengths compared with CR (Figure 2c to 2e). Tree diameter was overpredicted by less than 1 in. at all projection lengths (Figure 2c), whereas precision (MAD) was a minimum of 0.5 in. for the 3-year projection length; it then increased to just under 2 in. for the 6-year projection length and remained relatively constant through 24 years (Figure 2d).

To summarize the validation results, PA and CR produced similar predictions for the 3-year projection length. For longer projection lengths, the most notable differences can be seen in the tree diameter predictions. For tree diameter bias, CR differed from PA by 0.2 in. for loblolly and slash pine (Figures 1c and 2c, respectively) at the 3-year projection length. By the 24-year projection length, the difference increased to 9.4 and 9.9 in. for loblolly and slash pine, respectively. Precision for tree diameter followed a similar pattern. At the 24-year projection length, the percentage differences reached 6.7 and 8.2 in. for loblolly and slash pine, respectively (Figures 1d and 2d). Clearly, the predicting attributes method should be preferred over the constant rate method for annual predictions of tree diameter. However, for tree survival predictions, the constant rate method was more precise than the predicting attributes method for both loblolly and slash pine (Figures 1b, 1f, 2b, and 2f), which
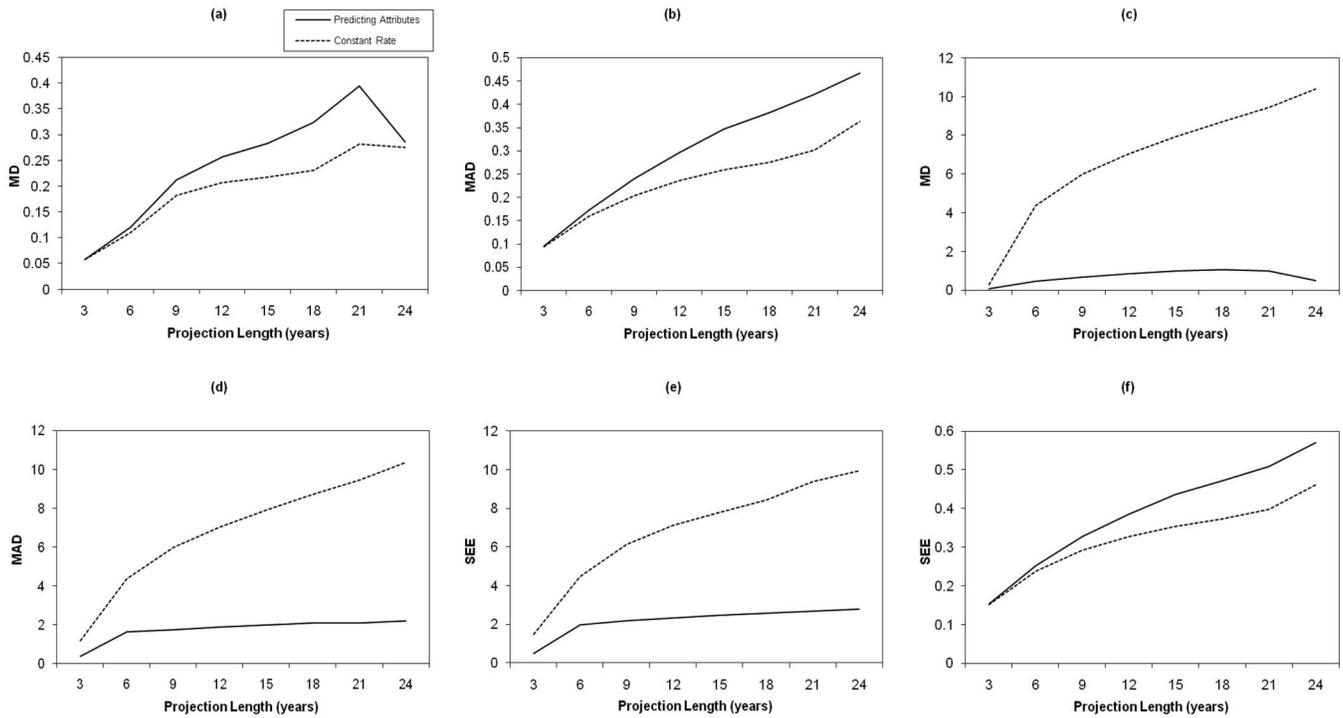
Figure 2. Slash pine evaluation results for the two methods used in this study (predicting attributes and constant rate). a, Mean difference (MD) for tree survival versus projection length; b, mean absolute difference (MAD) for tree survival versus projection length; c, MD for tree diameter versus projection length; d, MAD for tree diameter versus projection length; e, standard error of the estimate (SEE) for tree diameter versus projection length; f, SEE for tree survival versus projection length.

supports the assumption of constant annual mortality in the pine plantations represented in this study. On the basis of the validation results, we recommend the predicting attributes method for annual diameter growth predictions and the constant rate method for annual tree survival predictions in East Texas loblolly and slash pine plantations.

After validation was completed, the data were pooled to produce the final parameter estimates of Equations 2 and 3 for loblolly and slash pine (Table 2). For loblolly pine, the parameter estimates for all independent variables in Equations 2 and 3 were significantly different from zero ($P < 0.001$ for all β_k, except $P = 0.03$ for β_{13}) (Table 2). For Equation 2, the positive sign for β_8 indicates that diameter growth slows as the tree gets older. This may seem counterintuitive, but recall that the variable is the reciprocal of age, so the sign for β_8 is positive rather than negative. The negative sign for β_9 indicates that diameter growth slows as dominant height increases. This may also seem counterintuitive, but it is related to how diameter growth slows as the trees get older as much as it is related to tree height and site quality. In a plantation, tree height is relatively uniform, because overtopped trees die from competition, which occurs relatively early in the life of the plantation (i.e., the stem exclusion stage; Oliver and Larson 1990). Thus, dominant height increases as the plantation gets older, which causes diameter growth to slow. This process occurs at a faster rate for higher quality sites than for lower quality sites. The negative sign for β_{10} indicates that diameter growth slows as the stand density (basal area in ft²/ac) increases. The positive sign for β_{10} indicates that diameter growth is higher for larger trees versus smaller trees. Larger trees typically have larger crowns and root systems, so they are able to secure more site resources than smaller trees. For Equation 3, the expression containing the exponential function inside the brackets is raised to a nega-

Table 2. Final parameter estimates and fit statistics of East Texas loblolly and slash pine plantation predictive equations for individual tree annual diameter growth (Equation 2) and individual tree annual survival (Equation 3) based on the pooled data.

Species	Equation	Parameter	Parameter estimate	Standard error	P value
Loblolly pine	2	β_7	−0.6174	0.02394	<0.001
		β_8	2.9993	0.07640	<0.001
		β_9	−0.02444	0.000828	<0.001
		β_{10}	−0.0064	0.000249	<0.001
		β_{11}	0.3988	0.007354	<0.001
	3	β_{12}	−1.8421	0.0349	<0.001
		β_{13}	0.00674	0.00308	0.03
		β_{14}	0.0255	0.00164	<0.001
		β_{15}	0.00858	0.000435	<0.001
		β_{16}	−5.7783	0.0436	<0.001
		β_{17}	0.9549	0.0275	<0.001
Slash pine	2	β_7	−0.7804	0.03077	<0.001
		β_8	2.4932	0.08608	<0.001
		β_9	−0.02444	0.000878	<0.001
		β_{10}	−0.00456	0.00313	<0.001
		β_{11}	0.3975	0.01476	<0.001
	3	β_{12}	−2.1525	0.0620	<0.001
		β_{13}	0.0153	0.00768	0.04
		β_{14}	0.0287	0.00305	<0.001
		β_{15}	−0.00603	0.000623	<0.001
		β_{16}	−3.3500	0.0647	<0.001
		β_{17}	1.2745	0.0325	<0.001

tive power (i.e., q is negative), so negative signs for the parameter estimates result in an increase of survival probability, whereas positive signs result in a decrease of survival probability. The positive sign for β_{13} indicates that the probability of survival decreases as the trees get older. The positive sign for β_{14} indicates that the probability of survival decreases as dominant height increases. This may seem

counterintuitive, but the reason for this decrease is similar to the decrease of diameter growth as dominant height increases. Taller dominant height is related to older plantations, and survival probability decreases as the trees get older. As with diameter growth, this process occurs at a faster rate for higher versus lower quality sites. The positive sign for β_{15} indicates that the probability of survival decreases as stand density (basal area in ft^2/ac) increases. The negative sign for β_{16} indicates that the probability of survival increases as individual tree diameter approaches the average tree diameter of the plantation (i.e., as the ratio d/Dq approaches or exceeds 1). A larger tree has a greater probability of survival because it can secure more site resources than smaller trees. The positive sign for β_{17} indicates that the probability of survival decreases if a tree stem is infected by fusiform rust.

For slash pine, the final parameter estimates for all independent variables in Equations 2 and 3 were significantly different from zero ($P < 0.001$ for all β_k, except $P = 0.04$ for β_{13}) (Table 2). The only difference in signs from the loblolly pine parameter estimates was found with β_{15}. The negative sign for β_{15} indicates that the probability of survival increases as stand density increases. This result is opposite that of loblolly pine and is difficult to explain. For this data set, the average basal area per acre for slash pine is 68 ft^2/ac versus 92 ft^2/ac for loblolly pine (Table 1). Slash pine plantations typically have fewer trees per acre because of higher stem fusiform rust infection, which results in greater mortality rates versus loblolly pine plantations (Coble and Lee 2004). Perhaps the slash pine plantations used in this study do not reach high enough basal areas to decrease the probability of survival simply because so much mortality has already occurred in these plantations, even though the basal areas can get high enough to slow diameter growth.

Conclusion

We recommend the predicting attributes method for annual growth predictions of tree diameter (Equation 2) and the constant rate method for annual growth predictions of tree survival (Equation 3) in East Texas loblolly and slash pine plantations. For the plantation data used in this study, most mortality occurred as a result of competition and, for slash pine, fusiform rust, which resulted in a uniform reduction in trees per acre over time. Diameter growth, however, was nonlinear rather than uniform over time. Comparison with other studies of these species in this region, such as that of Cao and Strub (2008), was problematic because different variables and equations were used in the different studies. The individual-tree and whole-stand estimates in this study are not compatible, which means that the individual-tree estimates when summed on a per-acre basis do not necessarily equal the whole-stand estimates. If users wish to reconcile the potential differences between the two levels of resolution, they can (1) assume that the whole-stand estimates of basal area per acre and survival are more reliable and use the disaggregation method to update the individual trees (Ritchie and Hann 1997, Qin and Cao 2006) or (2) compute the weighted average of individual-tree and whole-stand estimates (Yue et al. 2008). Any whole-stand growth model can be used with these individual-tree models. We recommend Allen et al. (2010) because their model is best suited for East Texas loblolly and slash pine plantations.

A Numerical Example

This example shows how to use Equations 2 and 3 to grow an individual loblolly pine tree from the $A_1 = 14$ years to future $A_2 =$

Table 3. Numerical example for an individual loblolly pine tree projection from age 14 to 25 years, where Hd = dominant height (feet), N = trees per acre, B = basal area (ft^2/ac), Dq = quadratic mean diameter (in.), d = diameter breast height (in.), and p = probability of survival. Diameter and p are calculated from Equations 2 and 3 with coefficient values from Table 2. Hd, N, and B are calculated from equations in Allen et al. (2010).

Age	Hd	N	B	Dq	d	p
14	51.0	366.0	94.0	6.9	5.7	1.000
15	53.9	360.1	100.7	7.2	5.9	0.989
16	56.7	353.8	106.7	7.4	6.1	0.986
17	59.3	347.3	112.0	7.7	6.3	0.984
18	61.7	340.5	116.6	7.9	6.5	0.982
19	64.0	333.5	120.6	8.1	6.8	0.980
20	66.2	326.3	124.0	8.3	7.0	0.979
21	68.2	318.8	126.8	8.5	7.2	0.978
22	70.1	311.2	129.1	8.7	7.4	0.977
23	71.9	303.5	131.0	8.9	7.6	0.976
24	73.5	295.5	132.3	9.1	7.8	0.975
25	75.1	287.5	133.3	9.2	8.0	0.975

25 years (typical rotation age in East Texas). At the current age = 14 years, the following information is available from a standard inventory: $Hd_1 = 51$ ft, site index (25 year index age) = 72 ft, trees per acre$_1$ = N_1 = 366, B_1 = 94 ft^2/ac, Dq_1 = 6.9 in., d_1 = 5.7 in., and the tree has no stem fusiform rust. The values for this projection can be found in Table 3. The steps below illustrate how to calculate the values for age = 15 years. Once a year is completed, the values are then used to make the next year's calculations.

1. Calculate Hd_2 from Equation 2 in Allen et al. (2010):

$$Hd_2 = 51\left[\frac{1 - e^{-0.073835*15}}{1 - e^{-0.073835*14}}\right]^{1.445436} = 53.9 \text{ feet.}$$

2. Calculate N_2 from Equation 3 in Allen et al. (2010):

$$N_2 = 366 * e^{-0.00000797*72*(15^{1.994295} - 14^{1.994295})} = 360.1 \text{ tpa.}$$

3. Calculate B_2 from Equation 1 in Allen et al. (2010) using projected values of Hd_2 and N_2:

$$\ln B_2 = -3.11553 + \frac{14}{15}\Bigg(\ln(94) + 3.11553$$

$$- 0.574834 * \ln(366) - 1.081493 * \ln(51)$$

$$- 0.849356 * \frac{\ln(366)}{14} - 5.384194 * \frac{\ln(51)}{14}\Bigg)$$

$$+ 0.574834 * \ln(360.1) + 1.081493 * \ln(53.9)$$

$$+ 0.849356 * \frac{\ln(360.1)}{15} + 5.384194 * \frac{\ln(53.9)}{15}$$

$$= 4.61180.$$

Thus, $B_2 = e^{4.61180} = 100.66$ or 100.7 ft^2/ac.

4. Calculate Dq_2:

$$Dq_2 = \sqrt{\frac{B_2}{0.00545415 * N_2}} = \sqrt{\frac{100.7}{0.00545415 * 360.1}}$$

$$= 7.2 \text{ inches.}$$

5. Calculate the d age $= 15$ years using Equation 2 and the parameter estimates from Table 2:

$$d_2 = 5.7 + e^{(-0.6174+2.9993/14-0.02444*51-0.0064*94+0.3988*\ln(12.5))}$$

$$= 5.9 \text{ inches.}$$

6. Calculate the probability of survival for age $= 15$ years using Equation 3 and the parameter estimates from Table 2:

$$p_2 = \left[1 + e^{(-1.8421+0.00674*14+0.0255*51+0.00858*94-5.7783*(5.7/6.9)+0.9549*0)}\right]^{-1}$$

$$= 0.989.$$

The value of 0.989 can be applied to the expansion factor of each individual tree, thereby reducing the stand density (i.e., trees per acre) as the plantation age increases. For example, if the plot size is 0.1 ac, each individual tree represents 1/0.1 or 10 trees per acre at the beginning of the growth period and $10 \times (0.989)$ or 9.89 trees per acre at the end of the growth period.

7. The next step is to calculate steps 1 to 6 for age $= 16$, using the values calculated for age $= 15$ as starting values in steps 1 to 6. This process is repeated for subsequent years. Each year's values are presented in Table 3.

Literature Cited

ALLEN, M.G. II, D.W. COBLE, I. HUNG, AND J. YEISER. 2010. *A whole-stand growth and yield model for unmanaged east Texas loblolly and slash pine plantations.* East Texas Pine Plantation Research Project Report No. 67. Arthur Temple College of Forestry and Agriculture, Stephen F. Austin State University, Nacogdoches, TX. 12 p.

BURKHART, H.E., K.D. FARRAR, R.L. AMATEIS, AND R.F. DANIELS. 1987. *Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas.* Pub. No. FWS-l-87. School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, VA. 47 p.

CAO, Q.V. 2000. Prediction of annual diameter growth and survival for individual trees from periodic measurements. *For. Sci.* 46(1):127–131.

CAO, Q.V. 2002. Annual tree growth predictions based on periodic measurements. P. 7–13 in *Proc. of the IUFRO symposium on statistics and information technology in forestry.* IUFRO, Blacksburg, VA.

CAO, Q.V. 2004. Annual tree growth predictions from periodic measurements. P. 212–215 in *Proc. of the 12th biennial southern silvicultural research conference, 2004,* K.F. Connor (ed.). US For. Serv. Gen. Tech. Rep. SRS-71.

CAO, Q.V., S. LI, AND M.E. MCDILL. 2002. Developing a system of annual tree growth equations for the loblolly pine-shortleaf pine type in Louisiana. *Can. J. For. Res.* 32(11):2051–2059.

CAO, Q.V., AND M. STRUB. 2008. Evaluation of four methods to estimate parameters of an annual tree survival and diameter growth model. *For. Sci.* 54(6):617–624.

COBLE, D.W., AND Y.J. LEE. 2004. Fusiform rust trends in east Texas: 1969 to 2002. P. 153–157 in *Proc. of the 12th biennial southern silvicultural research conference, 2004,* K.F. Connor (ed.). US For. Serv. Gen. Tech. Rep. SRS-71.

HILT, D.E. 1985. *OAKSIM: An individual-tree growth and yield simulator for managed, even-aged, upland oak stands.* US For. Serv. Res. Pap. NE-562. 21 p.

HILT, D.E., AND R.M. TECK. 1989. NE-TWIGS: An individual-tree growth and yield projection system for the northeastern United States. *Compiler* 7(2):10–16.

LARSEN, D.R. AND D.W. HANN. 1985. *Equations for predicting diameter and squared diameter inside bark at breast height for six major conifers of southwest Oregon.* Res. Note 77. Forest Research Laboratory, Oregon State Univ., Corvallis, OR. 4 p.

LENHART, J.D., E.V. HUNT JR., AND J.A. BLACKARD. 1985. Establishment of permanent growth and yield plots in loblolly and slash pine plantations in east Texas. P. 436–437 in *Proc. of the 3rd biennial southern silvicultural research conference, 1985,* E. Shoulders (ed.). US For. Serv. Gen. Tech. Rep. SO-54.

MARQUIS, D.A., AND R.L. ERNST. 1992. *User's guide to SILVAH.* US For. Serv. Gen. Tech. Rep. NE-162. 124 p.

MCDILL, M.E., AND R.L. AMATEIS. 1993. Fitting discrete-time dynamic models having any time interval. *For. Sci.* 39(3):499–519.

MITCHELL, K.J. 1975. *Dynamics and simulated yield of Douglas-fir.* For. Sci. Monograph No. 17. 39 p.

NORD-LARSEN, T. 2006. Modeling individual-tree growth from data with highly irregular measurement intervals. *For. Sci.* 52(2):198–208.

OLIVER, C.D., AND B.C. LARSON. 1990. *Forest stand dynamics.* McGraw-Hill, New York, NY. 467 p.

QIN, J., AND Q.V. CAO. 2006. Using disaggregation to link individual-tree and whole-stand growth models. *Can. J. For. Res.* 36(4):953–960.

RITCHIE, M.W., AND D.W. HANN. 1997. Evaluation of individual-tree and disaggregative prediction methods for Douglas-fir stands in western Oregon. *Can. J. For. Res.* 27(2):207–216.

SAS INSTITUTE. 2004. *SAS/STAT 9.1 user's guide.* SAS Institute, Inc., Cary, NC. 5121 p.

STAGE, A.R. 1973. *Prognosis model for stand development.* US For. Serv. Res. Pap. INT-137. 32 p.

WENSEL, L.C., AND G.S. BIGING. 1987. The CACTOS system for individual-tree growth simulation in the mixed-conifer forests of California. P. 175–183 in *Proc. of IUFRO forest growth modelling and prediction conference.* Ek, A., and T. Burk (eds.). US Forest Service Gen. Tech. Rep. NC-120. Minneapolis, MN.

YUE, C, U. KOHNLE, AND S. HEIN. 2008. Combining tree- and stand-level models: A new approach to growth prediction. *For. Sci.* 54(5):553–566.