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A New Site Index Model for Intensively Managed Loblolly Pine (Pinus taeda) Plantations in the West Gulf Coastal Plain

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A New Site Index Model for Intensively Managed Loblolly Pine (*Pinus taeda*) Plantations in the West Gulf Coastal Plain

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Site index (SI) estimation for loblolly pine (*Pinus taeda* L.) plantations is important for the successful management of this important commercial tree species in the West Gulf Coastal Plain of the United States. This study evaluated various SI models for intensively managed loblolly plantations in the West Gulf Coastal Plain using data collected from permanent plots installed in intensively managed loblolly pine plantations across east Texas and western Louisiana. Six commonly used SI models (Cieszewski GADA model, both Chapman-Richards ADA and GADA models, both Schumacher ADA and GADA models, and McDill-Amateis GADA model) were fit to the data and compared. The Chapman-Richards GADA model and the McDill-Amateis GADA model were similar and best in their fit statistics. These two models were further compared to the existing models (Diéguez-Aranda et al. 2006 (DA2006), Coble and Lee 2010 (CL2010)) commonly used in the region. Both the Chapman-Richards GADA and the McDill-Amateis GADA models consistently predicted greater heights up to age 25 than the models of DA2006 and CL2010, with larger height differences for the higher quality sites, but predicted shorter heights thereafter. Ultimately, the McDill-Amateis GADA model was chosen as the best model for its consistency in predicting reasonable heights extrapolated beyond the range of the data. Foresters can use this model to make more informed silvicultural prescriptions for intensively managed loblolly pine plantations in the West Gulf Coastal Plain.

Keywords: dominant height, growth and yield, nonlinear models, generalized algebraic 31 difference approach, base age invariant equation

Loblolly pine (*Pinus taeda* L.) is the most extensively planted commercial pine species in the southern US, including the West Gulf Coastal Plain (WGCP) region. In Louisiana, the growing stock of loblolly pine forests is nearly 7 billion cubic feet. In east Texas, forestland occupies about 12.1 million acres, of which 2.9 million acres (24%) are classified as pine plantations, with most being composed of loblolly pine (Miles 2013). Due to its economic importance and large reforestation area, developing optimal forest management plans is crucial to the success of these plantations. Site index, an indirect measure of site quality, is an essential tool for developing forest management plans.

Site index (SI) models relate tree age, height, and site quality in even-aged, single-species stands (Carmean 1978). Numerous mathematic equations have been used to develop SI models, with each having specific biological explanations (Weiskittel et al. 2011, Burkhart and Tomé 2012). Efforts have been made to identify the optimal equations for developing SI models, which vary depending on region, species and other factors (Palahí et al. 2004, Diéguez-Aranda et al. 2005, 2006). Differential approaches such as the Algebraic Difference Approach (ADA) (Bailey and Clutter 1974) and the Generalized Algebraic Difference Approach (GADA) (Cieszewski and Bailey 2000) have been introduced to achieve desirable properties of SI models such as base-age invariance and polymorphism. ADA assigns one parameter in the base model, sometimes called the "Guide Curve", as a site-specific (local) parameter with the other parameters assigned as common (global) parameters. ADA site index models are typically anamorphic with a single asymptote. GADA assigns more than one parameter in the base model as local parameters, with the remaining parameters assigned as global parameters. GADA site index models can be polymorphic with multiple asymptotes, which is the main advantage over ADA.

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Due to its wide application in forest management, extensive studies have been carried out in developing SI models for loblolly pine forests in the south of the US. The development of SI models generally trails slightly behind advances in pine silviculture (Fox et al. 2007). Between 1930 and 1950, managing naturally regenerated, second growth loblolly pine forests was the focus. Corresponding SI models were developed (US Forest Service 1929). With more loblolly pine plantations being established on old-field and then on cut-over sites, the subsequent work on SI model development focused on old-field pine plantations (Bennett 1963, Coile and Schumacher 1964, Newberry and Pienaar 1978) and then on cut-over sites (Amateis and Burkhart 1985, Sharma et al. 2002, Diéguez-Aranda et al. 2006). Overall, most loblolly pine plantations established during 1980s or before were established using low-intensity establishment practices such as mechanical piling, burning of logging slash and using unimproved (woods-run) seed lots (referred to as extensively managed plantations), while those established thereafter were established using intensive management practices such as planting genetically-improved seedlings and applying thinning, herbicidal competition control, fertilizer amendments, and other treatments (referred to as intensively managed plantations). Site index often is very sensitive to forest management practices. The intensive silvicultural practices could double yields of loblolly pine plantations and decrease rotation lengths from 40–50 years to 20–30 years in some cases (Jokela et al. 2004, Eisenbies 2006, Fox et al. 2007). Bedding, for example, an intensive silvicultural practice that benefits drainage, reduces soil compaction, and provides some level of competition control, has been well implemented (Eisenbies 2006). Similarly, planting genetically-improved loblolly pine seed lots have significantly enhanced plantation productivity (Li et al. 1999, McKeand et al. 2003). Both bedding and genetics may alter SI. Most currently available SI models were developed for managing extensively managed loblolly pine plantations (Amateis and Burkhart 1985, Sharma et al. 2002, Diéguez-Aranda et al. 2006). SI models for intensively managed plantations have been rarely reported although effects of specific silvicultural practices on SI models have been reported and incorporated into available SI models (Hynynen et al. 1998, Gyawali and Burkhart 2015).

Much research has also been carried out on loblolly pine in the WGCP region. Various model forms have been applied (Coble and Lee 2006), but their relative fitness, in particular when they pair with ADA or GADA have not been investigated. SI models for extensively managed plantations such as those on abandoned agricultural land, old-field loblolly pine plantations (Lenhart and Fields 1970, Lenhart 1971), and cutover sites (Lenhart et al. 1986, Coble and Lee 2006, 2010) have been developed. These models, without accounting for substantial change in productivity, have still been widely applied in managing the intensively managed pine plantations since there is no available SI model specifically developed for intensively managed loblolly pine plantations in the region. More recently, Priest et al. (2015) published a SI model for the region, but their model was specifically for loblolly pine plantations on reclaimed mine land. There is a high demand from landowners and forest management organizations to develop SI models for intensively managed loblolly pine plantations.

To address this demand, beginning in 2004 and as part of the East Texas Pine Plantation Research Project (ETPPRP), permanent plots were established in young cutover, intensively managed loblolly pine plantations across east Texas and western Louisiana. The repeated measurements of these plots have accumulated sufficient data for developing SI models. The purpose of this study was to develop a suitable SI model for intensively managed loblolly pine plantations growing on cutover sites in the WGCP region, specifically east Texas and western Louisiana.

Material and Methods

Materials

Starting in 2004, permanent plots were installed in intensively managed loblolly pine plantations across east Texas and into western Louisiana to best represent the growing conditions unique to the Western Gulf region (e.g., greater drought severity and duration). At each study location, one square plot measuring approximately 0.25 acres (approximately 100 foot by 100 foot) was installed. Details for plot installation can be found in Trim (2018). As of 2017, 133 plots were established; among them 125 plots were actively measured and eight were lost or harvested, although some data from these plots were available for analysis.

At each plot, both planted loblolly pine trees and selected non-planted trees with diameter at breast height (dbh) larger than four inches were permanently tagged and measured when the plot was installed and every three years thereafter for diameter at breast height (dbh) (to the nearest 0.1 inch), height (to the nearest 1.0 foot), and crown length (to the nearest 1.0 foot). The trees' crown class (dominant, co-dominant, intermediate, and overtopped) as well as the presence of fusiform rust (yes or no, on stem or branch), and any damage to the tree were also recorded (Coble and Pendergast 2013).

This study used 469 longitudinal observations from 132 unthinned plots, which are summarized in Table 1 and Figure 1. Individual tree measurements from each plot were summarized to obtain dominant height by measurement cycle. Dominant height was determined by averaging the total heights of the dominant and co-dominant trees that were free of damage that affected

Management and Policy Implications

Loblolly pine (*Pinus taeda* L.) is the most widely planted commercial timber species in West Gulf Coastal Plain (WGCP), particularly in east Texas and western Louisiana. Developing a suitable site index (SI) model, the most widely used method for evaluating site quality, is a necessary component of developing sound management plans for plantations. In east Texas and western Louisiana, foresters currently use either the region-specific SI model developed by Coble and Lee (2010) or the south-wide SI model developed by Diéguez-Aranda et al. (2006) to predict SI. Both models were developed based on data collected from extensively managed loblolly pine plantations. In the past 20 years, pine silvicultural activities have been advanced substantially with most loblolly pine plantations established since 1990 having been intensively managed, resulting in substantial increase in productivity (Fox et al. 2007). Both models could potentially bias the SI prediction for intensively managed loblolly pine plantations in the region, as they were parameterized using data from stands managed with fewer silvicultural inputs (e.g., fertilizer, competition control). This study, by using data collected from the intensively managed loblolly pine plantations across the region and utilizing different models, developed a suitable SI model, filling this need. Foresters will be able to use this model to make more accurate silvicultural prescriptions for intensively managed loblolly pine plantations in the region. The results aid our understanding in height growth and management in pine plantations in WGCP.

Table 1. Observed stand characteristics for east Texas and western Louisiana loblolly pine plantations established on cutover sites. Based on N = 469 observations made from 132 plots in the ETPPRP database.

Variables	Mean	Standard Deviation	Minimum	Maximum
Age (years)	8.1	3.6	2.0	19.0
Dominant Height (feet)	32.0	12.3	6.5	63.4
Density (trees ac^{-1})	524.2	100.0	326.7	858.1
Basal Area (ft^2 ac^{-1})	79.5	44.8	1.2	184.3
Quadratic Mean Diameter (inches)	5.3	1.9	0.5	11.5

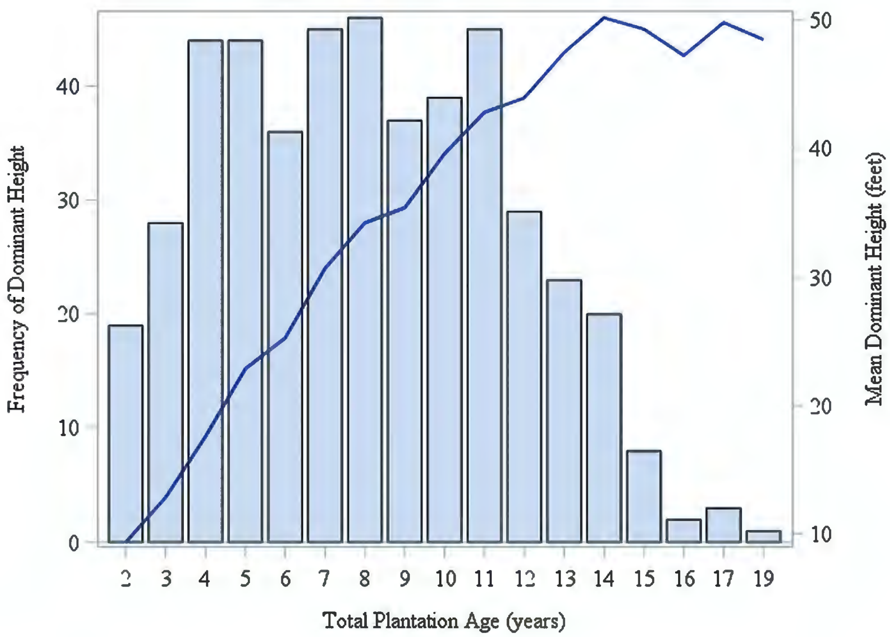

Figure 1. Frequency (bars) and mean height (line) of dominant height by plantation age of the ETPPRP data used to develop the site index models.

height growth (e.g., broken tops, dead tops, forks, fusiform rust). Plantation age (in years) was determined as the time between the current measurement date and the plantation establishment date derived from stand records.

Methods

This study fit two anamorphic base-age invariant models and four polymorphic base-age invariant models to the ETPPRP dominant height-age data (Trim 2018). ADA was used to derive the anamorphic models, and GADA was used to derive the polymorphic models. Each model is described in detail below:

Schumacher ADA Model: The model includes a logarithmic transformation on height to create a linear function with the reciprocal of age Schumacher (1939). The base form or guide curve equation of this model is:

$$H = e^{\left(\beta_0+\beta_1*A^{-1}\right)},$$

where H = total height (feet), A = total age (years), and β_0, β_1 = regression parameters to be estimated. Taking the natural logarithm of this equation gives:

$$\ln(H) = \beta_0 + \beta_1*A^{-1}.$$

To develop the ADA Schumacher site index model, first substitute the index age for age in the base model. Thus, the height at the index age is site index:

$$\ln(H_0) = \beta_0 + \beta_1*A_0^{-1},$$

where H_0 = site index (feet), A_0 = index age (years), and all other variables are defined as before. The regression parameter, β_0, is the intercept of the equation also known as the site-specific or local parameter while β_1 is the slope of the equation also known as the global parameter. Solving for β_0 gives:

$$\beta_0 = \ln(H_0) - \beta_1*A_0^{-1}.$$

Substituting β_0 into the original equation gives the Schumacher anamorphic, base-age invariant height-age model:

$$\ln(H) = \ln(H_0) + \beta_1\left(A^{-1} - A_0^{-1}\right). \tag{1}$$

Chapman-Richards ADA Model: One of the most widely used site index models today is the Chapman-Richards model (Richards 1959, Chapman 1961). The base form or guide curve equation of this model is:

$$H = \beta_1\left(1 - e^{-\beta_2 A}\right)^{\beta_3},$$

where all other variables are defined as before. The parameter, β_1, defines the asymptotic or maximum site index while the parameter, β_2, describes the rate, and the parameter, β_3, describes the shape of the curve.

To develop the ADA Chapman-Richards site index model, first substitute the index age for age in the base model. Thus, the height at the index age is site index:

$$H_0 = \beta_1 \left(1 - e^{-\beta_2 A_0}\right)^{\beta_3},$$

where all other variables are defined as before. The asymptote can be considered to vary across sites, so it can be isolated to allow site index to vary across sites while keeping the curve shape constant. Solving for β_1 gives:

$$\beta_1 = H_0 \left(1 - e^{-\beta_2 A_0}\right)^{-\beta_3}.$$

Substituting β_1 into the original equation gives the Chapman-Richards anamorphic, base-age invariant height-age model:

$$H = H_0 \left(\frac{1 - e^{-\beta_2 A}}{1 - e^{-\beta_2 A_0}} \right)^{\beta_3} \tag{2}$$

Schumacher GADA Model: As stated before, the base form or guide curve equation of Schumacher's model is:

$$H = \mathbf{e}^{\left(\delta_0 + \delta_1 * \left({}^{1}\!/\!_{\mathbf{A}}\right)\right)} = \mathbf{e}^{\left(\delta_0 + \delta_1 * \mathbf{A}^{-1}\right)}$$

where δ_0 and δ_1 are model parameters and all other variables defined as before.

To create the GADA solution of the Schumacher model, first make δ_0 and δ_1 both local parameters by replacing δ_0 with an unobserved site quality variable, X, and δ_1 with a linear function of X, $\beta_1 + \beta_2$*X:

$$H = e^{\left(X + (\beta_1 + \beta_2 * X) A^{-1}\right)}.$$

Taking the natural logarithm and solving for X gives:

$$X = \frac{\left(\ln(H) - \beta_1 A^{-1}\right)}{1 + \beta_2 A^{-1}}.$$

Substituting the initial conditions H_0 and A_0 for site index and index age, respectively, into the equation for X gives:

$$X_0 = \frac{\left(\ln(H_0) - \beta_1 A_0^{-1}\right)}{1 + \beta_2 A_0^{-1}}. \tag{3}$$

Replace X in the GADA solution with X_0 to create a polymorphic, base-age invariant formulation of the Schumacher height-age model:

$$H = e^{\left(X_0 + (\beta_1 + \beta_2 * X_0) A^{-1}\right)} \tag{4}$$

where X_0 is defined by equation (3) and all other variables are defined as before.

Chapman-Richards GADA Model: As stated before, the base form or guide curve equation of the Chapman-Richards model is:

$$H = \delta_1 \left(1 - e^{-\delta_2 A}\right)^{\delta_3},$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables are defined as before.

To create the GADA solution of the Chapman-Richards model, first make δ_1 and δ_3 both local parameters by replacing δ_1 with an exponential function of the unobserved site quality variable, X, and δ_3 with a linear inverse function of X or $\beta_3 + \beta_4$ / X. The parameter δ_2 is estimated as a global parameter, β_2. The GADA formulation is:

$$H = e^X \left(1 - e^{-\beta_2 A}\right)^{\left(\beta_3 + \beta_4 X^{-1}\right)}.$$

Taking the natural logarithm of both sides of the equation and solve for X:

$$\ln(H) = ln\left(e^X \left(1 - e^{\beta_2 A}\right)^{\left(\beta_3 + \beta_4 X^{-1}\right)}\right),$$

$$\ln(H) = ln\left(e^X\right) + \ln\left(\left(1 - e^{\beta_2 A}\right)^{\left(\beta_3 + \beta_4 X^{-1}\right)}\right),$$

$$\ln(H) = X + \left(\beta_3 + \beta_4 X^{-1}\right) * \ln\left(1 - e^{\beta_2 A}\right),$$

$$\ln(H) = X + \beta_3 \ln\left(1 - e^{\beta_2 A}\right) + \beta_4 X^{-1} \ln\left(1 - e^{\beta_2 \mathrm{A}}\right),$$

$$\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right) = X + \beta_4 X^{-1} \ln\left(1 - e^{\beta_2 A}\right),$$

$$\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right) = X^{-1}\left(X^2 + \beta_4 \ln\left(1 - e^{\beta_2 A}\right)\right),$$

$$X\left(\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right)\right) = X^2 + \beta_4 \ln\left(1 - e^{\beta_2 A}\right),$$

$$X^2 - \left(ln(H) - \beta_3 ln\left(1 - e^{\beta_2 A}\right)\right) X + \beta_4 ln\left(1 - e^{\beta_2 A}\right) = 0.$$

Solving for X requires a quadratic solution. First, let $a = 1$, $b = -\left(ln(H) - \beta_3 ln\left(1 - e^{\beta_2 A}\right)\right)$, and $c = \beta_4 ln\left(1 - e^{\beta_2 A}\right)$. Then, use the quadratic formula to find the solution:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{\left(ln(H) - \beta_3 ln\left(1 - e^{\beta_2 A}\right)\right) \pm \sqrt{\left(ln(H) - \beta_3 ln\left(1 - e^{\beta_2 A}\right)\right)^2 - 4\beta_4 ln\left(1 - e^{\beta_2 A}\right)}}{2}$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X and take the roots most likely to be positive and real:

$$X_0 = \frac{\left(ln(H_0) - \beta_3 ln\left(1 - e^{\beta_2 A_0}\right)\right) + \sqrt{\left(ln(H_0) - \beta_3 ln\left(1 - e^{\beta_2 A_0}\right)\right)^2 - 4\beta_4 ln\left(1 - e^{\beta_2 A_0}\right)}}{2} \tag{5}$$

Solve for δ_1 in the initial condition formulation of the model, and express in terms of the GADA formulation:

$$H_0 = \delta_1 \left(1 - e^{-\delta_2 A_0}\right)^{\delta_3},$$

$$\delta_1 = H_0 \left(1 - e^{\delta_2 A_0}\right)^{-\delta_3},$$

$$\delta_1 = e^X = H_0 \left(1 - e^{\delta_2 A_0}\right)^{-\left(\beta_3 + \beta_4 X_0^{-1}\right)}.$$

Then, substitute this initial condition for δ_1 into the original GADA formulation of the model to create a polymorphic, base-age invariant formulation of the Chapman-Richards height-age model:

$$H = H_0 \left(\frac{1 - e^{\beta_2 A}}{1 - e^{\beta_2 A_0}} \right)^{\left(\beta_3 + \beta_4 X_0^{-1}\right)}, \tag{6}$$

where X_0 is defined as equation (5) and all other variables are defined as before.

Cieszewski GADA Model: Cieszewski (2001, 2002, 2003) examined several GADA formulations of Hossfeld models, also known as log-logistic models. The base form of the Hossfeld equation that performed best (Cieszewski 2002) is:

$$H = \frac{\delta_1}{1 + e^{\delta_2 + \delta_3 \ln(A)}} = \frac{\delta_1}{1 + e^{\delta_2} A^{\delta_3}}.$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables defined as before.

To create the GADA solution of the Hossfeld model, first make δ_1 and δ_2 both local parameters by replacing δ_1 with a constant plus the unobserved site quality variable, X, and e^{δ_2} with β_2 / X. The parameter δ_3 is estimated as a global parameter, β_3. The GADA formulation is:

$$H = \frac{\beta_1 + X}{1 + \frac{\beta_2}{X} A^{\beta_3}} = \frac{\beta_1 + X}{1 + \beta_2 X^{-1} A^{\beta_3}}.$$

Next solve for X:

$$H = \frac{\beta_1 + X}{X^{-1}\left(X + \beta_2 A^{\beta_3}\right)},$$

$$H = \frac{X\left(\beta_1 + X\right)}{X + \beta_2 A^{\beta_3}},$$

$$H\left(X + \beta_2 A^{\beta_3}\right) = X\left(\beta_1 + X\right),$$

$$HX + H\beta_2 A^{\beta_3} = \beta_1 X + X^2,$$

$$X^2 + \beta_1 X - HX - H\beta_2 A^{\beta_3} = 0,$$

$$X^2 + \left(\beta_1 - H\right) X - H\beta_2 A^{\beta_3} = 0,$$

$$X^2 - \left(H - \beta_1\right) X - H\beta_2 A^{\beta_3} = 0.$$

Solving for X requires a quadratic solution. First, let $a = 1$, $b = -\left(H - \beta_1\right)$, and $c = -H\beta_2 A^{\beta_3}$. Then, use the quadratic formula to find the solution:

$$X = \frac{\left(H - \beta_1\right) \pm \sqrt{\left(H - \beta_1\right)^2 + 4H\beta_2 A^{\beta_3}}}{2}.$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X and take the roots most likely to be positive and real:

$$X_0 = \frac{\left(H_0 - \beta_1\right) + \sqrt{\left(H_0 - \beta_1\right)^2 + 4H_0\beta_2 A_0^{\beta_3}}}{2}. \quad (7)$$

Replace X in the GADA solution with X_0 and simplify to create a polymorphic, base-age invariant formulation of the Cieszewski-Hossfeld height-age model:

$$H = \frac{\beta_1 + X_0}{1 + \beta_2 X_0^{-1} A^{\beta_3}}, \quad (8)$$

where X_0 is defined as equation (7) and all other variables are defined as before.

McDill and Amateis GADA Model: McDill and Amateis (1992) proposed another variant of the Hossfeld model that only considers δ_2 as the local parameter in the Cieszewski (2002) GADA model. As before, the base form of the Cieszewski (2002) model is:

$$H = \frac{\delta_1}{1 + e^{\delta_2} A^{\delta_3}}.$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables defined as before.

To create the GADA solution of the McDill-Amateis model, first make δ_2 the local parameter by replacing e^{δ_2} with β_2 / X, where X is the unobserved site quality variable. The parameters δ_1 and δ_3 are estimated as global parameters, β_1 and -β_3, respectively. The GADA formulation is:

$$H = \frac{\beta_1}{1 + \beta_2 X^{-1} A^{-\beta_3}}.$$

Next solve for X:

$$H = \frac{\beta_1}{X^{-1}\left(X + \beta_2 A^{-\beta_3}\right)},$$

$$H = \frac{X\beta_1}{X + \beta_2 A^{-\beta_3}},$$

$$H\left(X + \beta_2 A^{-\beta_3}\right) = X\beta_1,$$

$$HX + H\beta_2 A^{-\beta_3} = X\beta_1,$$

$$HX - X\beta_1 = -H\beta_2 A^{-\beta_3},$$

$$X\left(H - \beta_1\right) = -H\beta_2 A^{-\beta_3},$$

$$X = \frac{-H\beta_2 A^{-\beta_3}}{H - \beta_1},$$

$$X = \frac{-H\beta_2 A^{-\beta_3}}{H\left(1 - \beta_1 H^{-1}\right)},$$

$$X = \frac{-\beta_2 A^{-\beta_3}}{1 - \beta_1 H^{-1}}.$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X:

$$X_0 = \frac{-\beta_2 A_0^{-\beta_3}}{1 - \beta_1 H_0^{-1}}.$$

Replace X in the GADA solution with X_0 and simplify to create a polymorphic, base-age invariant formulation of the McDill-Amateis height-age model:

$$H = \frac{\beta_1}{1 + \beta_2 \left(\frac{-\beta_2 A_0^{-\beta_3}}{1 - \beta_1 H_0^{-1}}\right)^{-1} A^{-\beta_3}},$$

$$H = \frac{\beta_1}{1 + \beta_2 \left(\frac{1 - \beta_1 H_0^{-1}}{-\beta_2 A_0^{-\beta_3}}\right) A^{-\beta_3}},$$

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A^{-\beta_3}}{A_0^{-\beta_3}}\right)},$$

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A}{A_0}\right)^{-\beta_3}},$$

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A_0}{A}\right)^{\beta_3}}, \quad (9)$$

where all other variables are defined as before.

Since data were repeatedly collected, they were not independent of each other. Additionally, heteroscedasticity could be a problem that inflates variances with measurements made over time. Ignoring either of these in SI model development could result in biased estimates of model coefficients and their statistical inferences. Another issue arises when using SI as an independent variable in the model because it is not usually measured. Bias could be introduced into the model depending on how SI is estimated in the model fitting procedure. Northway (1985) described an iterative procedure to generate unbiased estimates of SI to use in the development of SI models. Strub and Cieszewski (2002) improved on Northway's procedure to also address the potential bias introduced by the repeated measurements in the data used to fit ADA/GADA models. This iterative procedure required an observed growth series from which estimates of SI were calculated during the iterative nonlinear fitting process. In our study, each record in the dataset contained a single height-age pair from a plot, along with its entire growth series, which was every height-age pair from the plot measured over time. To estimate SI for each height-age pair, initial estimates of the regression parameters were first set equal to the starting values in the iterative nonlinear fitting process, and they were changed with successive iterations. Within each iteration, conditional site index estimates (CSI) were used in the equation being fit. Heights were predicted for the entire growth series for the CSIs. The squared differences (observed – predicted) in height were then calculated. The values of CSI for the current iteration that minimized the squared differences were used as final estimates to calculate new values of the regression parameters for the next iteration. This process was repeated until the least squares error for the overall regression was minimized (i.e., lowest SSE). Thus, CSI was the estimate of SI that minimized squared differences of serially correlated observations, given the current coefficient estimates. Therefore, the procedure simultaneously estimated SI for the growth series and CSI used in the model. Final CSI values were basically local estimates of the height at the index age (25 years in this study) for each growth series.

This iterative procedure recognizes that some parameters in the model are local and others are global, depending on how the ADA/GADA models were derived. Nonlinear mixed effects models have also been used to develop site models by treating the local parameters as random effects (Wang et al. 2014). We chose to use the iterative approach of Strub and Cieszewski (2002) in this study because of the improved performance over models developed with nonlinear mixed model techniques (Cieszewski and Strub 2018), and because of its success in other studies (Krumland and Eng 2005, Diéguez-Aranda et al. 2006, Coble and Lee 2010).

Residual plots were viewed to verify that this procedure minimized the effects of autocorrelation and heteroscedasticity. The Durbin-Watson statistic (DW) was also calculated to assess autocorrelation in the models. When DW = 2, autocorrelation was not detected in the model. Values between zero and 2 are indicative of positive autocorrelation, and values between 2 and 4 are indicative of negative autocorrelation. So, when DW is close to 2, autocorrelation is not considered an issue in the model. All non-linear procedures were run in PROC NLIN of SAS version 9.4.

The best models were selected based on statistical and visual analyses of the model residuals. Three fit statistics (Burkhart and Tomé 2012) were used: root mean square error (RMSE) which measures model precision, the coefficient of determination for nonlinear models (pseudo-R^2) which measures the amount of variability in the dependent variable explained by the independent variable, and Akaike's information criterion (AIC) which measures the goodness of fit of an estimated statistical model. AIC is a tool that allows the selection of the best fit model from a pool of candidate models (Akaike 1974). We also calculated the AIC differences to aid in selection of the best fit model. The formulas for the fit criteria can be expressed as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n - p}}.$$

$$pseudo - R^2 = r^2_{Y_i \hat{Y}_i},$$

$$AIC = n \ln\left(\hat{\sigma}^2\right) + 2(p + 1),$$

$$\Delta AIC = n \log\left(\hat{\sigma}^2\right) + 2\,(p + 1) - \text{minimum}(n \log\left(\hat{\sigma}^2\right) + 2(p + 1)),$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n},$$

where $r^2_{Y_i \hat{Y}_i}$ is the Pearson's correlation coefficient between the observed (Y_i) and predicted ($\hat{Y}_i$) dependent variable or dominant height in this study, n is the total number of observations or height-age pairs in this study, p is the number of parameters in the model, and $\hat{\sigma}^2$ is the estimated error variance of the model or mean square error of the model.

We used a relative ranking system proposed by Poudel and Cao (2013). Their system not only considered the ordinal rank of a fit statistic for a model, but also the exact position of a model's fit statistic relative to the other models under consideration. This relative ranking system provided an objective way to determine the best fit model. The relative rank for a fit statistic was calculated as:

$$R_i = 1 + \frac{(m - 1)\,(S_i - S_{min})}{(S_{max} - S_{min})},$$

where R_i was the relative rank of the model i (i = 1, 2, …, m), m = 6 models for this study, S_i is the fit statistic under consideration (i.e., RMSE, Pseudo-R^2, AIC), S_{min} is the minimum S_i, and S_{max} is the maximum S_i. For this study, the best model has the relative rank of 1 and the worst model has the relative rank of 6. The remaining ranks were expressed as real numbers between one and six, which depicts the order and magnitude of the fit statistics under consideration.

Ideally, each model would be evaluated based on its ability to predict responses for a set of independent data (i.e., model validation).

Since no independent data were available for this project, the data could have been split into a model fitting data set and a model validation data set. The model would be fit with the former and validated with the latter. However, Kozak and Kozak (2003) showed that this splitting technique as well as cross-validation techniques for model validation did not provide any additional information about the model beyond what ordinary fit statistics provide from the model fit with the entire data set. Therefore, data splitting or cross-validation techniques were not used in this study.

The selected best fit models in this study were further compared to two currently used SI models in the region: DA2006 (Diéguez-Aranda et al. 2006) and CL2010 (Coble and Lee 2010). DA2006 is a base-age invariant, polymorphic model for extensively managed loblolly pine growing throughout the South of the US. CL2010 is a base-age invariant, anamorphic model specifically for extensively managed loblolly pine plantations in east Texas, which was developed based on the Schnute (1981) equation. These comparisons were conducted to examine potential differences arising from factors such as different height growth rates between regions, variations in silvicultural intensity, and deployment of improved genetics. Comparisons were made at four SI values representing a common range in the region: 50, 60, 70 and 80 feet at an index age of 25 years, and were based on biological judgement and visual analysis since no independent data were available to evaluate the models across a range of site index and plantation age values.

Results

All stand variables ranged substantially across plots measured (Table 1). These plantations were young in that they were two to 19 years old with an average age of 8.1 years. The plots on average had 524 trees per acre, a dominant height of 32 feet, a basal area 79.5 $ft^2\ ac^{-1}$, and a quadratic mean diameter of 5.3 inches.

All model parameter estimates were significantly ($p < 0.05$) different from zero other than the β_2 parameter estimate for the Cieszewski GADA model (Table 2). All models resulted in high pseudo-R^2 values (> 96%), although the Cieszewski GADA model, the Chapman-Richards ADA and GADA models, as well as the McDill-Amateis GADA model were comparably the better (> 98.4%). In terms of RMSE, all the models had low values (around 1.5 feet) other than the Schumacher models, which had RMSE values of around 2.3 feet. Similar results were obtained in model ranking based on the AIC and AIC differences (ΔAIC). The Cieszewski GADA model was the best; however, both the Chapman-Richards GADA and McDill-Amateis GADA models received substantial support, with ΔAIC being less than < 2, suggesting that the levels of empirical support of both models were substantial (Burnham and Anderson 2002). The Schumacher ADA and GADA models had the poorest fit with ΔAIC being larger than 10, essentially no support of selecting these models. In terms of total relative rank, which considers the order and magnitude of pseudo-R^2, RMSE, and AIC, the Chapman-Richards GADA model was designated the best fit model, followed closely by the McDill-Amateis GADA model (Table 3).

The residuals for all the models other than the Schumacher models indicated no evidence of bias, autocorrelation, or heteroscedasticity (Figure 2). The Durbin-Watson statistics for all models except the Schumacher models were close to 2.0 (Table 2), which can be interpreted to mean that serial autocorrelation is minimal for these models. For both Schumacher models, the residuals exhibited curvilinear trends, which are indicative of bias from serial autocorrelation. The Durbin-Watson statistics for the Schumacher models were around 1.7, which can be interpreted to mean that positive autocorrelation exists in these models. This corroborates with the upward curvature observed in their residual plots (Figure 2). The iterative fitting methodology used in this study was not adequate to overcome serial

Table 2. Parameter estimates, standard errors, 95% confidence limits, and Durbin-Watson statistics for the equations.

Model	Parameter	Estimate	Standard Error	95% Confidence Limits		Durbin-Watson
Chapman-Richards ADA	β_2	1.1222	0.0342	1.0549	1.1895	1.86
	β_3	0.0737	0.00692	0.0601	0.0681	
Chapman-Richards GADA	β_2	0.0794	0.00658	0.0665	0.0924	1.94
	β_3	-1.9088	0.7730	-3.4278	-0.3898	
	β_4	13.4184	3.3953	6.7464	20.0904	
Cieszewski GADA	β_1	92.2730	19.8997	53.1688	131.4000	
	β_2	892.3000	904.3000	-884.7000	2669.4000	
	β_3	1.1844	0.0322	1.1211	1.2476	1.94
McDill-Amateis GADA	β_1	112.10000	6.5480	99.2160	125.0000	1.94
	β_3	1.1729	0.0300	1.1139	1.2319	
Schumacher ADA	β_1	-5.7647	0.0774	-5.9168	-5.6126	1.69
Schumacher GADA	β_1	-34.4077	9.0506	-52.1927	-16.6228	1.73
	β_2	6.6633	2.1058	2.5253	10.8013	

Table 3. Fit statistics, where Pseudo-R^2 = coefficient of determination, RMSE = root mean square error, AIC = Akaike Information Criterion, and ΔAIC = difference in AIC from smallest value.

Model	Pseudo-R^2	RMSE	AIC	ΔAIC	Relative Rank Pseudo-R^2	Relative Rank RMSE	Relative Rank AIC	Relative Rank Total
Chapman-Richards ADA	0.98360	1.58420	436.514	27.954	1.244	1.277	1.246	3.767
Chapman-Richards GADA	0.98446	1.54004	410.025	1.465	1.011	1.000	1.013	3.024
Cieszewski GADA	0.98450	1.61660	408.560	0	1.000	1.480	1.000	3.480
McDill-Amateis GADA	0.98445	1.54038	410.237	1.677	1.014	1.002	1.015	3.030
Schumacher ADA	0.96607	2.33763	801.483	392.923	6.000	6.000	4.459	16.459
Schumacher GADA	0.96620	2.33763	976.574	568.014	5.965	6.000	6.000	17.965

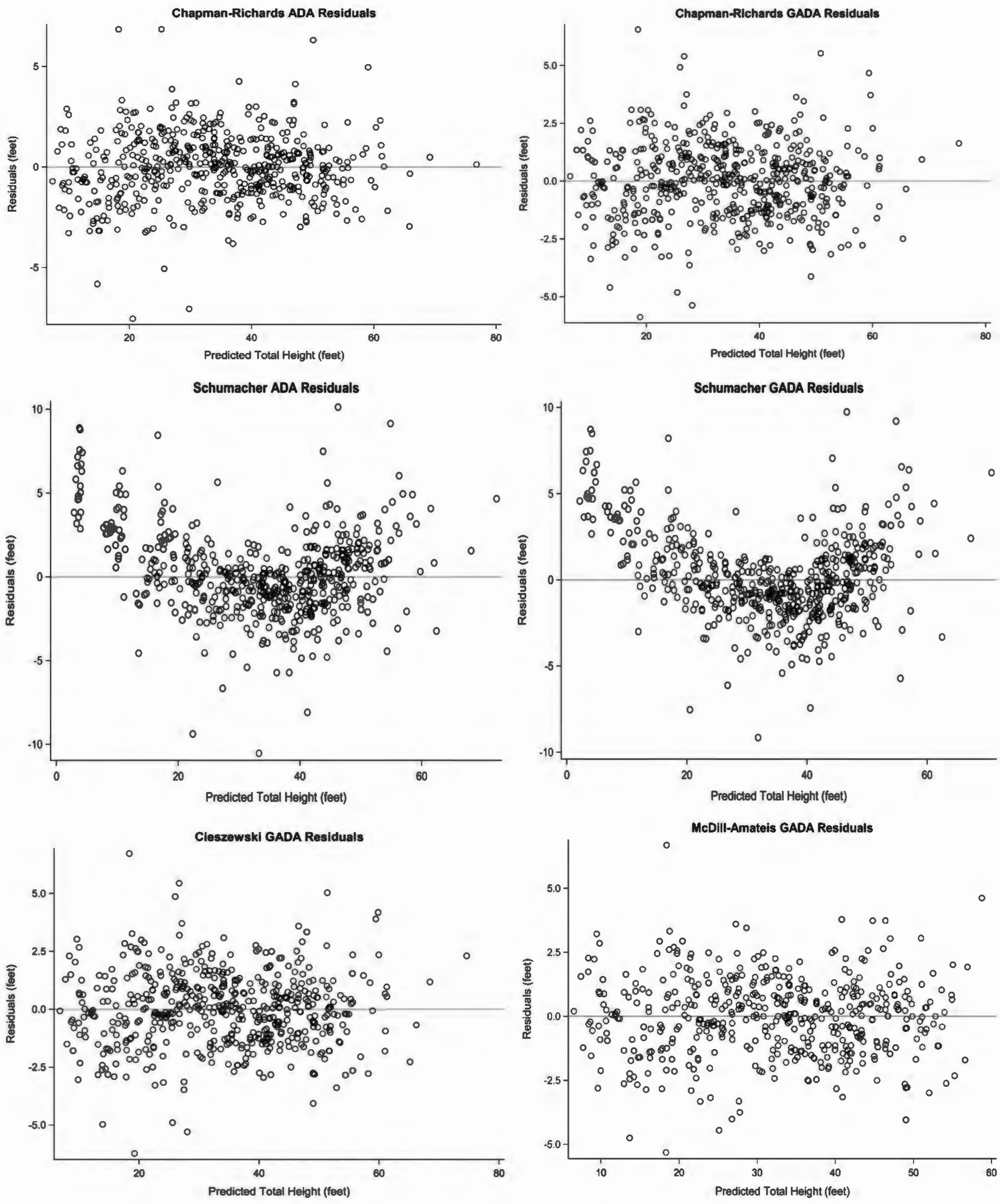

Figure 2. Plot of residuals against predicted dominant height for six site index models.

autocorrelation in either Schumacher model. A remedial step would be to incorporate an autoregressive error term, such as AR(1), in the model fitting process, which we did not attempt in this study since the other models did not exhibit problems with serial autocorrelation.

Since the Chapman-Richards GADA model and the McDill-Amateis GADA model were ranked similarly and neither exhibited problems with serial autocorrelation or heteroscedasticity, they were selected for further comparison against the DA2006 and CL2010 models. Even though maximum age for the data used in this study was only 19 years, plantation age was plotted up to 50 years to examine the extrapolative behavior of all the models (Figure 3). The DA2006 and CL2010 models predicted similar heights at ages less than 25 years, but thereafter, the former consistently estimated greater heights than the latter. Across the range

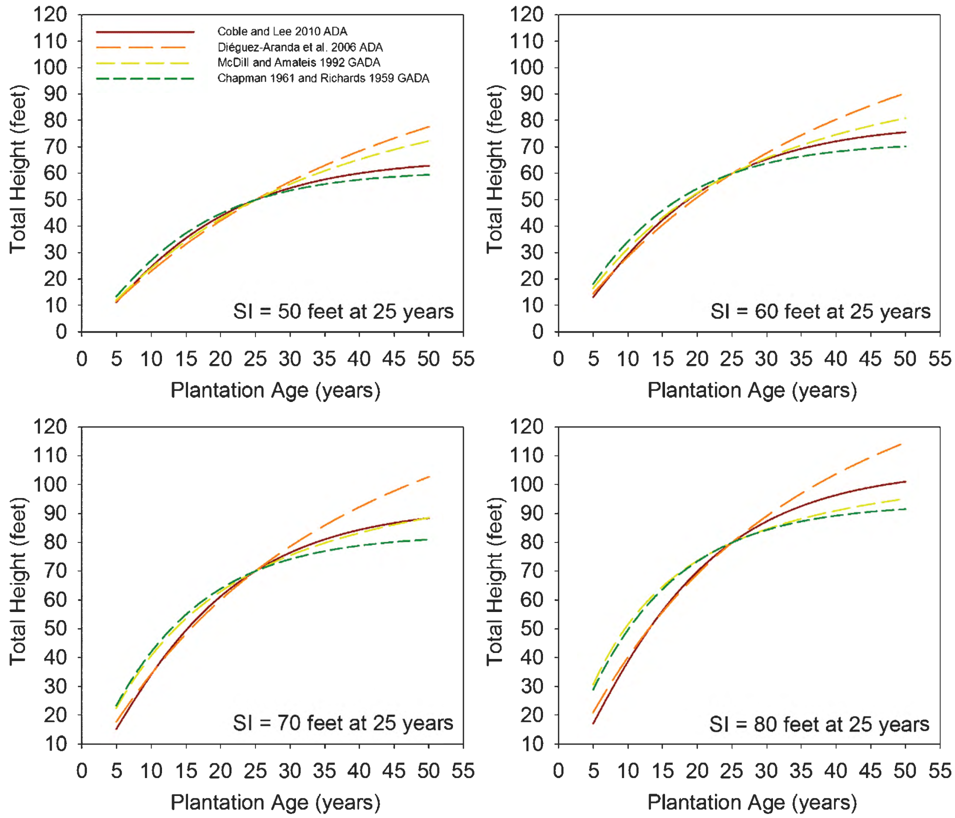

Figure 3. Comparison of site index models for site index 50, 60, 70 and 80 feet.

of site indices examined, both the Chapman-Richards GADA model and the McDill-Amateis GADA model predicted greater heights at ages less than 25 years than the DA2006 and CL2010 models, and the differences increased with increasing SI. After age 25, the DA2006 model predicted the greatest heights, followed by the McDill-Amateis GADA, CL2010, and Chapman-Richards GADA in all SI cases other than SI = 80 feet. At SI = 80 feet, the DA2006 and CL2010 models predicted greater heights than the selected models. Although the Chapman-Richards GADA model had a slightly better relative rank than the McDill-Amateis GADA model, the McDill-Amateis GADA model extrapolated heights more consistently for plantations at 30 and 50 years and was more parsimonious than the Chapman-Richards GADA model since the latter requires a sophisticated calculation of Xo (equation 5). We, therefore, recommend the McDill-Amateis GADA model for use in the region. The derived site index curves from the McDill-Amateis GADA model can be found in Figure 4. The derived site index curves from the McDill-Amateis GADA model with the observed height-age data can be found in Figure 5 to show the suitability of the model.

The model can be applied easily. For an example dominant height calculation, let the total age (A) of a hypothetical loblolly pine plantation be 12 years and the site index (H_0) be 65 feet. The index age (A_0) is 25 years. Dominant height (H) can be calculated from equation (9):

$$H = \frac{112.1}{1-(1-112.1/\,65)\left(\frac{25}{12}\right)^{1.1729}} = 41.3 \text{ feet}$$

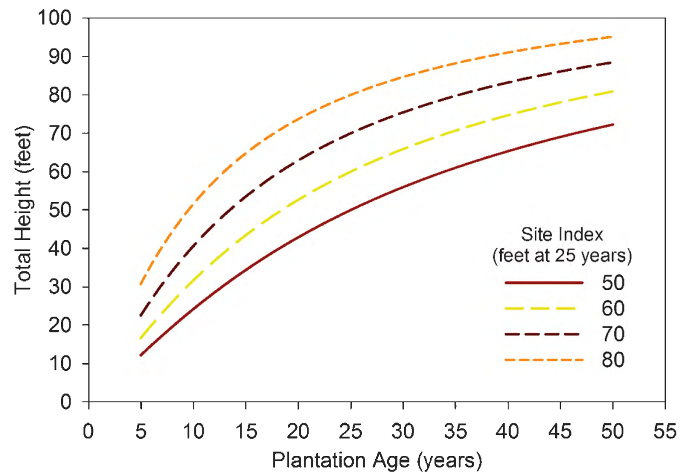

Figure 4. Site index curves for the McDill-Amateis GADA model.

For an example site index calculation, let the total age (A) of a hypothetical loblolly pine plantation be 15 years and the dominant height (H) be 54 feet. The index age (A_0) is 25 years. Site index can be calculated by solving equation (9) for H_0:

$$H_0 = \frac{\beta_1}{1-(1-\beta_1 H^{-1})\left(\frac{A}{A_0}\right)^{\beta_3}},$$

$$H = \frac{112.1}{1-(1-112.1/\,54)\left(\frac{15}{25}\right)^{1.1729}} = 70.5 \text{ feet}$$

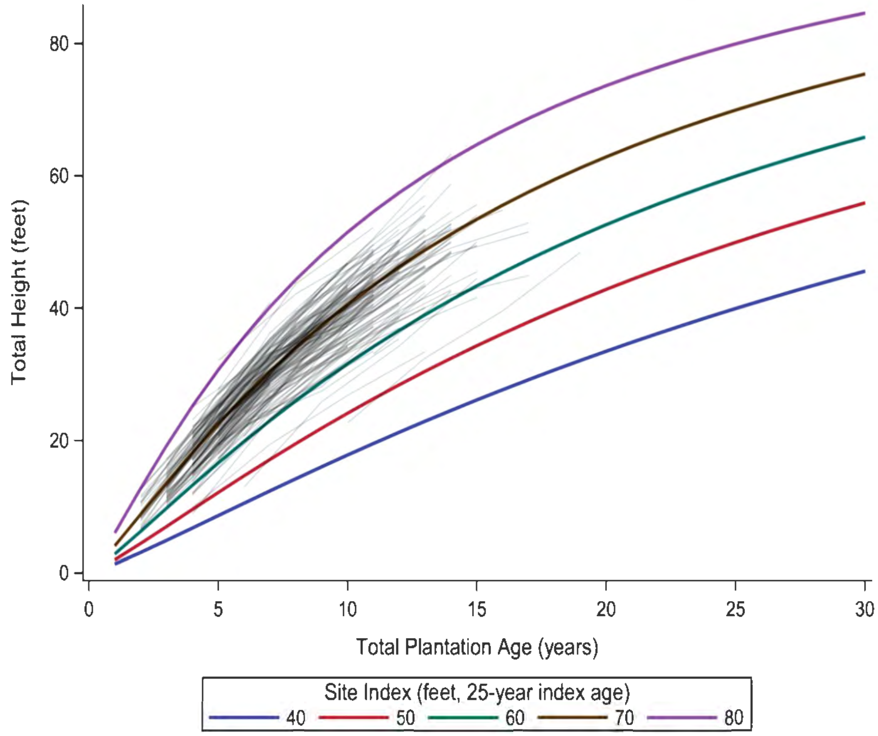

Figure 5. Site index curves for the McDill-Amateis GADA model with observed height-age data included.

Discussion

Numerous model forms have been applied to develop SI curves (Burkhart and Tomé 2012) and selecting the best to describe tree height-age relationships for a region is of great interest for growth and yield modelers. Studies that compared various functions to model height and age reported varying results. For estimating dominant height growth of Scots pine, the polymorphic Hossfeld difference equation produced the most adequate site curves for plantations in northeastern Spain (Palahí et al. 2004), but the McDill-Amateis (1992) model was preferred for populations in northwestern Spain (Diéguez-Aranda et al. 2005). Diéguez-Aranda et al. (2006), using data mostly from loblolly pine plantations in the southeastern US, evaluated four dynamic site equations derived with GADA methods and found that Cieszekski's (2002) model best described height growth. In this study, six SI models were fitted to data collected from east Texas and western Louisiana. Results suggest that the Chapman-Richards and McDill-Amateis models paired with the GADA approach outperformed the others (Table 3), inconsistent with Diéguez-Aranda et al. (2006) or Coble and Lee (2010). In this study only commonly used SI models were compared using the iterative ADA and GADA approaches. Performance of other models not used in this study as well as mixed effects modeling techniques with autoregressive error structures could be further investigated.

Estimates of current site index are a complex result related to genetics, climate, past management practices, and their interactions. Site index has proven to be extremely sensitive to many commonly used silvicultural activities, such as soil bedding, competing vegetation control, fertilization, and genetic selection (Burkhart et al. 1981, Nance and Wells 1981, Monserud and Rehfeldt 1990, Shiver and Martin 2002, Zhao et al. 2009a, b, 2016, Weiskittel et al. 2011). These silvicultural activities and genetic selection can substantially enhance plantation productivity, which may change (increase) site index. In one study in the southeast US, after 25 years, fertilizer and competition control treatments increased site index (index age = 25 years) from 64 to 87 feet in loblolly pine (Jokela et al. 2010). Compared to unimproved seed lots, the improved seed lots changed the asymptotic coefficient of the height-age relationship significantly (Buford and Burkhart 1987).

Both CL2010 and DA2006 were developed for extensively managed loblolly pine plantations, with the former developed specifically for east Texas and the latter for the southeastern US. Differences are expected to exist between them due to regional differences in climate, genetics and other site related factors. Surprisingly, both models predicted similar heights for plantations younger than 25 years, although thereafter DA2006 predicted greater heights than CL2010 (Figure 3). This result may reflect the fact that loblolly pine trees growing further east, which are represented by DA2006, reach greater heights than loblolly pine trees growing in the WGCP region (Wells 1983), which are represented by CL2010.

Statistically the Chapman-Richards GADA model and the McDill-Amateis GADA model provided similar predictions that outperformed the other four models. It is equally important to see their performances by comparing them to the currently used SI models in the region such as CL2010 and DA2006. Data used to develop the Chapman-Richards GADA model and the McDill-Amateis GADA model in this study were collected from intensively managed plantations in east Texas and western Louisiana. Therefore, heights predicted from the Chapman-Richards GADA model or the McDill-Amateis GADA model would be expected to be greater than those obtained from the DA2006 and CL2010 models, since the former two models were parameterized using data from intensively, rather than extensively, managed stands. This was confirmed when plantations were less than 25 years old (Figure 3), suggesting both selected models are better than DA2006 and CL2010 in calculating heights approaching 25 years for intensively managed loblolly pine plantations in the region. The height differences became more pronounced as site index increased (Figure 3), suggesting that effects of intensive management activities are not constant across site quality. This result supports investing in improved genetics and intensive silvicultural practices on better sites to achieve taller dominant heights. Without doubt, these differences are a result of a complex byproduct of genetics, climate, applied silvicultural treatments and even their interactions. The differences with CL2010 could possibly be a result of genetics, silviculture, and their interaction, whereas regional climatic differences in addition to silviculture and genetics might strongly contribute to the differences with DA2006. However, the Chapman-Richards GADA and McDill-Amateis GADA models predicted shorter heights than DA2006 and CL2010 for plantations older than 25 years (Figure 3). These unexpected results may be due to the lack of data from older ages in this study, since the oldest stand measurement in this study was 19 years old. Both the CL2010 and DA2006 models were created using data available from unthinned loblolly pine plots that ranged in age up to 37 years or older, well beyond the typical rotation age of 25 years (Lenhart et al. 1986, Coble and Lee 2006, 2010). It is important to maintain and continue to measure these plots so both the Chapman-Richards GADA and McDill-Amateis GADA models can be updated in the future. Nonetheless, these models are still useful for intensively managed stands which are typically managed in short rotations such as 25 years or less.

Although both the Chapman-Richards GADA model and the McDill-Amateis GADA model can predict early height growth well, we recommend the McDill-Amateis GADA model since it seemed to extrapolate better after age 25 years and was more parsimonious than the Chapman-Richards GADA model (Figure 3). Both models need to be refit to older data, but until such data become available, our SI models can be used in conjunction with growth and yield models for the WGCP region (Coble et al. 2016). Estimates of future yields utilizing the models should be more realistic and will aid foresters in making more appropriate silvicultural prescriptions for intensively managed loblolly pine plantations in this unique region.

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