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Total and Partial Stand-Level Yield Prediction for Loblolly and Slash Pine Plantations in East Texas

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ABSTRACT. *Observations from East Texas Pine Plantation Research Project permanent plots in loblolly (*Pinus taeda *L.) and slash (*Pinus elliottii *Engelm.) pine plantations throughout East Texas were utilized to develop methods to estimate stand-level yield values. Predicted yields are volume in cubic feet and green weight in pounds. Predictor variables for total yield are plantation age, site index (base age 25 yr) and surviving trees per acre. Partial yield is derived using total yield and plantation quadratic mean diameter in conjunction with specified threshold dbh and upper stem dob values. Expected total yield per acre is converted to partial yield per acre by considering combinations of threshold dbh and upper stem dob values. For each combination, total yield per acre is reduced by a proportional value to estimate a partial yield per acre. South. J. Appl. For. 20(1):36–41.*

A diameter distribution yield prediction system for loblolly (*Pinus taeda* L.) and slash (*Pinus elliottii* Engelm.) pine plantations in East Texas was developed by Lenhart (1988) with subsequent updates and modifications by Taylor (1990) and Lapongan (1993). The system, comprised of a series of equations, is specifically designed to provide yield estimates by tree size classes, and by summing across all classes, a stand-level estimate can be obtained. However, in some cases, it may be useful to directly estimate stand-level yield values in a simple straightforward manner.

Stand-level prediction methods have been determined for loblolly pine plantations in the Atlantic Coastal Plain (Burkhart et al. 1972), southwide (Burkhart et al. 1985 and Amateis et al. 1986) and Gulf Coastal Plain (Ledbetter et al. 1986). In order to meet specified merchandizing standards, it may be necessary to reduce the total yield estimate to reflect limits on diameter at breast height and diameter on the upper stem. A concept developed by Amateis et al. (1986) and used for planted loblolly pine trees southwide and utilized by Pienaar and Rheney (1993) for slash pine plantations in the southeastern coastal plain provides for partial yield estimation in an equable manner.

This paper presents equations to directly estimate total yield of loblolly and slash pine plantations in East Texas plus an application of the Amateis et al. (1986) concept for converting total yield to partial yield.

Converting Total Yield to Partial Yield

Amateis et al. (1986) developed a model that converts total yield to partial yield for various combinations of threshold dbh values and upper stem diameter values in a consistent and logical manner. A proportion or ratio is calculated that is a function of a minimum dbh limit, an upper stem diameter outside bark limit, and quadratic mean stand diameter. In this manner, partial yield does not exceed total yield for a given plantation. The expected proportion is multiplied by the total yield as:

$$PY = TYe^{\left(b1(t/D)^{b2} + b3(d/D)^{b4}\right)} \tag{1}$$

where

PY	=	partial yield per acre for trees of d and larger to a upper stem diameter of t
TY	=	total yield per acre
D	=	quadratic mean dbh (in.)
t	=	upper stem diameter outside bark limit (in.)
d	=	threshold dbh value (in.)
e	=	base of the natural logarithm
$b1$–$b4$	=	parameters to be estimated

Properties of Equation (1) are:

- When both t and d equal zero, partial yield equals total yield
- When t equals zero, partial yield is determined by the minimum dbh value d.

• When d equals zero, partial yield is determined by the upper stem diameter value t.

The Amateis proportion model was utilized in this study.

Plantation Measurements

Source of data for this yield study was the East Texas Pine Plantation Research Project (ETPPRP). The ETPPRP was initiated in 1982 by the College of Forestry at SFASU with assistance from participating forest industries in East Texas.[1] In this continuing study, 170 and 76 permanent plots are located in industrial loblolly and slash pine plantations, respectively, throughout East Texas.

At a plot, there are two subplots—one for model development and the other for model evaluation. Within each subplot, all planted pines are tagged and numbered. On a 3 yr cycle, dbh, total height, crown class, and presence of fusiform rust, among other values, are determined for each of the planted pines. A minimum age limit of 5 yr was set on ETPPRP data to be included in this yield analysis to allow for an accurate assessment of fusiform rust by the field crews. Data from 12 years of sampling were available for analysis. All values derived from the four repeated measurements were treated as independent observations in the various ensuring regression analyses.

For each observation for each subplot, the following plantation parameters were determined:

A = number of years since planting

H = average total height of the 10 tallest trees (nearest ft)

T = number of surviving trees per acre

S = predicted site index base age 25 yr (nearest ft)

D = quadratic mean diameter at breast height (nearest 0.1 in.)

Using tree content prediction equations developed by Lenhart et al. (1987) and updated by Lapongan et al. (1993), the following stand-level yield values were calculated for each observation:

$TYCFWB$ = total volume wood and bark per acre (nearest ft^3)

$TYCFW$ = total volume wood only per acre (nearest ft^3)

$TYGWWB$ = total green weight wood and bark per acre (nearest lb)

$TYGWW$ = total green weight wood only per acre (nearest lb)

Total yield includes the content of the complete stem of all the trees from the stump to the tip of the terminal bud. The total yield data set consisted of 541 and 250 observations for loblolly and slash pine, respectively.

Tables 1 and 2 present means and ranges for observed plantation parameters from the development and evaluation subplots for loblolly and slash pine, respectively. Age characteristics of the sample observations for each species are similar. Loblolly plots tended to have more trees per acre than slash pine plots, while slash plots appeared to be located on more productive sites, as measured by site index, than loblolly plots.

Partial yields per acre were computed for each observation for 27 combinations of minimum dbh values and upper stem diameter limits. Minimum dbh ranged from 2–7 in., and for a given dbh class, upper stem diameter values ranged from 0–6 in., as appropriate. Using the tree content functions by Lapongan et al. (1993), 27 stand-level partial yield per acre values were calculated for each of the following:

$PYCFWB$ = partial volume wood and bark per acre (nearest ft^3)

$PYCFW$ = partial volume wood only per acre (nearest ft^3)

$PYGWWB$ = partial green weight wood and bark per acre (nearest lb)

$PYGWW$ = partial green weight wood only per acre (nearest lb)

Partial yield included the content of the stem from the stump to the specified upper stem diameter outside bark for those trees exceeding the specified minimum dbh. After calculations were completed, a total of 12,396 and 5,369 observations from the development subplots were available for loblolly and slash pine, respectively, to fit the Amateis model [Equation (1)], using nonlinear regression analyses. From the evaluation subplots, a total of 12,453 and 5,388 observations were determined for assessing the stand-level yield prediction equations.

Yield Prediction

Total Yield

Examinations of plottings of total yield over plantation parameters for both species indicated that a variant of the Schumacher model (1939) was appropriate for predicting total yield per acre as:

$$\ln(TY) = b1 + b2(1/A) + b3[\ln(S)] + b4[\ln(T)] \quad (2)$$

where ln = natural logarithm, and other variables defined above.

Multiple linear regression analyses were used to fit Equation (2) to the model development total yield data sets for both species. The loblolly and slash pine total yield prediction

[1] Support from Champion International Corporation, International Paper Company, Louisiana-Pacific Corp. and Temple-Inland Forest Products Corporation is appreciated.

Table 2 Observed stand structure characteristics for the East Texas slash pine data by subplot

Characteristic	Subplot: Development	Subplot: Evaluation
Age (yr)		
Mean	10.6	10.6
Range	5–24	5–24
Plantation height (ft)		
Mean	35.7	35.6
Range	8–73	8–80
Surviving trees/ac		
Mean	388	398
Range	91–1,002	91–1,032
Site index (ft, base age 25 yr)		
Mean	73.8	73.8
Range	22–96	30–97
Quadratic mean diameter (in.)		
Mean	4.90	4.89
Range	0.73–9.06	0.52–9.33
Total volume wood and bark/ac (ft^3)		
Mean	959	981
Range	4–4,072	3–4,873
Total volume wood only/ac (ft^3)		
Mean	692	709
Range	2–3,114	1–3,889
Total green weight wood and bark/ac (lb)		
Mean	52,312	53,527
Range	224–225,736	142–273,655
Total green weight wood only/ac (lb)		
Mean	45,559	46,629
Range	178–199,123	113–244,134

equations for each of the four measures are listed in Tables 3 and 4, respectively. Plottings of residuals indicated no adverse trends. Each of the eight equations for yields accounted for at least 96% of the total variation in yield.

Model evaluation total yield data sets provided an additional opportunity to assess total yield prediction models for bias and adverse trends. For each species and for each unit of measure, an examination of the difference between observed and predicted total yields showed no adverse trends relative to plantation parameters. An analysis of the mean differences indicated that none were significantly different from zero at the 0.05 level.

Table 1. Observed stand structure characteristics for the East Texas loblolly pine data by subplot.

Characteristic	Subplot: Development	Subplot: Evaluation
Age (yr)		
Mean	10.9	10.9
Range	5–24	5–24
Plantation height (ft)		
Mean	37.0	36.7
Range	9–76	9–79
Surviving trees/ac		
Mean	466	462
Range	87–993	144–937
Site index (ft, base age 25 yr)		
Mean	69.3	69.1
Range	28–99	30–99
Quadratic mean diameter (in.)		
Mean	5.05	5.05
Range	0.49–9.19	0.62–10.28
Total volume wood and bark/ac (ft^3)		
Mean	1,249	1,238
Range	2–5,615	3–5,480
Total volume wood only/ac (ft^3)		
Mean	985	975
Range	1–4,729	2–4,686
Total green weight wood and bark/ac (lb)		
Mean	70,287	69,570
Range	103–338,537	146–330,472
Total green weight wood only/ac (lb)		
Mean	64,556	63,891
Range	87–313,413	123–308,296

Table 3. Estimating total yield per acre, quadratic mean diameter, and portion of total yield to any desired upper stem diameter and considering trees above a threshold dbh for loblolly pine plantations in East Texas.

$$\mathit{TYCFWB} = e^{(-7.48981 - 28.79181/A + 3.30880\ln(S) + 0.50762\ln(I))} \quad \text{(a)}$$

$$\mathit{TYCFW} = e^{(-8.32285 - 30.39674/A + 3.49114\ln(S) + 0.49756\ln(T))} \quad \text{(b)}$$

$$\mathit{TYGWWB} = e^{(-3.95467 - 29.89577/A + 3.42761\ln(S) + 0.51860\ln(T))} \quad \text{(c)}$$

$$\mathit{TYGWW} = e^{(-4.13681 - 30.29124/A + 3.47130\ln(S) + 0.50900\ln(T))} \quad \text{(d)}$$

$$D = e^{(-0.85021 - 9.62461/A + 1.12761\ln(S) - 0.22497\ln(T))} \quad \text{(e)}$$

$$\mathit{PYCFWB} = \mathit{TYCFWB}\ e^{(-0.56203(t/D)^{2.78244} - 0.35030(d/D)^{6.15691})} \quad \text{(f)}$$

$$\mathit{PYCFW} = \mathit{TYCFW}\ e^{(-0.61040(t/D)^{2.76686} - 0.34395(d/D)^{6.19911})} \quad \text{(g)}$$

$$\mathit{PYGWWB} = \mathit{TYGWWB}\ e^{(-0.61487(t/D)^{2.70988} - 0.35411(d/D)^{6.14521})} \quad \text{(h)}$$

$$\mathit{PYGWW} = \mathit{TYGWW}\ e^{(-0.62440(t/D)^{2.71697} - 0.34930(d/D)^{6.17180})} \quad \text{(i)}$$

where:

TYCFWB = total volume wood and bark/ac (ft^3)
TYCFW = total volume wood only/ac (ft^3)
TYGWWB = total green weight wood and bark/ac (lb)
TYGWW = total green weight wood only/ac (lb)
PYCFWB = partial volume wood and bark/ac (ft^3)
PYCFW = partial volume wood only/ac (ft)
PYGWWB = partial green weight wood and bark/ac (lb)
PYGWW = partial green weight wood only/ac (lb)
A = number of years since planting
S = site index base age 25 yr (ft)
T = number of surviving trees/ac
D = quadratic mean dbh (in.)
t = upper stem diameter limit outside bark (in.)
d = threshold dbh (in.)
ln = natural logarithm
e = base of the natural logarithm

Table 4. Estimating total yield per acre, quadratic mean diameter, and portion of total yield to any desired upper stem diameter and considering trees above a threshold dbh for slash pine plantations in East Texas.

$$\mathit{TYCFWB} = e^{(-8.84214 - 25.08303/A + 3.28506\ln(S) + 0.65438\ln(I))} \quad \text{(a)}$$

$$\mathit{TYCFW} = e^{(-9.80662 - 26.74539/A + 3.48994\ln(S) + 0.63441\ln(T))} \quad \text{(b)}$$

$$\mathit{TYGWWB} = e^{(-4.96066 - 25.56821/A + 3.32657\ln(S) + 0.65023\ln(T))} \quad \text{(c)}$$

$$\mathit{TYGWW} = e^{(-5.25272 - 26.01654/A + 3.38002\ln(S) + 0.64309\ln(T))} \quad \text{(d)}$$

$$D = e^{(-1.60900 - 8.26246/A + 1.14790\ln(S) - 0.15495\ln(T))} \quad \text{(e)}$$

$$\mathit{PYCFWB} = \mathit{TYCFWB}\ e^{(-0.54376(t/D)^{3.10297} - 0.36724(d/D)^{6.69198})} \quad \text{(f)}$$

$$\mathit{PYCFW} = \mathit{TYCFW}\ e^{(-0.58027(t/D)^{3.10649} - 0.35403(d/D)^{6.77884})} \quad \text{(g)}$$

$$\mathit{PYGWWB} = \mathit{TYGWWB}\ e^{(-0.58477(t/D)^{3.04979} - 0.36266(d/D)^{6.71817})} \quad \text{(h)}$$

$$\mathit{PYGWW} = \mathit{TYGWW}\ e^{(-0.58964(t/D)^{3.06732} - 0.35775(d/D)^{6.74950})} \quad \text{(i)}$$

where:

TYCFWB = total volume wood and bark/ac (ft^3)
TYCFW = total volume wood only/ac (ft^3)
TYGWWB = total green weight wood and bark/ac (lb)
TYGWW = total green weight wood only/ac (lb)
PYCFWB = partial volume wood and bark/ac (ft^3)
PYCFW = partial volume wood only/ac (ft)
PYGWWB = partial green weight wood and bark/ac (lb)
PYGWW = partial green weight wood only/ac (lb)
A = number of years since planting
S = site index base age 25 yr (ft)
T = number of surviving trees/ac
D = quadratic mean dbh (in.)
t = upper stem diameter limit outside bark (in.)
d = threshold dbh (in.)
ln = natural logarithm
e = base of the natural logarithm

Partial Yield

For each species and each of the four units of measure, the partial yield data set from the model development subplots was utilized in nonlinear regression analyses to fit Equation (1). The results of the analyses are shown in Tables 3 and 4 for loblolly and slash pine, respectively. None of the asymptotic 95% confidence intervals for the four coefficients in each of the eight partial yield prediction equations contained zero. Plottings of residuals over plantation parameters indicated no adverse trends.

In addition, the performance of the partial yield prediction equations was assessed using the model evaluation partial yield data sets. For each observation, a predicted partial yield was computed and compared to the actual yield. Plottings of the differences over plantation parameters showed no adverse trends. None of the mean differences were significantly different from zero at the 0.05 level.

Quadratic Mean Diameter

If partial yield estimates are desired and quadratic mean diameter is not known, then it will be necessary to predict D. Plottings of observed quadratic mean diameter over plantation parameters for each species suggested that Equation (2) was also appropriate for predicting $\ln(D)$. Using multiple linear regression analyses, observations from the total yield data sets were used to fit Equation (2) for estimating $\ln(D)$ for each species. The quadratic mean diameter prediction equations for loblolly and slash pine plantations are listed in Tables 3 and 4, respectively. Residual analysis indicated no adverse trends, and each equation accounted for at least 92% of the total variation in $\ln(D)$.

The two quadratic mean diameter prediction equations were evaluated using the total yield model evaluation data sets. No adverse trends were seen, and none of the mean differences were significantly different from zero at the 0.05 level.

Site Index

Since site index (base age 25 yr) is a component in estimating yield values, an assessment of the productivity of a loblolly or slash pine plantation is needed prior to yield prediction. The ability of land in East Texas to grow planted loblolly and slash pine trees can be determined using site index equations developed by Vaughn et al. (1993), which are an update of Lenhart et al. (1986):

$$\text{Loblolly site index} = H\{0.88439/[1 - e^{(-0.08630(A))}]\}^{1.59587} \quad (3)$$

and

$$\text{Slash site index} = H\{0.73156/[1 - e^{(-0.05261(A))}]\}^{1.31659} \quad (4)$$

and all variables defined above.

Surviving Number of Trees perAc

Another component in this yield prediction process is the number of trees per acre. For a current plantation, the value for T can usually be obtained from inventory records. For a specified future plantation, the number of trees per acre may be anticipated. However, for projecting current number of trees per acre into the future, survival functions may be required.

In East Texas pine plantations, fusiform rust incidence can affect survival rates. Recent mensurational work[2] resulted in survival estimation models that consider fusiform rust:

Loblolly

$$Nu_2 = Nu_1 e^{(-0.01298(A_2 - A_1))} \quad (5)$$

and

$$Ni_2 = (Ni_1 - 0.13072 Nu_1) e^{(-0.04839(A_2 - A_1))} + 0.13072 Nu_1 e^{(-0.01298(A_2 - A_1))} \quad (6)$$

Slash

$$Nu_2 = Nu_1 e^{(-0.03465(A_2 + A_1))} \quad (7)$$

and

$$Ni_2 = (Ni_1 - 0.89135 Nu_1) e^{(-0.07625(A_2 - A_1))} + 0.89135 Nu_1 e^{(-0.03465(A_2 - A_1))} \quad (8)$$

where

A_1 = initial plantation age (yr)
A_2 = projected plantation age (yr)
Nu_1 = number of uninfected trees per acre at A_1
Nu_2 = number of uninfected trees per acre at A_2
Ni_1 = number of infected trees per acre at A_1
Ni_2 = number of infected trees per acre at A_2

Applications

An Existing Plantation

From an inventory of an existing loblolly pine plantation, the following values are available:

- Plantation age = 12 yr
- Average total height of the 10 tallest trees = 40.5 ft
- Using Equation (3), site index = 67 ft
- Surviving number of trees per acre = 524
- Quadratic mean diameter = 5 in.

Estimates of total yields can be calculated using Equations (a, b, c, and d) from Table 3 as:

- Total volume wood and bark per acre = 1,342 ft^3
- Total volume wood only per acre = 1,031 ft^3

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2 An article compiling this work appears on p. 30–35 of this issue of the *Southern Journal of Applied Forestry*.

- Total green weight wood and bark per acre = 74,107 lb
- Total green weight wood only per acre = 67,627 lb

If partial yield values for those trees in this plantation with dbh greater than or equal to the quadratic mean diameter are needed, Equations (f, g, h, and i) from Table 3 can be used. The four partial yield values are:

- Partial volume wood and bark per acre = 945 ft^3
- Partial volume wood only per acre = 731 ft^3
- Partial green weight wood and bark per acre = 52,008 lb
- Partial green weight wood only = 47,689 lb

If partial yield values for those trees in this plantation with dbh greater than or equal to quadratic mean diameter plus consideration of an upper stem dob utilization limit of 4 in. are needed, Equations (f, g, h, and i) from Table 3 can be used. The four partial yield values are:

- Partial volume wood and bark per acre = 699 ft^3
- Partial wood only per acre = 526 ft^3
- Partial green weight wood and bark per acre = 37,171 lb
- Partial green weight wood only = 33,926 lb

Projected Yields

After an inventory of an existing slash pine plantation, several values are determined as:

- Plantation age = 5 yr
- Site index = 73 ft
- Surviving trees per acre clear of fusiform rust = 290
- Surviving trees per acre with fusiform rust galls on stem = 124
- Total number of surviving trees = 414

It is anticipated that this plantation will be harvested at 22 yr of age. The projected cubic feet of wood only per acre in 17 yr hence can be estimated in the following manner:

- Use Equation (7) and predict surviving trees per acre clear of rust galls on stem = 161.
- Use Equation (8) and predict surviving trees per acre not clear of rust galls on stem = 107.
- Expected total number of surviving trees per acre = 268, or about 1 of 3 trees projected to die during the 17 yr projection period.
- Use Equation (e) from Table 4 to predict quadratic mean diameter = 7.96 in.
- Use Equation (b) from Table 4 to predict total cubic feet wood only per acre = 1,805
- If d = 7 in. and t = 4 in., use Equation (g) from Table 4 to predict partial cubic feet wood only per acre = 1,454, or about 20% of the total yield will not meet these utilization standards.

Summary

A versatile and easy to use stand-level yield prediction method is presented in this paper. Readily available stand variables—age, site index, and number of trees per acre, are utilized to estimate the total yield (*TYCFWB, TYCFW, TGWWB,* and *TGWW*) per acre. A merchandising of the total yields into partial yields is possible by setting utilization standards of a threshold dbh *(d)* and upper stem diameter limit outside bark *(t)*. A component of the merchandising process requires a value of quadratic mean diameter *(D)*, which can also be estimated using age, site index and number of trees per acre.

This stand-level prediction method should provide useful estimates of current and future yields of unthinned loblolly and slash pine plantations in East Texas. The method is enhanced by providing a procedure to impose merchantability standards on the yield estimation process. As a result, foresters in East Texas should be able to develop useful wood flow schedules to incorporate into their plantation management plans

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