Mathematics of the Significant Tornado Parameter

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Meteorology: A Division of Math and Physics

Meteorology is a division of physics that involves thermodynamics as well as the movement of fluid. As more research has been conducted on the atmosphere, weather predicting has become more advanced and reliable. One outcome of this research is a multi-valued parameter known as the Significant Tornado Parameter (STP). The STP looks at the current energy state of the atmosphere and predicts the likelihood of a tornado of rating EF2 or greater forming in that area.

Main Project

The idea of this project came from my curiosity of weather and especially how severe storms are predicted. There are many parameters used to predict severe weather that use mathmematical formulas. After sorting through the parameters, the STP was the one selected due to its use of other parameters for prediction of tornadoes, as well as concepts used in Single and Multi-Variable Calculus, such as areas between curves and vector magnitude and direction, and learning how to use new graphs than what is normally used.

Graphs and Vectors

The data is collected from the atmosphere from an instrument strapped onto a weather balloon. The instrument measures the temperature, dew point, wind speed, and wind direction. These are plotted on a graph known as a SkewT-log p Diagram. The y-axis of a SkewT-log p is the logarithmic pressure given in millibars (mb). The elevation markers, with units of kilometers, are labeled on the right side of this axis. The temperature markers, with units in degrees Celsius, are skewed 45° and the labels are located along the x-axis. To find the temperature at a certian point on a line, follow the skewed line to the x-axis and that is the temperature. Winds are displayed as vectors on the right-hand side of the graph. The direction of the vector is the direction of the wind with special barbs on the vector representing wind speed in knots. To show how the wind is circulating with respect to height, the wind is plotted on a circular graph known as a hodograph. The circular axis on the graph is the wind speed, and the direction is the angle measured from the positive x-axis. The altitude in kilometers is given as data points on the graph.

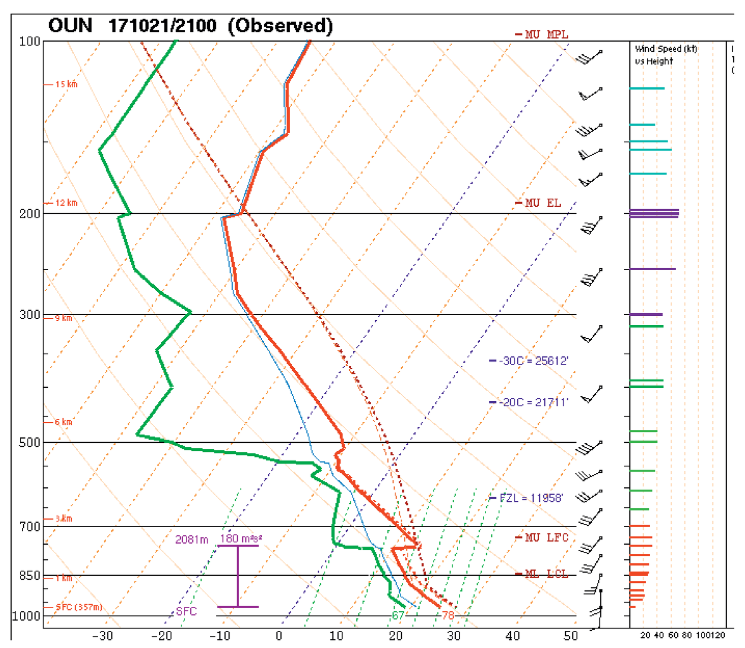

Figure 1: SkewT-log p Diagram of an Observed Sounding. Source: National Weather Service Storm Prediction Center

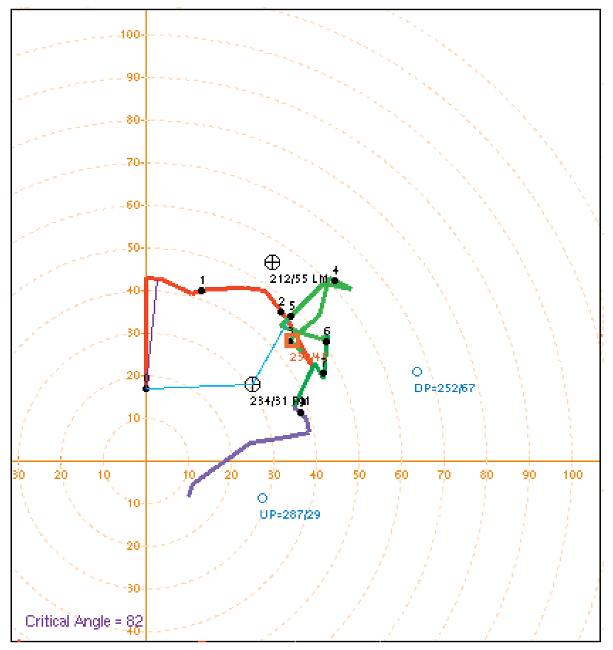

Figure 2: Hodograph of an Observed Sounding. Source: National Weather Service Storm Prediction Center

Lifted Condensation Level

The Lifted Condensation Level (LCL) is the height where a parcel of air reaches a relative humidity of 100%. While there have been many attempts to create an equation to calcuate the LCL, the equations created have always contained a high percent error. The best and easiest way to determine the LCL is from the SkewT-log p diagram where it is marked on the right-hand side of the graph (See Figure 1).

Wind Shear

Wind shear is the way the wind changes speed and direction along a plane. Rotating thunderstorms are usually caused by high levels of vertical shear. Updrafts in a rotating thunderstorm are often stronger than the buoyant forces of air. On a hodograph, if the winds are shown in a straight line, then the vertical shear, as shown in Figure 3, provide a source for the rotating updraft. This initial rotation is weak, but due to change in pressure, new updrafts are created until the rotation is strong. If there is a definite, clockwise curve to the winds on the hodograph, as seen in Figure 2, shows the initial updraft is already rotating and favors the creation of right-moving supercells. A calculation of shear used is the bulk wind difference (BWD), which is the magnitude of the wind velocity vector from the inflow base of a thunderstorm to the equilibrium level height of a thunderstorm.

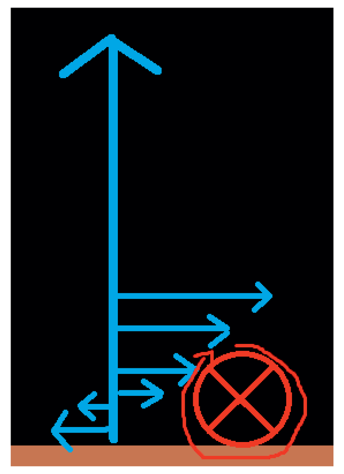

Figure 3: Roation (red) caused by wind shear (blue)

Convective Available Potential Energy

Convective Available Potential Energy (CAPE) is a vertically integrated index that measures the buoyant energy in the free convective layer (FCL) from the level of free convection (LFC), where a parcel of air has free acceleration upward, to the equilibrium level (EL), where a parcel of air is colder than the enviroment around it; usually the top of a thunderstorm. This equation for CAPE is

$$\text{CAPE} = g\int_{z_{LFC}}^{z_{EL}} \left(\frac{T_{V_p} - T_{V_e}}{T_{V_e}}\right) dz$$

Where g is the acceleration due to gravity, T_{V_p} is the temperature of the parcel of air being measured, and T_{V_e} is the temperature of the enviroment and is constrained by the depth of the FCL and the average magnitude of buoyancy. While this does not measure the instability of the atmosphere, the aspect ratio of the positive area of CAPE from the SkewT-log p graph should be considered. To find these values from the SkewT-log p graph (as seen in Figure 1), the LFC and EL are marked on the right-hand side of the graph, and the temperature of the enviroment is the dotted brown line. The area to integrate is where this line and the red parcel temperature line intersect on both sides.

Storm-Relative Helicity

Storm-Relative Helicity is a type a wind shear where the winds change from a horizontal flow to a vertical flow within a thunderstorm. Strong updrafts tilt the horizontal winds and stretching begins to form. The equation for this stretching, also known as streamwise vorticity, is another vertically integrated index from 0 to the height of the storm, h, and measures the dot product of the velocity vector of the wind, $\mathbf{v}$, with the horizontal vorticity, $\omega \equiv \mathbf{k} \times \left(\frac{\partial \mathbf{v}}{\partial z}\right)$. This area can be found from a hodograph rather than a SkewT-log p diagram (See Figure 2).

Figure 4: Helicity showing rotaing horizontal winds being tilted vertical in an updraft. Source: Wunderground/Jessica Higgs

$$\text{SRH} = -\int_0^h \left[\mathbf{v} \cdot \mathbf{k} \times \left(\frac{\partial \mathbf{v}}{\partial z}\right)\right] dz$$

Convective Inhibition

Convective Inhibition (CIN) is the amount of work required to lift a parcel through a layer of the atmosphere that is warmer than the parcel itself. This is sometimes called negative CAPE as it's properties follow closely to CAPE. The only difference in the calculation between CIN and CAPE is the location in the atmosphere. CIN is measured from the surface (SFC) to the Level of Free Convection (LFC). Thus, the CAPE equation becomes

$$\text{CIN} = g\int_{z_{SFC}}^{z_{LFC}} \left(\frac{T_{V_p} - T_{V_e}}{T_{V_e}}\right) dz$$

Significant Tornado Paramenter

Research on tornadoes have shown that multiple conditions appear to favor supercells that produce significant tornadoes. Because of this, the National Storm Prediction Center (SPC) modified their Supercell Composite Number into the Sigificant Tornado Parameter. The STP uses normalized versions of BWD, CAPE, Helicity, LCL, and CIN and the normalized vales are changed periodically due to findings from new tornado research. The current version is defined as

$$\mathbf{STP} = \frac{\text{MLCAPE}}{1500\ \text{J kg}^{-1}} \times \frac{\text{EBWD}}{20\ \text{m s}^{-1}} \times \frac{\text{ESRH}}{150\ \text{m}^2\,\text{s}^{-2}} \times \frac{2000 - \text{MLLCL}}{1000\ \text{m}} \times \frac{200 + \text{MLCIN}}{150\ \text{J kg}^{-1}}$$

Where ML is the mixed-layer or the mean conditions found in the lowest 100 mb of the atmosphere for CAPE, LCL, and CIN and E is the Effective, or from the base of the storm to the height of the storm, for BWD and SRH. Since this is a probability, this will return a value between 0 and 1 with closer to 1 being the higher chance of a significant tornado. However, some computer generated models have been known to calculate values greater than 1, which in many cases have predicted several severe outbreaks. While the STP is a good measure to calculate the chance of having tornadoes rated EF2 or greater, this is only a conditional parameter that is a simplified estimate of environments favoring both supercell initiation and sustenance and recently has seem to only work during certain times of the year for different areas of the country.

Conclusion

The STP is a conditional probability that uses multiple parameters that has basic mathematical and physical calculations and in areas where all conditions are met, strong tornadoes are more probable. The STP is still being refined today. As our knowledge of tornadoes change, the understanding of how to predict them improve.

Acknowledgements and References

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