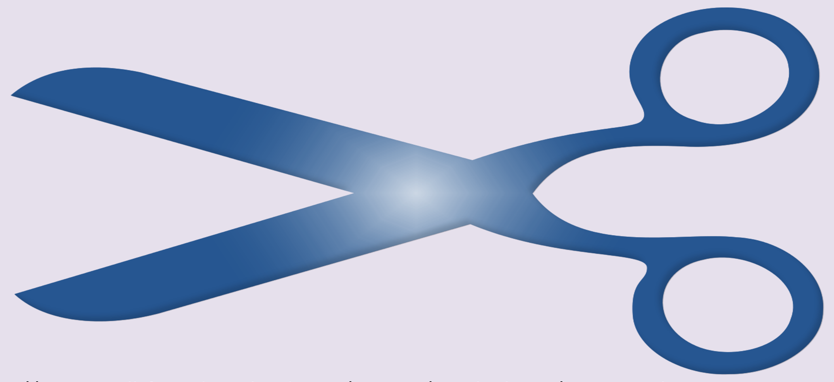

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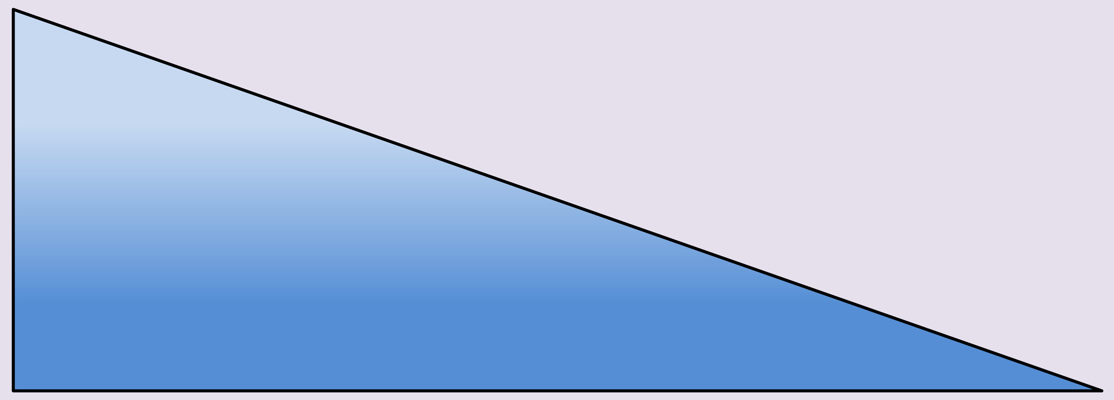
CUTTING TRIANGLES



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Abstract

The idea of this project began with a question:
Draw any triangle on a piece of paper, is it possible to fold the paper so that the triangle can be cut out in one slice?

Basics

What is a Triangle?
A triangle is a two-dimensional figure with three sides and three angles.

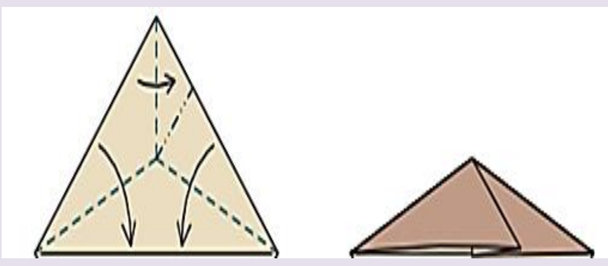
Types of Triangles:
Equilateral, isosceles, scalene, right, and obtuse triangles

Idea Formulation:
To cut out the triangle in one slice means that I need to separate what I want to cut out and what I don't on two sides of a line.

Method Development

Symmetry Method:
My first attempts involved folding triangles that were symmetrical because I intuitively drew symmetrical triangles such as equilateral and isosceles triangles, and I successfully developed a method for triangles that have an axis of symmetry. However, right and obtuse triangles do not necessarily have axes of symmetry, so the symmetry method failed for those triangles. Then, after folding many triangles, I was reminded of a triangular origami base I used to fold as a child using a swivel fold. Thus, I began exploiting the idea behind the swivel fold to develop the Rabbit Ear Method below.

Rabbit Ear Method:



https://upload.wikimedia.org/wikipedia/commons/2/2e/Origami-Basisfaltungen.png

1. Draw the triangle on a piece of paper
2. Using a compass, find the 3 angle bisectors of the triangle
   a. Place the compass point on a vertex of the triangle
   b. Set the compass to any width
   c. Draw two arcs crossing each adjacent side of the triangle
   d. From where each previous arc crosses a side, make a new arc in the triangle's interior for both previous arcs
   e. Draw a line from the vertex to where the two new arcs cross
   f. Repeat 2a – 2e to find the other 2 angle bisectors of the triangle
3. Make a swivel fold using the 3 angle bisectors
4. Use a pair of scissors to cut at the line

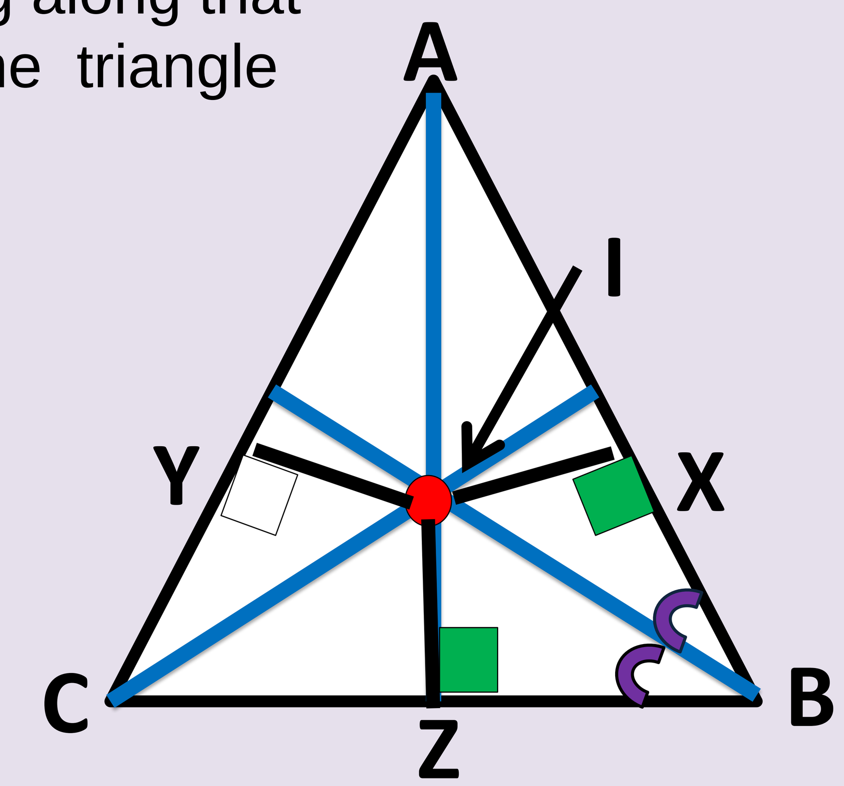
Theorems

Concurrence Theorems - the bisectors of the angles of a triangle meet at a point (incenter) that is equidistant from the sides of the triangle

Angle-Angle-Side Congruence Theorem (AAS) - if two angles and any side of one triangle are congruent to two angles and any side of another triangle, then the triangles are congruent.

Proof

Let ABC be a triangle. We can find the 3 angle bisectors of the triangle. By the Concurrence Theorem, the angle bisectors intersect at the incenter, labeled I, of the triangle where the distances from the sides of the triangle (called points X, Y, and Z) to the incenter are equal. Hence, the line segments IX, IY, and IZ are all equal. From Calculus, for any point not on a line, there exists another line containing that point that is perpendicular to the line. Thus, we have that IX is perpendicular to AB, IY perpendicular to AC, and IZ perpendicular to BC. By the Angle-Angle-Side Congruence Theorem, since the angles IXB and IZB are 90°, the angles XBI and ZBI are equal (from bisecting angle XBZ), and the side IB is shared between triangles XIB and ZIB, those two interior triangles are congruent. A similar argument can be made for the other two pairs of interior triangles. Now, the swivel fold will collapse the three pairs of congruent interior triangles onto a straight line separating the paper into two regions: one region makes up the triangle that we want to cut out and the other region the leftover area of the paper not making up the triangle. Therefore, cutting along that straight line will yield us the triangle we originally drew on the piece of paper.



Extensions

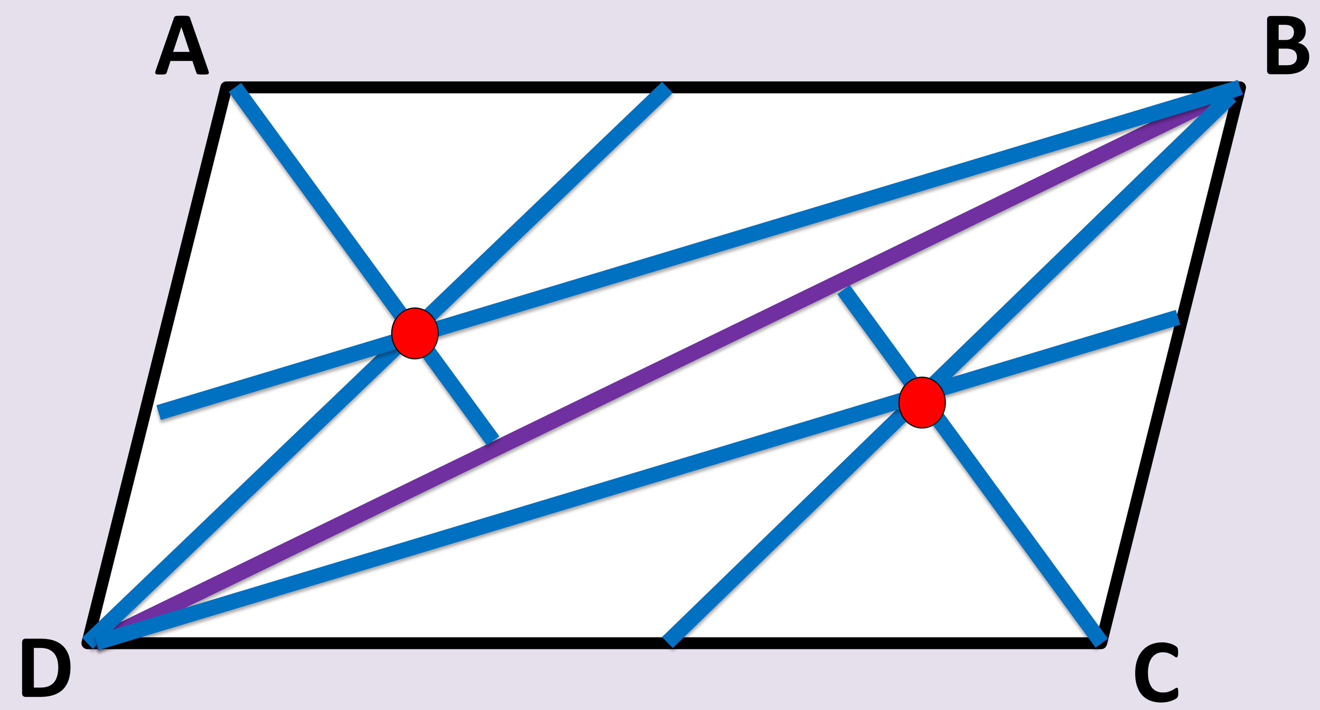
What about Quadrilaterals?
The natural next question we would like to answer is: Is it possible to fold the paper so that a quadrilateral drawn on the piece of paper can be cut out in one slice since the next size of polygon is a quadrilateral.

Types of Quadrilaterals:
Square, rectangle, trapezoid, parallelogram, rhombus, kite, etc.

Convex vs. Non-convex :
A convex polygon has all interior angles measuring less than 180°.

Does the method still work for quadrilaterals?
Yes! Including convex and non-convex quadrilaterals.

Proof:
Divides the arbitrary quadrilateral into two triangles with a diagonal of the quadrilateral, and the result follows from the proof for triangles. Here, the straight line to which we collapse the triangles will be the diagonal we chose earlier.



Note: Either choice for the diagonal will work!

Result

We can fold a piece of paper so that any convex polygon can be cut out using the Rabbit Ear Method. However, complications arise with non-convex polygons with more than 4 sides because the piece of paper could be ripped apart when we try to perform the swivel fold.