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A NEW SITE INDEX MODEL FOR INTENSIVELY MANAGED LOBLOLLY PINE (Pinus taeda) PLANTATIONS IN THE WEST GULF COASTAL PLAIN

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ABSTRACT

Loblolly pine (*Pinus taeda)* is the most important commercial species in the southern United States and as such, foresters must choose the most appropriate silvicultural prescriptions to maintain or improve the site quality of plantations and natural forests. The quality of a forest site can be estimated by several means, however, the most commonly used method is site index (SI). To date, there is no available SI model for intensively managed loblolly pine plantations in the Western Gulf Coastal Plain. To fill the gap, the scope of the East Texas Pine Plantation Research Project (ETPPRP) was expanded and permanent plots were established in intensively managed loblolly pine plantations across east Texas and western Louisiana. Using the data collected, this study developed a SI model specific to intensively managed loblolly pine plantations in the West Gulf Coastal Plain region. Data were fitted to six commonly used SI models: Schumacher Algebraic Difference Approach (ADA) model, Chapman-Richards ADA model, Schumacher Generalized Algebraic Difference Approach (GADA) model, Chapman-Richards GADA model, Cieszewski GADA model, and McDill-Amateis GADA model. Results showed that the Chapman-Richards GADA model and the McDill-Amateis GADA model were similar and best in their fit statistics. These two models were further compared to the existing models of Diéguez-Aranda et al. (2006) and Coble and Lee (2010), both of which were developed using data from extensively managed plantations and are currently utilized in forest management in the region. Both Chapman-Richards GADA and McDill-Amateis GADA models consistently predicted greater heights at younger ages on higher quality sites than the models of Diéguez-Aranda et al. (2006) and Coble and Lee (2010), however, the GADA models predicted shorter heights at older ages. Ultimately, the McDill-Amateis GADA model was chosen as the best model for its good fit statistics and ease of use. Foresters will be able to use this model to make silvicultural prescriptions better suited for intensively managed loblolly pine plantations in the West Gulf Coastal Plain.

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INTRODUCTION

The West Gulf Coastal Plain spans parts of four southern states: east Texas, western Louisiana, southeastern Oklahoma and southwestern Arkansas (The Nature Conservancy 2003). In east Texas and western Louisiana, the West Gulf Coastal Plain stretches from the western edge of the Mississippi River floodplain in Louisiana to the Trinity River in east Texas and north from the Gulf Coast to the rolling hills of northeast Texas and northern Louisiana (The Nature Conservancy 2003). Texas alone has 63.4 million acres of forestland with 12.3 million acres (approximately 20%) of that forestland located in east Texas (Parajuli et al. 2017). According to a 2015 study by the Texas A&M Forest Service, in 28 of the 43 east Texas counties, wood-based industries were in the top five employers within the manufacturing sector (Parajuli et al. 2017).

Forest industry is vital to the Texas economy, making the management of forests for sustainable output of products important as well. Sustainability is often described as the steady output of forest commodities in a non-declining flow over time (Clutter et al. 1983). The quality of a forest site is linked to sustainability through biotic and abiotic factors (Avery and Burkhart 2002), and the role of the forester is to ensure the site quality is not degraded through management. Direct measurements of site quality such as culmination of mean annual increment are difficult to quantify, so foresters often measure site quality indirectly as site index. Site index (SI) is an indirect relative measure of site quality in that it uses the height of dominant and codominant trees at a specific point in the life of the forest (i.e., index age or base age) as a surrogate for site quality. The primary reason to use height in developing SI is that height, particularly that of dominant and codominant trees, is minimally affected by stand density within the normal density range of operational forestry and is also related to the yield of a stand. The height and age of trees are also relatively easy to estimate.

In east Texas, SI models were first developed for loblolly pine (*Pinus taeda* L.) planted in old-field sites (i.e., abandoned agricultural land) (Lenhart 1971; Lenhart and Fields 1970). Subsequently, site index models were developed for loblolly pine and slash pine (*Pinus elliottii* Engelm.) planted on non-old-field sites (i.e., natural mixed pine hardwood forests converted to pine plantations) (Coble and Lee 2006; Coble and Lee 2010; Lenhart et al. 1986). These plantations were managed extensively in that low-intensity establishment practices were used such as mechanical piling and burning of logging slash and woods-run genetics (genetically unimproved) seedlings were planted. Extensive forest management aims to keep operating and investment costs low on a per acre basis. It involves the use of management procedures that ultimately encourage natural regeneration of a stand. Intensive forest management, however, attempts to maximize productivity on a per acre basis by utilizing a variety of forest management and silvicultural techniques. Intensive silvicultural activities such as planting genetically improved seedlings, applying fertilizer, bedding, and other procedures have been frequently applied to pine plantations (referred to as intensively managed plantations) across the southern United States, including the West Gulf Coastal Plain region. By the late 1990s and early 2000s, east Texas pine plantations were being re-established on cut-over sites that used more intensive establishment practices. Recently, Priest et al. (2016) published site indices for loblolly pine in east Texas, however, their work was limited to reclaimed mineland in only one county, and those stands were managed with relatively few silvicultural inputs. Though site index data for intensively-managed loblolly pine plantations have been collected by the East Texas Pine Plantation Research Project (ETPPRP) since 2004, no site index model has been developed for the West Gulf Coastal Plain. The purpose of this research project was to develop a new site index model for intensively-managed loblolly pine plantations growing on cut-over sites in the West Gulf Coastal Plain, specifically east Texas and western Louisiana.

OBJECTIVES

The purpose of this research project was to develop a new site index model for intensively-managed loblolly pine plantations growing on cut-over sites in the West Gulf Coastal Plain. The specific objectives are:

1. Organize the data that will be used to develop the new site index model from the ETPPRP database.
2. Develop a new site index model for intensively managed loblolly pine plantations in the West Gulf Coastal Plain.
3. Compare the new site index model to previously published models for the east Texas region and the south-wide range.
4. Prepare site index graphs that can be used by foresters working in the east Texas and western Louisiana regions.

LITERATURE REVIEW

Site Quality

Site quality commonly refers to the capacity of a site to grow trees and/or other vegetation. Forest or site productivity refers to the capacity of a site to aggrade biomass. Biotic and abiotic factors which could include wildlife, insects, microbes, soil parent material, soil chemistry, precipitation, sunlight exposure, existing vegetation, and climate play a role in determining site quality or forest productivity. Site quality has been assessed utilizing a variety of methods. Ideally, site quality would be measured directly using yields in the same way agricultural crops like wheat, corn, or soybeans are quantified. With agricultural crops, the crop is planted then harvested at the end of its growing season or rotation. This rotation is usually less than a year, and yield can be determined after harvest in terms of bushels per acre or a similar measure. In forestry, however, the crops, or trees, are grown on much longer rotations, typically anywhere from 25 to 80 years depending on the species of tree. Therefore, direct measurements are difficult if not impossible to obtain and foresters must rely on indirect methods of evaluating site quality. Various indirect measurements have been developed, such as plant associations and habitat types, soil-site relationships, biogeophysical relationships, and potential productivity such as culmination of mean annual increment (MAI). However, site index is the most widely used method (Avery and Burkhart 2002).

Site Index

Foresters most often use site index (SI) to indirectly measure forest or site quality. This is a vegetation-based method of assessing site quality defined as the average height (feet) of the dominant and codominant trees of a given species at an index age or base age (Avery and Burkhart 2002; Clutter et al. 1983). SI is based on Eichorn's Law which states that canopy height is strongly correlated with yield (volume) (Skovsgaard and Vanclay 2013). Trees are assumed to be phytometers in that they integrate all the biotic and abiotic site factors affecting growth and yield of the site. Height of trees in the upper canopy is minimally affected by stand density within the normal density range of forest management. Therefore, for even-aged pure stands, dominant and codominant tree height can be used as a surrogate for site quality because higher quality sites will have taller trees compared to poorer quality sites at an index age. Because of this strong correlation, SI is the most widely accepted and simplest method for estimating site quality (Diéguez-Aranda et al. 2006; Sharma et al. 2002).

Site index is an expression of the interaction of trees with edaphic factors (Krumland and Eng 2005). Selected trees, termed "site trees," are used to estimate site index. Site trees are chosen based on explicit criteria: they must be of the target species, in the dominant or codominant height class, free-growing having never been suppressed by overstory trees, and free of damage that would inhibit height growth (Avery and Burkhart 2002; Carmean and Hahn 1981; Carmean 1971, 1978; Hanson et al. 2003). The use of genetically-improved seedlings and improved cultural measures to increase tree growth can and does increase site index (Burger 2009; Eisenbies 2006; Krumland and Eng 2005; Zhao et al. 2016). However, cultural treatments can negatively alter the SI as well (Krumland and Eng 2005). Nevertheless, SI is still the most common measure of site quality and a critical component of growth and yield models.

Site Index Models

Site index has been used in North American forest management since the early 1900s. Miscellaneous Publication 50 (US Forest Service 1929, revised 1976) was the first comprehensive site index system published for major southern pines in second-growth natural forests across the South. Schnur (1937) developed site index curves for upland oaks in the central United States. McArdle and Meyer (1930) first published site index curves for Douglas-fir (*Pseudotsuga menziesii* Mirb.) in the Pacific Northwest in 1930, then revised them in 1949 and again in 1961 (McArdle et al. 1949; 1961). King (1966) developed site index curves for second-growth Douglas-fir in the Pacific Northwest. More site index curves were developed for other western tree species (Brickell 1966; Chojnacky 1986; Cochran 1979, 1985; Curtis et al. 1974; DeLasaux and Pillsbury 1987; DeMars and Herman 1987; Dolph 1983, 1987, 1991; Edminster and Jump 1976; Edminster et al. 1985). Additional site index curves were developed for other tree species in the central and eastern United States (Beck 1971; Carmean 1971, 1972, 1978; Cooley 1958; Doolittle and Vimmerstedt 1960; Kulow et al. 1966) and the southern United States (Bennett 1963, Coile and Schumacher 1964, Newberry and Pienaar 1978, Borders et al. 1984). Other site index curves can be found at the National Register of Site

Index Curves References (https://esi.sc.egov.usda.gov/html/fsregref.htm. Accessed 28 April 2015).

In the early 1970s, site index models for abandoned agricultural land or old-field loblolly pine plantations in northeast Texas and the Interior West Gulf Coastal Plain were developed (Lenhart 1971; Lenhart and Fields 1970). By 1986, approximately two million acres of mixed pine-hardwood had been harvested in east Texas and then replanted with loblolly and slash pine (Lenhart et al. 1986). Lenhart et al. (1986) published site index models for these non-old-field loblolly and slash pine plantations in east Texas. Coble and Lee (2006; 2010) published two sets of site index models for non-old-field loblolly and slash pine plantations in east Texas. They improved the models developed by Lenhart et al. (1986) and added newly acquired data from older plantations. While their models had been widely used in developing forest management plans, their new models still applied to only non-old-field loblolly and slash pine plantations established with low-intensity establishment practices (e.g., piling and burning slash piles, woods-run (genetically unimproved) seedlings). By the late 1990s and early 2000s, east Texas pine plantations were being established on cut-over pine plantation sites with more intensive silvicultural practices (e.g. chemical site preparation to control competing vegetation, bedding, mid-rotation thinning, and establishment fertilization) and genetically-improved pine seedlings. Eisenbies (2006) found that intensive silvicultural practices had doubled yields of loblolly pine plantations and decreased rotation lengths from 20-30 years to 15-20 years in some cases. At the end of World War II, large amounts of agricultural land had been abandoned due to poor soil productivity brought about by abusive agricultural practices. During the 1920's and 1930's, the United States Forest Service and other entities successfully replanted well over 1 million acres in the south and demonstrated the conservation value of trees. Forest nurseries run by states, industrial organizations and government agencies such as the United States Forest Service, provided high quality seedlings needed to continue reforestation efforts. Seed orchards and tree improvement programs were established in order to provide seedlings with improved volume growth, tree form, disease resistance, and wood quality. Prior to the 1950's, planting was mostly done on old fields while the seed tree method was used to regenerate cut-over sites albeit with poor success rates. Mechanical site preparation procedures attempted to replicate old field conditions and control competing hardwood vegetation. These methods increased the success of plantations established on cut-over sites, however, concerns of the impact on long-term site productivity led to chemical site preparation procedures gaining favor over mechanical procedures. Herbicides targeting herbaceous vegetation without harming pine seedlings were later developed and improved plantation success along with mid-rotation fertilization typically using nitrogen and phosphorous (Fox et al. 2007).

Bedding, for example, is an intensive silvicultural practice whose benefits in drainage, reduction of soil compaction, and competition control had been well-established (Eisenbies 2006). The rate at which site resources are made available to a plantation is one factor that governs stand growth, therefore, practices that increased the availability of those site resources effectively elevated site index (Eisenbies 2006). The long-term breeding programs of loblolly pine had significantly enhanced plantation productivity which may alter SI (Li et al. 1999; McKeand et al. 2003). Current genetic improvement programs are typically aimed at increasing the productivity of loblolly pine stands (Antony et al. 2013; McKeand et al. 2003). Significant gains in productivity had been realized with first generation and second generation planting stock (Antony et al. 2013; Li et al. 1999; McKeand et al. 2003). These gains effectively altered the site index of stands planted with these improved seedlings. Site index can also be affected by 1) increased carrying capacity with the use of P fertilizer at stand establishment on a P deficient site, and 2) accelerated stand development with mid-rotation N fertilization and/or bedding. The site index models developed by Lenhart et al. (1986) as well as Coble and Lee (2006; 2010) do not capture these improvements in genetics and silvicultural treatments now used in loblolly pine plantations established in the West Gulf Coastal Plain.

Site index models mathematically describe the height development of the upper-canopy trees across stand age for a range of site qualities. All site index models incorporate a base age or index age that is typically chosen to coincide with the average stand rotation. Commonly used index ages include 25 years, 50 years, or even 100 years for some western species. Statistical regression analysis procedures are used to estimate the coefficients of site index models (Clutter et al. 1983). There are two types of models or curves: anamorphic and polymorphic (Avery and Burkhart 2002; Clutter et al. 1983). Anamorphic site index curves have the same shape because they are proportional to each other. The assumption with anamorphic curves is that the height-age relationship is constant across the range of sites. Polymorphic site index curves are not proportional so they can differ in shape. This allows polymorphic curves to be more flexible to better represent different height-age relationships that apply to different sites.

Anamorphic, Base-Age Invariant Site Index Models

Bailey and Clutter (1974) introduced base-age invariance as a property of site index models using the Algebraic Difference Approach (ADA) method to derive base-age invariant site index models. ADA assigns one parameter in the base model, sometimes called the “Guide Curve”, as site-specific (local) parameter with the other parameters assigned as common (global) parameters. ADA site index models are typically anamorphic with a single asymptote.

Schumacher ADA Model

Schumacher (1939) proposed the first mathematical site index model used in North America. Schumacher (1939) included a logarithmic transformation on height to create a linear function with the reciprocal of age. The base form or guide curve equation of his model is:

$$\boldsymbol{H = e^{(\beta_0 + \beta_1 * A^{-1})}} \tag{1}$$

where,

H = total height (feet),

A = total age (years),

e = Euler's number = 2.71828, truncated at five decimal places, and

β_0, β_1 = regression parameters to be estimated.

Taking the natural logarithm of this equation gives:

$$\ln(H) = \beta_0 + \beta_1 * A^{-1} \tag{2}$$

To develop the Schumacher ADA site index model, first substitute the index age for age in the base model. Thus, the height at the index age is site index (S):

$$\ln(S) = \beta_0 + \beta_1 * A_i^{-1} \tag{3}$$

where A_i = index age (years), and all other variables are defined as before. The regression parameter, β_0, is the intercept of the equation also known as the site-specific or local parameter while β_1 is the slope of the equation also known as the global parameter. Solving for β_0 gives:

$$\beta_0 = \ln(S) - \beta_1 * A_i^{-1} \tag{4}$$

Substituting b_0 into the original equation gives the Schumacher anamorphic, base-age invariant height-age model:

$$\ln(H) = \ln(S) + \beta_1(A^{-1} - A_i^{-1}). \tag{5}$$

Inverting this equation to solve for site index gives the Schumacher anamorphic, base-age invariant site index model:

$$\mathbf{\ln(\boldsymbol{S}) = \ln(\boldsymbol{H}) + \boldsymbol{\beta_1}\left(\boldsymbol{A_i^{-1}} - \boldsymbol{A^{-1}}\right)}. \tag{6}$$

Coile and Schumacher (1964) used this anamorphic site index model for loblolly pine plantations growing on the Piedmont Plateau in the southeastern United States and obtained the following SI model:

$$\log(S) = \log(H) + 5.190(A^{-1} - A_i^{-1}) \tag{7}$$

where,

S = site index (feet),

H = total height of tree (feet),

A = total age (years),

A_i = Index or base age = 25 years, and

log = common log base 10.

Chapman-Richards ADA Model

One of the most widely used site index models today is the Chapman-Richards model. The base form or guide curve equation of this model is:

$$H = \beta_1 \left(1 - e^{-\beta_2 A}\right)^{\beta_3} \tag{8}$$

where all other variables are defined as before. The parameter, β_1, defines the asymptotic or maximum site index while the parameter, β_2, describes the rate, and the parameter, β_3, describes the shape of the curve.

To develop the ADA Chapman-Richards site index model, first substitute the index age for age in the base model. Thus, the height at the index age is site index (S):

$$S = \beta_1 \left(1 - e^{-\beta_2 A_i}\right)^{\beta_3}. \tag{9}$$

where all other variables are defined as before. The asymptote can be considered to vary across sites, so it can be isolated to allow site index to vary across sites while keeping the curve shape constant. Solving for β_1 gives:

$$\beta_1 = S \left(1 - e^{-\beta_2 A_i}\right)^{-\beta_3}. \tag{10}$$

Substituting β_1 into the original equation gives the Chapman-Richards anamorphic, base-age invariant height-age model:

$$H = S\left(\frac{1-e^{-\beta_2 A}}{1-e^{-\beta_2 A_i}}\right)^{\beta_3} . \qquad (11)$$

Inverting this equation to solve for site index gives the Chapman-Richards anamorphic, base-age invariant site index model:

$$S = H\left(\frac{1-e^{-\beta_2 A_i}}{1-e^{-\beta_2 A}}\right)^{\beta_3} . \qquad (12)$$

Newberry and Pienaar (1978) developed an equation for site-prepared slash pine plantations of the Atlantic and Gulf Coastal Plain of Georgia and Florida using this model:

$$S = H\left(\frac{1-e^{-0.100354(A_i)}}{1-e^{-0.100354(A)}}\right)^{1/1-0.516188} . \qquad (13)$$

Newberry and Pienaar (1978) found that the Chapman-Richards model proved to be more flexible than earlier models because it more closely followed the height growth pattern of trees over time.

Lenhart et al. (1986) used the Chapman-Richards model to describe site index for loblolly pine plantations growing in east Texas on non-old-field sites:

$$S = H\left(\frac{1-e^{-0.08005(A_i)}}{1-e^{-0.08005(A)}}\right)^{1.62857} . \qquad (14)$$

Lenhart et al. (1986) found that this site index model was a great improvement over earlier site index models for east Texas loblolly pine plantations.

Schnute ADA Model

Coble and Lee (2006) developed an equation for non-old-field loblolly pine plantations growing in east Texas. They used a generalized sigmoid growth model first presented by Schnute (1981):

$$S = \left(0.99476^{0.68232} + \left(H^{0.68232} - 0.99476^{0.68232} \right) \frac{1 - e^{-0.08036(A_i - 1)}}{1 - e^{-0.08036(A - 1)}} \right)^{1/0.68232} \quad (15)$$

where all variables are defined as before. The Schnute model was an improvement over the Chapman-Richards model of Lenhart et al. (1986). Coble and Lee (2010) improved on their 2006 model using a self-referencing technique described by Northway (1985) to account for serial autocorrelation, which is the relationship of repeated height observations on the same tree through time because they are not independent observations.

Polymorphic, Base-Age Invariant Models

The models described so far are all anamorphic (i.e., same shape) and base-age invariant (Bailey and Clutter 1974), which means the equation can be developed to compute predictions from any height-age pair without having to already know or previously measure the site index. Base-age invariance allows for all height-age pairs to be used in the model fitting process, not just those pairs with heights measured at an index age. Though base-age invariant, anamorphic models were an improvement in describing height development patterns over time, they still described curves with one shape. Polymorphic site index models (i.e., more than one shape) offer more flexibility than anamorphic models in describing height development patterns over time. Krumland and Eng (2005) developed several base-age invariant, polymorphic site index models for a variety of tree species in California. Diéguez-Aranda et al. (2006) developed a base-age invariant, polymorphic site index model for loblolly pine across its native range in the South. Cieszewski (2001) developed a base-age invariant, polymorphic site index model for Douglas-fir that performed better than anamorphic models.

Cieszewski and Bailey (2000) introduced the Generalized Algebraic Difference Approach (GADA) which allows more than one parameter to be local or site-specific. GADA site index models can be polymorphic with multiple asymptotes, which is the main advantage over ADA. In GADA, a variable is introduced that represents the unobservable site index value. This unobserved variable need not be defined because it is only used in intermediate steps and later replaced by a function of initial conditions and other global parameters.

Schumacher GADA Model

As stated before, the base form or guide curve equation of Schumacher's model is:

$$\boldsymbol{H} = \boldsymbol{e}^{\left(\boldsymbol{\delta_0}+\boldsymbol{\delta_1}*\left(\boldsymbol{1}/\boldsymbol{A}\right)\right)} = \boldsymbol{e}^{\left(\boldsymbol{\delta_0}+\boldsymbol{\delta_1}*\boldsymbol{A}^{-\boldsymbol{1}}\right)} \tag{16}$$

where δ_0 and δ_1 are model parameters and all other variables defined as before.

To create the GADA solution of the Schumacher model, first make δ_0 and δ_1 both local parameters by replacing δ_0 with an unobserved site quality variable, X, and δ_1 with a linear function of X, $\beta_1 + \beta_2$*X:

$$H = e^{\left(X+(\beta_1+\beta_2*X)A^{-1}\right)} \ . \tag{17}$$

Taking the natural logarithm and solving for X gives:

$$X = \frac{(\ln(H) - \beta_1 A^{-1})}{1 + \beta_2 A^{-1}}. \quad (18)$$

Substituting the initial conditions H_0 and A_0 into the equation for X gives:

$$X_0 = \frac{(\ln(H_0) - \beta_1 A_0^{-1})}{1 + \beta_2 A_0^{-1}}. \quad (19)$$

Replace X in the GADA solution with X_0 to create a polymorphic, base-age invariant formulation of the Schumacher height-age model:

$$\boldsymbol{H = e^{(X_0 + (\beta_1 + \beta_2 * X_0) A^{-1})}} \quad (20)$$

where X_0 is defined as the function above and all other variables are defined as before.

Chapman-Richards GADA Model

As stated before, the base form or guide curve equation of the Chapman-Richards model is:

$$H = \delta_1 \left(1 - e^{\delta_2 A}\right)^{\delta_3} \quad (21)$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables are defined as before.

To create the GADA solution of the Chapman-Richards model, first make δ_1 and δ_3 both local parameters by replacing δ_1 with an exponential function of the unobserved site quality variable, X, and δ_3 with a linear inverse function of X or $\beta_3 + \beta_4 / X$. The parameter δ_2 is estimated as a global parameter, β_2. The GADA formulation is:

$$H = e^X \left(1 - e^{\beta_2 A}\right)^{(\beta_3 + \beta_4 / X)}. \tag{22}$$

First, take the natural logarithm of both sides of the equation and solve for X:

$$\ln(H) = \ln\left(e^X \left(1 - e^{\beta_2 A}\right)^{(\beta_3 + \beta_4 X^{-1})}\right) \tag{23}$$

$$\ln(H) = \ln\left(e^X\right) + \ln\left(\left(1 - e^{\beta_2 A}\right)^{(\beta_3 + \beta_4 X^{-1})}\right) \tag{24}$$

$$\ln(H) = X + \left(\beta_3 + \beta_4 X^{-1}\right) * \ln\left(1 - e^{\beta_2 A}\right) \tag{25}$$

$$\ln(H) = X + \beta_3 \ln\left(1 - e^{\beta_2 A}\right) + \beta_4 X^{-1} \ln\left(1 - e^{\beta_2 A}\right) \tag{26}$$

$$\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right) = X + \beta_4 X^{-1} \ln\left(1 - e^{\beta_2 A}\right) \tag{27}$$

$$\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right) = X^{-1}\left(X^2 + \beta_4 \ln\left(1 - e^{\beta_2 A}\right)\right) \tag{28}$$

$$X\left(\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right)\right) = X^2 + \beta_4 \ln\left(1 - e^{\beta_2 A}\right) \tag{29}$$

$$X^2 - \left(\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right)\right)X + \beta_4 \ln\left(1 - e^{\beta_2 A}\right) = 0 \tag{30}$$

Solving for X requires a quadratic solution. First, let:

$$a = 1, \tag{31}$$

$$b = -\left(\ln(H) - \beta_3 \ln\left(1 - e^{\beta_2 A}\right)\right), \tag{32}$$

$$c = \beta_4 \ln\left(1 - e^{\beta_2 A}\right). \tag{33}$$

Then, use the quadratic formula to find the solution:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{34}$$

$$X = \frac{-(-(\ln H) - \beta_3 \ln(1 - e^{\beta_2 A})) \pm \sqrt{(-(\ln(H) - \beta_3 \ln(1 - e^{\beta_2 A}))^2 - 4 * 1 * \beta_4 \ln(1 - e^{\beta_2 A})}}{2 * 1} \tag{35}$$

$$X = \frac{(\ln(\mathrm{H}) - \beta_3 \ln(1 - e^{\beta_2 A})) \pm \sqrt{(\ln(\mathrm{H}) - \beta_3 \ln(1 - e^{\beta_2 A}))^2 - 4\beta_4 \ln(1 - e^{\beta_2 A})}}{2} \tag{36}$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X and take the roots most likely to be positive and real:

$$X_0 = \frac{(\ln(H_0) - \beta_3 \ln(1 - e^{\beta_2 A_0})) + \sqrt{(\ln(H_0) - \beta_3 \ln(1 - e^{\beta_2 A_0}))^2 - \beta_4 \ln(1 - e^{\beta_2 A_0})}}{2} \tag{37}$$

Solve for δ_1 in the initial condition formulation of the model, and express in terms of the GADA formulation:

$$H_0 = \delta_1 \left(1 - e^{\delta_2 A_0}\right)^{\delta_3}, \tag{38}$$

$$\delta_1 = H_0 \left(1 - e^{\delta_2 A_0}\right)^{-\delta_3}, \tag{39}$$

$$\delta_1 = e^X = H_0 \left(1 - e^{\beta_2 A_0}\right)^{-\left(\beta_2 + \beta_3 X_0^{-1}\right)}. \tag{40}$$

Then, substitute this initial condition for δ_1 into the original GADA formulation of the model to create a polymorphic, base-age invariant formulation of the Chapman-Richards height-age model:

$$H = H_0 \left(\frac{1 - e^{\beta_2 A}}{1 - e^{\beta_2 A_0}} \right)^{\left(\beta_2 + \beta_3 X_0^{-1}\right)}, \tag{41}$$

where X_0 is defined as the function above, $H_0 = S$, $A_0 = A_i$, and all other variables are defined as before.

<u>Cieszewski GADA Model</u>

Cieszewski (2001, 2002, 2003) examined several GADA formulations of Hossfeld models, also known as log-logistic models. The base form of the Hossfeld equation that performed best (Cieszewski 2002) is:

$$H = \frac{\delta_1}{1 + e^{\delta_2 + \delta_3 \ln(A)}} = \frac{\delta_1}{1 + e^{\delta_2} A^{\delta_3}} \tag{42}$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables defined as before.

To create the GADA solution of the Hossfeld model, first make δ_1 and δ_2 both local parameters by replacing δ_1 with a constant plus the unobserved site quality variable, X, and e^{δ_2} with β_2 / X. The parameter δ_3 is estimated as a global parameter, β_3. The GADA formulation is:

$$H = \frac{\beta_1 + X}{1 + \frac{\beta_2}{X} A^{\beta_3}} = \frac{\beta_1 + X}{1 + \beta_2 X^{-1} A^{\beta_3}} \tag{43}$$

To solve for X, first let $Y = A^{b_3}$:

$$H = \frac{\beta_1 + X}{1 + \beta_2 X^{-1} Y} \tag{44}$$

Then,

$$H = \frac{\beta_1 + X}{X^{-1}\left(X + \beta_2 Y\right)} \tag{45}$$

$$H = \frac{X\left(\beta_1 + X\right)}{X + \beta_2 Y} \tag{46}$$

$$H\left(X + \beta_2 Y\right) = X\left(\beta_1 + X\right) \tag{47}$$

$$HX + H\beta_2 Y = X\beta_1 + X^2 \tag{48}$$

$$X^2 + \beta_1 X - HX - H\beta_2 Y = 0 \tag{49}$$

$$X^2 + \left(\beta_1 - H\right)X - H\beta_2 Y = 0 \tag{50}$$

$$X^2 - \left(H - \beta_1\right)X - H\beta_2 Y = 0 \tag{51}$$

Solving for X requires a quadratic solution. First, let:

$$a = 1, \tag{52}$$

$$b = -\left(H - \beta_1\right), \tag{53}$$

$$c = -H\beta_2 Y . \tag{54}$$

Then, use the quadratic formula to find the solution:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{55}$$

$$X = \frac{-(-(H-\beta_1)) \pm \sqrt{(-(H-\beta_1))^2 - 4*1*-H\beta_2 Y}}{2*1}. \tag{56}$$

$$X = \frac{(H-\beta_1) \pm \sqrt{(H-\beta_1)^2 + 4H\beta_2 Y}}{2} = \frac{(H-\beta_1) \pm \sqrt{(H-\beta_1)^2 + 4H\beta_2 A^{\beta_3}}}{2}. \tag{57}$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X and take the roots most likely to be positive and real:

$$X_0 = \frac{(H_0-\beta_1) + \sqrt{(H_0-\beta_1)^2 + 4H_0\beta_2 A_0^{\beta_3}}}{2}. \tag{58}$$

Replace X in the GADA solution with X_0 and simplify to create a polymorphic, base-age invariant formulation of the Cieszewski-Hossfeld height-age model:

$$H = \frac{\beta_1 + X_0}{1 + \beta_2 X_0^{-1} A^{\beta_3}}, \tag{59}$$

where X_0 is defined as the function above, H_0 = S, A_0 = A_i, and all other variables are defined as before.

McDill and Amateis GADA Model

McDill and Amateis (1992) proposed another variant of the Hossfeld model that only considers δ_2 as the local parameter in the Cieszewski (2002) GADA model. As before, the base form of the Cieszewski (2002) model is:

$$H = \frac{\delta_1}{1 + e^{\delta_2} A^{\delta_3}} \tag{60}$$

where δ_1, δ_2, and δ_3 are model parameters and all other variables defined as before.

To create the GADA solution of the McDill-Amateis model, first make δ_2 the local parameter by replacing e^{δ_2} with β_2 / X, where X is the unobserved site quality variable. The parameters δ_1 and δ_3 are estimated as global parameters, β_1 and β_3, respectively. The GADA formulation is:

$$H = \frac{\beta_1}{1 + \frac{\beta_2}{X} A^{\beta_3}} = \frac{\beta_1}{1 + \beta_2 X^{-1} A^{\beta_3}} \tag{61}$$

To solve for X, first let $Y = A^{\beta_3}$:

$$H = \frac{\beta_1}{1 + \beta_2 X^{-1} Y}. \tag{62}$$

Then,

$$H = \frac{\beta_1}{X^{-1}\left(X + \beta_2 Y\right)} \tag{63}$$

$$H = \frac{X\beta_1}{X + \beta_2 Y} \tag{64}$$

$$H\left(X + \beta_2 Y\right) = X\beta_1 \tag{65}$$

$$HX + H\beta_2 Y = X\beta_1 \tag{66}$$

$$HX - X\beta_1 = -H\beta_2 Y \tag{67}$$

$$X\left(H - \beta_1\right) = -H\beta_2 Y \tag{68}$$

$$X = \frac{-H\beta_2 Y}{H - \beta_1} \tag{69}$$

$$X = \frac{-H\beta_2 Y}{H\left(1 - \beta_1 H^{-1}\right)} \tag{70}$$

$$X = \frac{-\beta_2 Y}{1 - \beta_1 H^{-1}} = \frac{-\beta_2 A^{\beta_3}}{1 - \beta_1 H^{-1}} \tag{71}$$

Next, substitute the initial conditions for X_0, A_0, and H_0 in the equation for X:

$$X_0 = \frac{-\beta_2 A_0^{\beta_3}}{1 - \beta_1 H_0^{-1}} \tag{72}$$

Replace X in the GADA solution with X_0 and simplify to create a polymorphic, base-age invariant formulation of the McDill-Amateis height-age model:

$$H = \frac{\beta_1}{1 + \beta_2 \left(\frac{-\beta_2 A_0^{\beta_3}}{1 - \beta_1 H_0^{-1}} \right)^{-1} A^{\beta_3}} \tag{73}$$

$$H = \frac{\beta_1}{1 + \beta_2 \left(\frac{1 - \beta_1 H_0^{-1}}{-\beta_2 A_0^{\beta_3}} \right) A^{\beta_3}} \tag{74}$$

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A^{\beta_3}}{A_0^{\beta_3}} \right)} \tag{75}$$

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A}{A_0} \right)^{\beta_3}} . \tag{76}$$

Since β_3 is a parameter to be estimated, the McDill-Amateis model can also be expressed as:

$$H = \frac{\beta_1}{1 - \left(1 - \beta_1 H_0^{-1}\right)\left(\frac{A_0}{A} \right)^{\beta_3}} \tag{77}$$

where H_0 = S, A_0 = A_i, and all other variables are defined as before.

East Texas Pine Plantation Research Project

The East Texas Pine Plantation Research Project (ETPPRP) was established in 1982 by J. David Lenhart in the Arthur Temple College of Forestry and Agriculture at Stephen F. Austin State University and four participating industrial collaborators. The purpose was to provide growth and yield information for planted pine plantations across east Texas (Lenhart et al. 1986). The intent of the ETPPRP was to install permanent plots in young pine plantations throughout east Texas so as to have a range of sites available for growth and yield research. Key components of the ETPPRP include: collect data during the life of the plantations, track wood production over time, and develop models to estimate growth and yield of these plantations. Regular data summaries and analyses are provided to ETPPRP participants while information and quantitative tools are provided to east Texas foresters and plantation landowners.

In the ETPPRP's Phase 1 study which initiated in 1982 and terminated in 2016, permanent plots were installed in extensively managed pine plantations across east Texas. To fill knowledge gaps in growth and yield models, beginning in 2004, ETPPRP researchers began to establish new permanent plots (Phase 2 plots) in young cut-over, intensively managed loblolly pine plantations across east Texas and western Louisiana. Plot locations were selected to represent geologic formations and a range of soil drainage classes common to the region. Furthermore, within geologic formations and drainage classes, plots were selected to represent a range of site establishment practices that included mechanical and chemical competition control, fertilization at site establishment, and bedding. The specific establishment practices and dates of application were available for each selected plantation for the project.

Similar to the Phase 1 plots, trees at each Phase 2 plot were measured when the plot was installed and remeasured every three years thereafter. Data collection is ongoing. The focus of this new effort was to determine the growth and yield of intensively managed pine plantations in the West Gulf Coastal Plain. These new data will deliver the necessary information to better provide growth and yield information for the management of genetically-improved, intensively-managed loblolly pine plantations in the West Gulf Coastal Plain. A 2017 study by Coble et al. found that loblolly pine trees planted in the intensively managed Phase 2 plots are reaching greater heights faster than those in the extensively managed Phase 1 plots (Figure 1). This further justifies the need for updated site index curves developed specifically for intensively managed plantations.

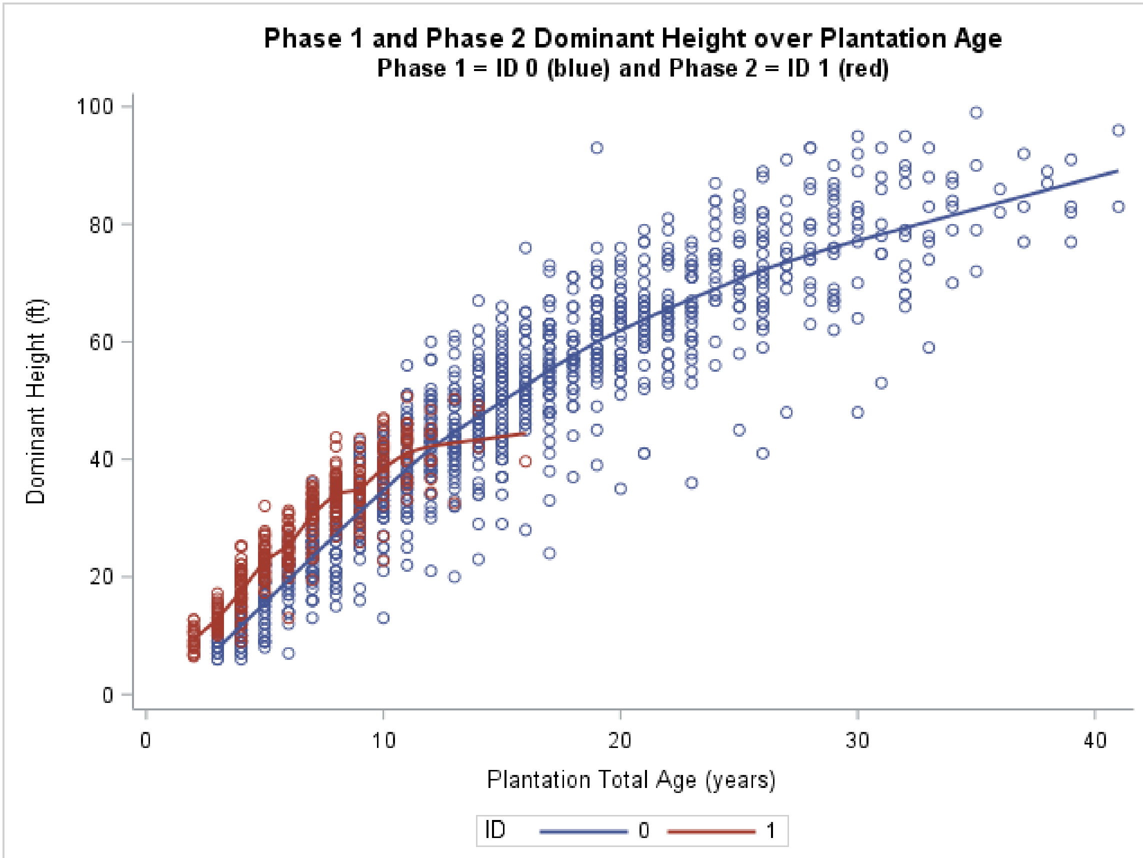

Figure 1. Average height for intensive (Phase 2) and extensive (Phase 1) ETPPRP plots with average trend lines.

METHODS

Data Description

The Phase 2 plots are distributed across east Texas and into western Louisiana to best represent the growing conditions unique to the Interior West Gulf Coastal Plain (Figure 2. At each study location, one permanent square plot measuring approximately 0.25 acres (approximately 100 foot by 100 foot) was installed (Appendix A). The following items were recorded for each plot: plantation establishment date, initial planting density, slope, aspect, landform, geographic location (UTM NAD 83 Zone 15 coordinates with GPS), and stand history. The plots were installed in such a way that plot boundaries run parallel/perpendicular to the planting rows. Plot boundaries running parallel to the planted rows were located at the midpoint of the rows, as much as possible. Some exceptions occurred where rows were established along contours. As of 2017, there were a total of 133 Phase 2 plots available for analysis. Of this total, 125 plots were actively measured and 8 had been logged or otherwise disturbed, but still provided data for analysis.

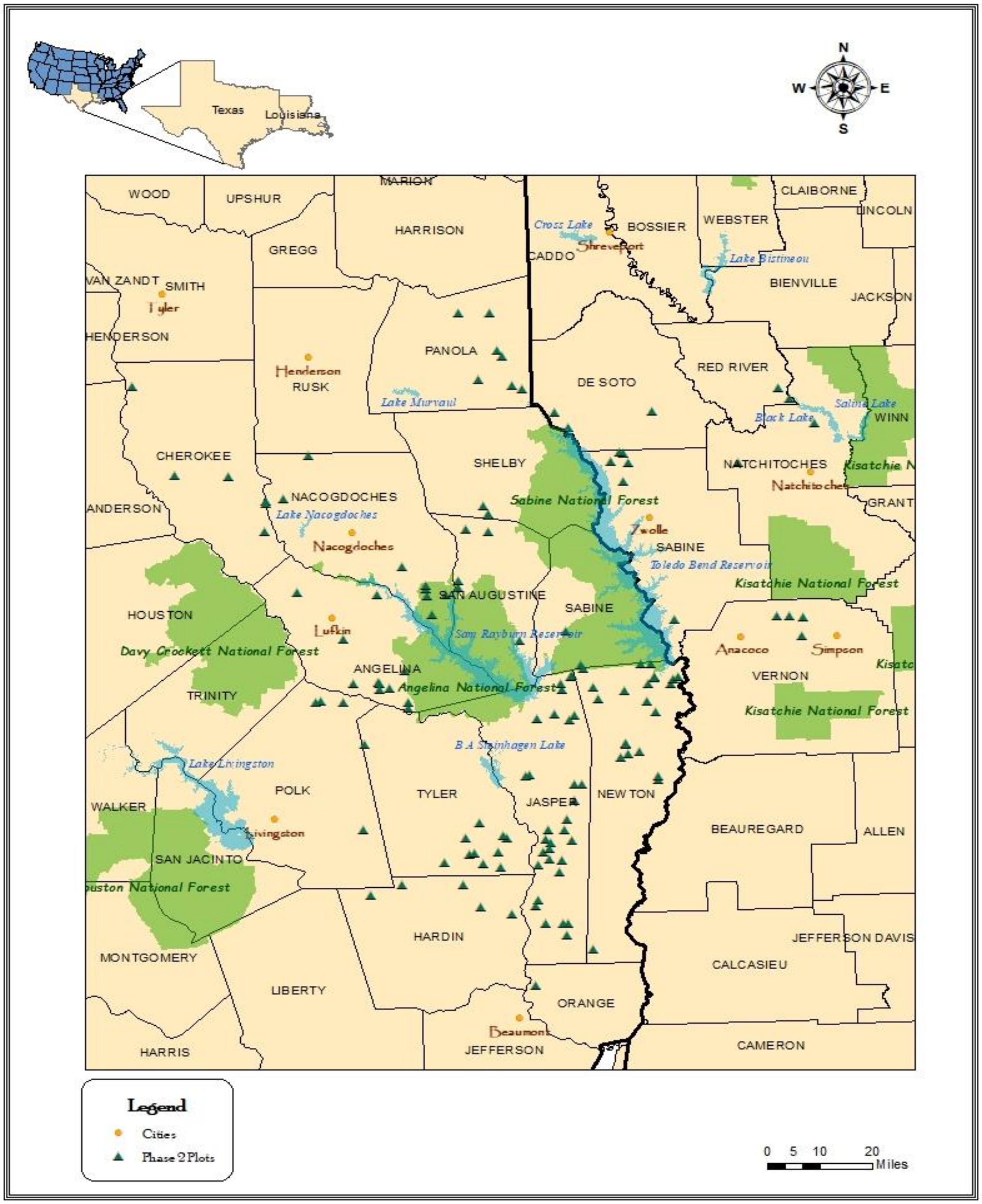

Figure 2. Map depicting the locations of Phase 2 plots of the East Texas Pine Plantation Research Project.

At each plot, both planted pine trees and non-planted trees (>4-inch dbh) were permanently marked with numbered aluminum tags. When the plot was installed and every three years thereafter, diameter at breast height (dbh; nearest 0.1 inch) was measured while height (nearest 1.0 foot), and crown length (nearest 1.0 foot) were estimated. The trees' crown class (dominant, codominant, intermediate, suppressed) as well as the presence of fusiform rust (yes or no, on stem or branch), and any damage to the tree were also recorded (Coble and Pendergast 2013).

This study used 469 longitudinal observations from 133 unthinned Phase 2 plots (Table 1). Individual tree measurements from each plot were summarized to obtain dominant height for each measurement cycle and plantation total age was obtained from known plantation history. Dominant height (feet) was determined by averaging the total heights of the dominant and co-dominant trees that were free of damage that affected height growth (e.g., broken tops, dead tops, forks, fusiform rust). Plantation total age (years) was determined as the total time between the current measurement date and the plantation establishment date.

Table 1. Observed stand characteristics for east Texas and western Louisiana loblolly pine plantations established on cut-over sites.
Based on N=469 observations made from 133 plots in the ETPPRP Phase 2 database.

Variables	Mean	SD	Minimum	Maximum
Age	8.1	3.6	2.0	19.0
Hd	32.0	12.3	6.5	63.4
TPA	524.2	100.0	326.7	858.1
BAPA	79.5	44.8	1.2	184.3

Note: Age=plantation age, Hd=height of dominant and codominant trees (feet), TPA=trees per acre, BAPA=basal area per acre (ft^2), SD=standard deviation

Data Analysis

Six models: Schumacher ADA, Chapman-Richards ADA, Schumacher GADA, Chapman-Richards GADA, Cieszewski GADA, and McDill-Amateis GADA were selected to model site index. This was a decision based on previous studies of the ETPPRP Phase 1 data (Lenhart et al. 1986). Data were fitted to each of the above models using PROC NLIN of SAS version 9.4 procedure (SAS 2017).

Serially correlated data, also known as longitudinal data, are repeated measurements made on the sampling unit over time. In this study, total heights of individual dominant and codominant site trees that were averaged to obtain a dominant height for each plot every three years represent the serially correlated data. These repeated measurements are not independent of each other because each plot contributed a series of measurements over time. Measurements that occurred closer together in time are more highly correlated than measurements farther apart in time. Special statistical procedures are needed when the independence condition is violated by the presence of serial correlation. If serial correlation was ignored, the coefficients in the model would still be unbiased, but their variances would be inflated which creates problems for statistical inference (e.g., is the coefficient significantly different from zero?). Other than the issue related to independence, heteroscedasticity could also be a problem that inflates variances with measurements made over time. Heteroscedasticity is present when the size of the error term differs across values of an independent variable.

Northway (1985) presented a methodology for fitting dynamic functions to serially correlated data that had been used in some form by others (Coble and Lee 2010; Krumland and Eng 2005; Strub and Cieszewski 2002) that could address both serial autocorrelation and heteroscedasticity. Northway's (1985) methodology required an estimate of H_0 at A_0 prior to the fitting process, which was a problem since H_0 at A_0 were rarely measured in the field. He referred to this estimate of H_0 and A_0 as site index (S) at the index age (A_i). Each remeasured ETPPRP plot provided a growth series from which estimates of S were calculated during the iterative nonlinear fitting process. Each record in the dataset contained a single height-age pair from a plot, along with its entire growth series, which was every height-age pair from a plot measured over time. This growth series was used to estimate S for each height-age pair. The GADA models of Schumacher, Chapman-Richards, Cieszewski, and McDill and Amateis were fitted to these height-age pairs. All non-linear procedures were run in PROC NLIN of SAS version 9.4. Residual plots were viewed to verify that this procedure minimized the effects of serial correlation and heteroscedasticity.

To estimate S for each height-age pair, initial estimates of the regression coefficients were first set equal to the starting values (obtained previously from an initial model fit using all the data) in the iterative nonlinear fitting process, and they were changed with successive iterations. Within each iteration, conditional site index estimates (CSI) were used in the equation being fit. Heights were predicted for the entire growth series for the CSIs. The squared differences (observed – predicted) in height were then calculated. The values of CSI for the current iteration that minimized the squared differences were used as final S estimates to estimate new values of the regression coefficients for the next iteration. This process was repeated until the least squares error for the overall regression was minimized. Thus, CSI was the estimate of site index that minimized squared differences of serially correlated observations, given the current coefficient estimates. Therefore, the procedure simultaneously estimated S for the growth series and CSI used in the function. Final CSI values were local estimates (i.e., plot or site-specific estimates) of the height at the index age (25 years in this study) for each growth series. Model parameter estimates were evaluated at alpha=0.05 to determine their significance.

Model Comparison

The data were fitted to the Cieszewski GADA model, both Chapman-Richards ADA and GADA models, both Schumacher ADA and GADA models, as well as the McDill-Amateis GADA model. These six models were evaluated to determine which model fit the Phase 2 data best. The models were compared based on statistical and visual analyses of the model residuals. Four fit statistics were used in this study: root mean square error (RMSE) which measured model precision, the coefficient of determination for nonlinear models (pseudo-R^2) which measured the amount of variability in the dependent variable explained by the independent variable, and Akaike's information criterion (AIC) (Akaike 1974) which measured the goodness of fit of an estimated statistical model. AIC was a tool that allowed the selection of the best fit model from a pool of candidate models. AIC differences (Δ*i*) allows candidate models to be ranked and thus compared to find the best fit model with larger Δ*i* denoting poorer models. Δ*i* are found by subtracting the lowest AIC value from the AIC of each model. A Δ*i* of 0-2 indicates a substantial level of support for the model, 4-7 indicates considerably less support, and >10 indicates essentially no support for the model (Burnham and Anderson 2003). The model with the lowest RMSE, AIC, and Δ*i* values as well as the highest pseudo-R^2 value was considered the best fit model.

Ideally, each model would be evaluated based on its ability to predict responses for a set of independent data. Since no independent data were available for this project, the ETPPRP data could have been split into a model fitting data set and a model validation data set. The model would have been fit with the former and validated with the latter. However, Kozak and Kozak (2003) showed that this splitting technique as well as cross-validation techniques for model validation did not provide any additional information about the model beyond what ordinary fit statistics provided from the model fit with the entire data set. Therefore, data splitting or cross-validation techniques were not used in this study to determine the best fit model.

Comparison to Other Studies

It was expected that the best fit model in this study would have better predictability than the models currently being used in east Texas, thus the best fit model was further compared to the base-age invariant, polymorphic model for loblolly pine throughout the south developed by Diéguez-Aranda et al. (2006). The purpose of this comparison was to determine which model best represents height development patterns in east Texas loblolly pine plantations. The best fit model of this study was also compared to the base-age invariant, anamorphic model for non-old-field loblolly pine plantations in east Texas developed by Coble and Lee (2010). The purpose of this comparison was only for a point of reference since the loblolly pine trees represented in the two studies characterize different silvicultural activities including genetically different populations. However, it was a useful comparison since the model of Coble and Lee (2010) is still widely used by foresters in east Texas. The comparison was based on visual analysis since no independent data were available to evaluate the models across a range of site index and plantation age values.

RESULTS

These plantations were young in that they range in age from 2 to 19 years with an average age of 8.1 years. The average height of the dominant and codominant trees was 32.0 feet, the average stand density was 524.2 trees per acre, and the average basal area was 79.5 ft^2 per acre (Table 1).

The R^2 values of all the models were >0.97 indicating all the models produce very good predictions of the height-age relationship (Table 2). All models displayed low RMSE values (<2.4 ft.) which again indicates that all models are good. The Chapman-Richards GADA, Cieszewski GADA, and McDill-Amateis GADA models resulted in the lowest values of 1.5 feet. The Chapman-Richards ADA model had a slightly larger value of 1.6 feet, and the Schumacher ADA and GADA models had the highest RMSE values of 2.3 feet. The differences between the models are best judged using the Akaike Information Criterion (AIC), a measure of model fit that considers bias and precision (low AIC scores = best fit model) and delta AIC. The Schumacher ADA and GADA models had the largest AIC values at 801.5 and 783.2, respectively. The AIC values for the other models were much lower; 436.5 for the Chapman-

Richards ADA model, 410.0 for the Chapman-Richards GADA model, and 410.2 McDill-Amateis GADA model, and 408.6 for the Cieszewski GADA model. The delta AIC (Δi) values were calculated for each model with the Cieszewski GADA model having the most support at 0.00. The Chapman-Richards GADA and McDill-Amateis GADA models had Δi values of 1.5 and 1.7 respectively which indicated a substantial level of support for the models. The remaining models all had Δi values greater than 10 meaning the models had very little support as candidates for the best fit model. However, for the Cieszewski GADA model, the parameter estimate for β_2 is not significant (the 95% confidence interval contains zero) which means the formulation for the local parameter δ_2 is not correct for these data.

Table 2. Parameter estimates and fit statistics for the equations where SE= standard error, CI= confidence interval, R^2= coefficient of determination, RMSE= root mean square error, AIC= Akaike information criterion, and Δ_i = AIC differences.

Model	Parameter	Estimate	SE	95% CI		R^2	RMSE (ft.)	AIC	Δ_i
Schumacher ADA Equation [6]	b1	-5.765	0.077	-5.917	-5.613	0.966	2.338	801.483	392.923
Chapman-Richards ADA Equation [12]	b2	1.122	0.034	1.055	1.190	0.984	1.584	436.514	27.954
	b3	0.074	0.007	0.060	0.087				
Schumacher GADA Equation [20]	b1	-34.408	9.051	-52.193	-16.623	0.968	2.293	783.237	374.677
	b2	6.663	2.106	2.525	10.801				
Chapman-Richards GADA Equation [41]	b1	0.079	0.007	0.067	0.092	0.984	1.540	410.025	1.465
	b2	-1.909	0.773	-3.428	-0.390				
	b3	13.418	3.395	6.746	20.090				
Cieszewski GADA Equation [59]	b1	92.273	19.900	53.169	131.400	0.985	1.538	408.560	0.000
	b2	892.300	904.300	-884.700	2669.400				
	b3	1.184	0.032	1.121	1.248				
McDill-Amateis GADA Equation [77]	b1	112.100	6.548	99.216	125.000	0.984	1.540	410.237	1.677
	b3	1.173	0.030	1.114	1.232				

Models were further compared with residual plots (Figures 2-7). The residuals for the Chapman-Richards ADA model (Figure 4), Chapman-Richards GADA model (Figure 6), Cieszewski GADA model (Figure 7), and McDill-Amateis GADA model (Figure 8) all indicated no evidence of bias, autocorrelation, or heteroscedasticity. The residuals for both Schumacher models (Figures 3 and 5), however, exhibited curvilinear trends, which was indicative of bias from serial autocorrelation.

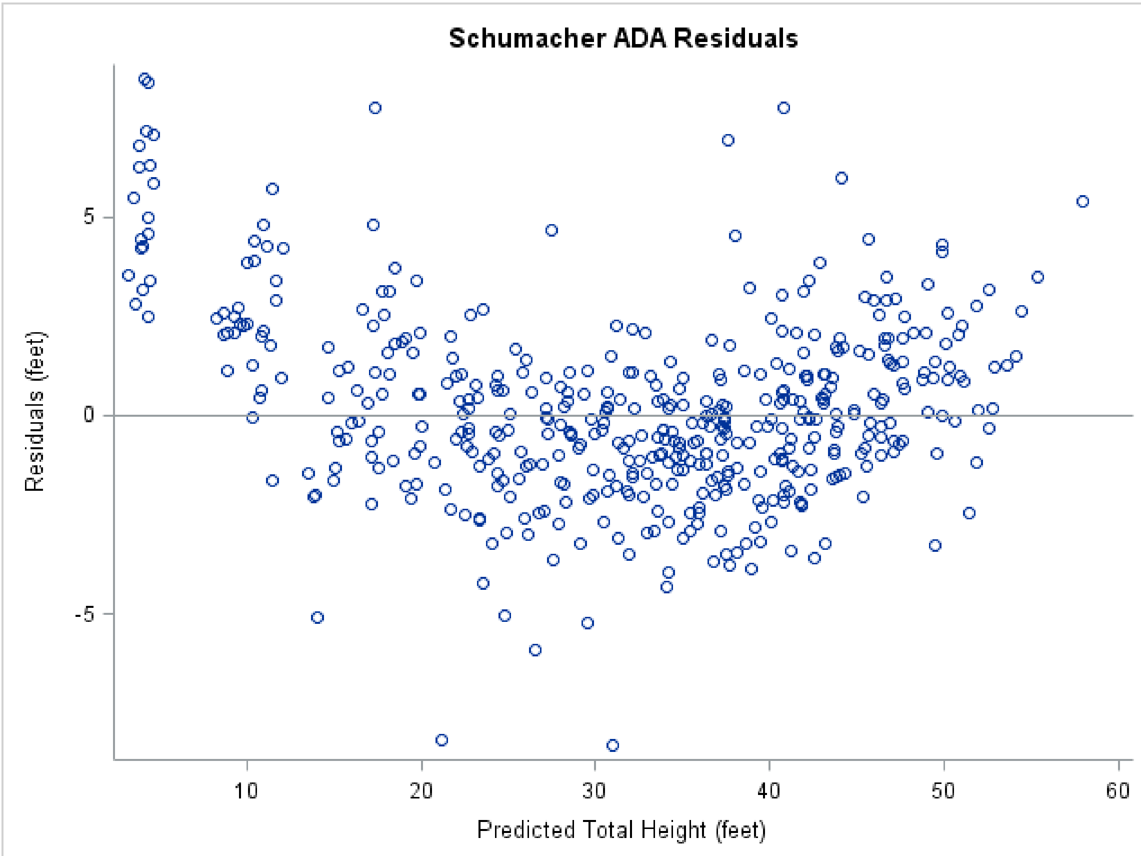

Figure 3. Plot of residuals against predicted total tree height for Schumacher ADA model Equation [6].

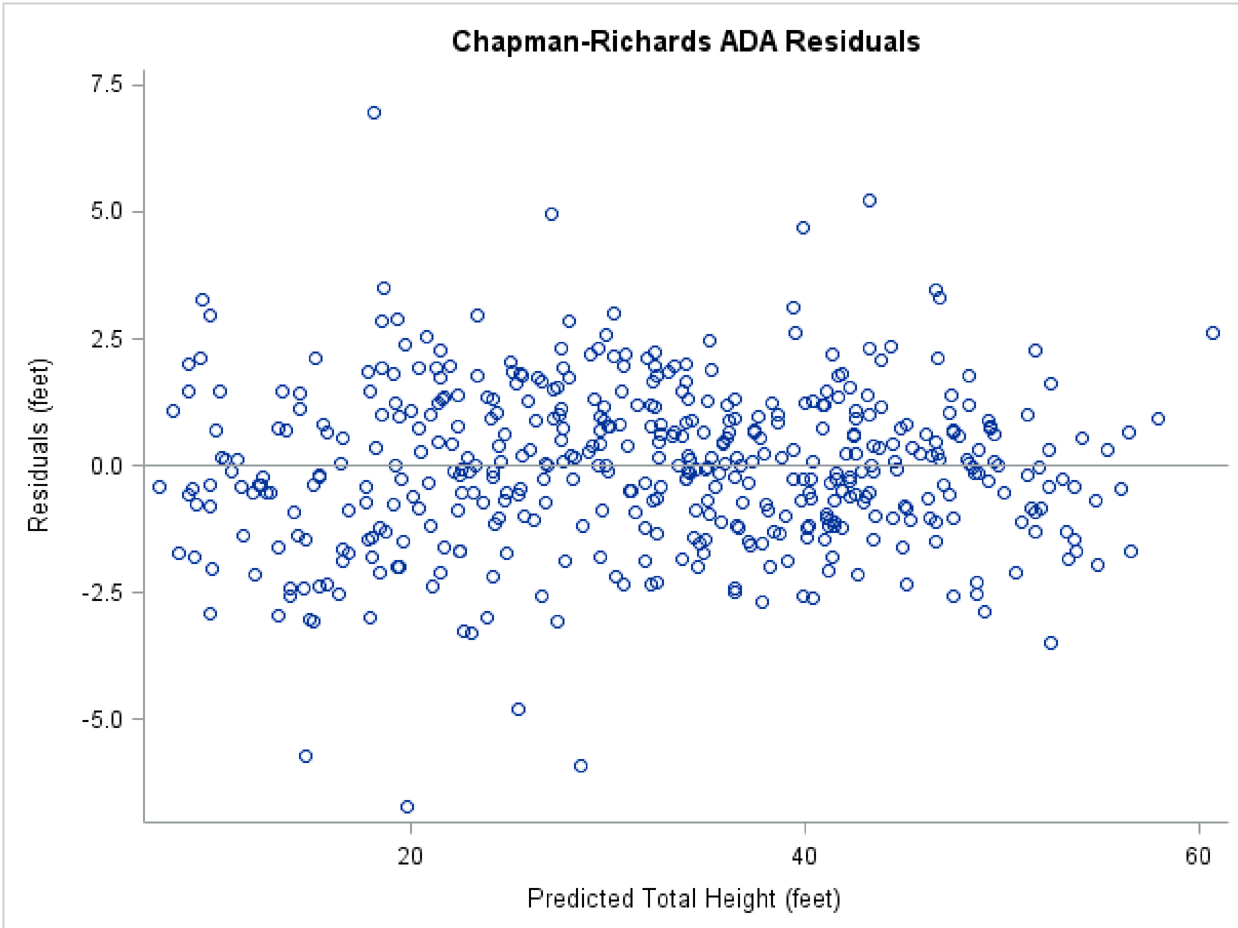

Figure 4. Plot of residuals against predicted total tree height for Chapman-Richards ADA model Equation [12].

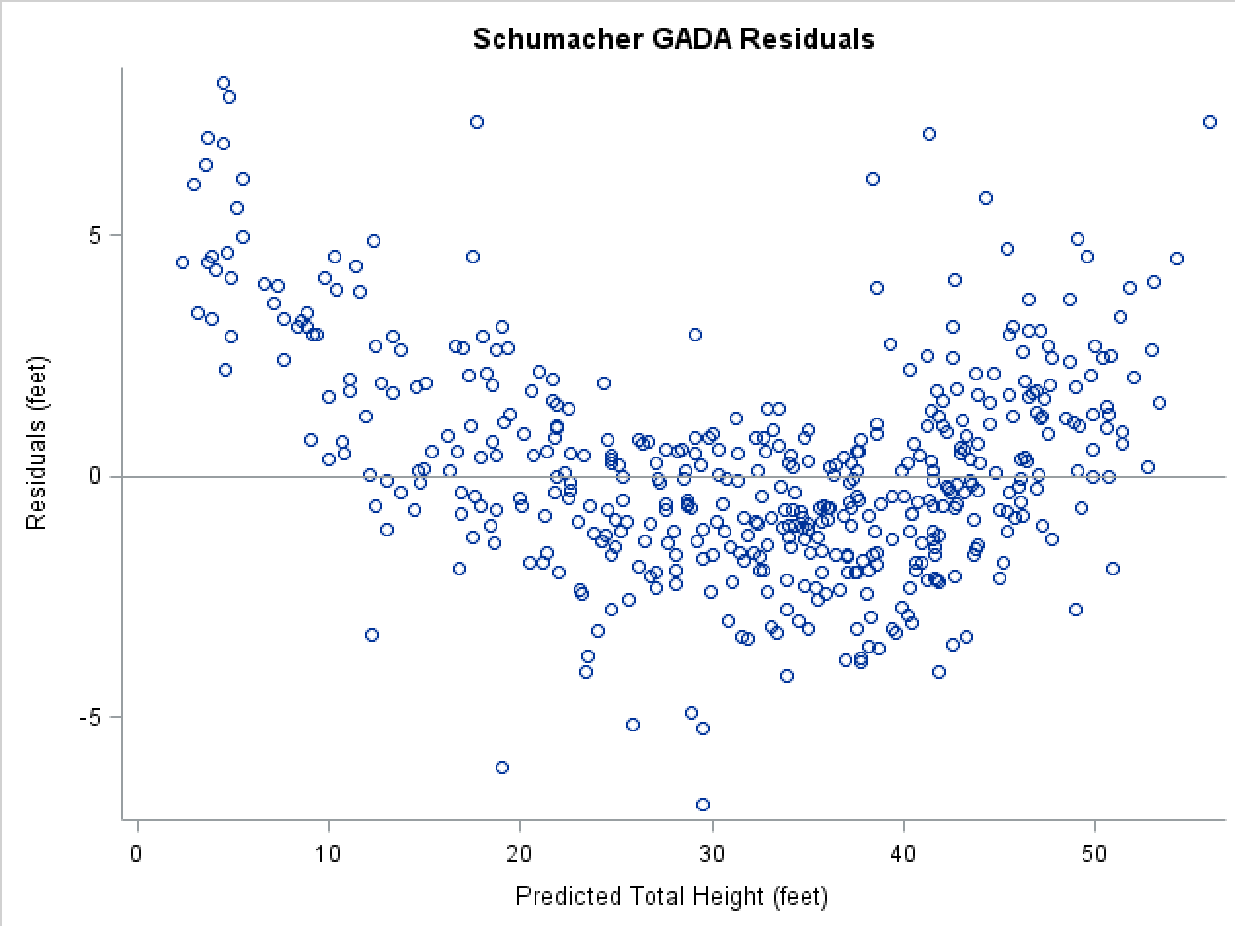

Figure 5. Plot of residuals against predicted total tree height for Schumacher GADA model Equation [20].

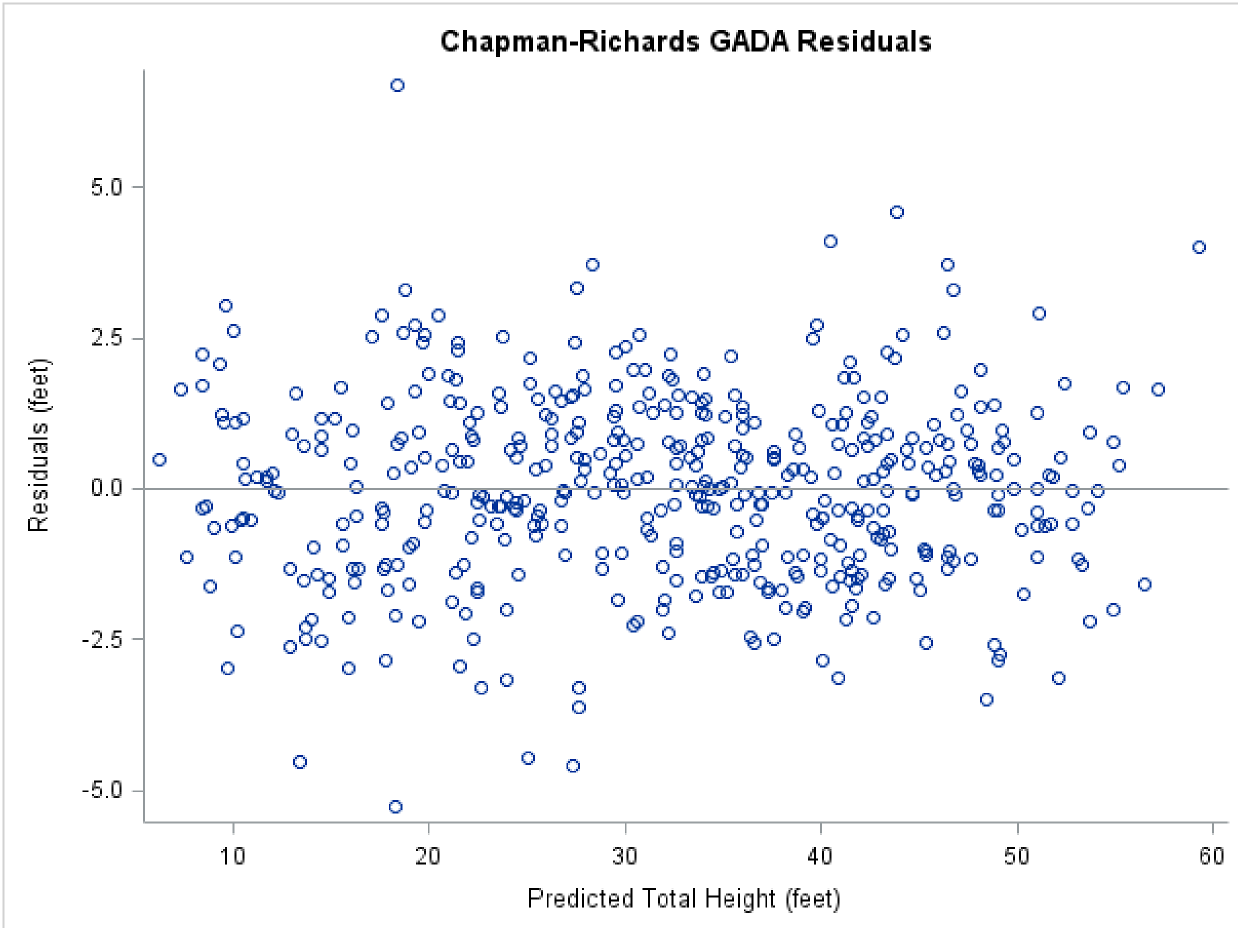

Figure 6. Plot of residuals against predicted total tree height for Chapman-Richards GADA model Equation [41].

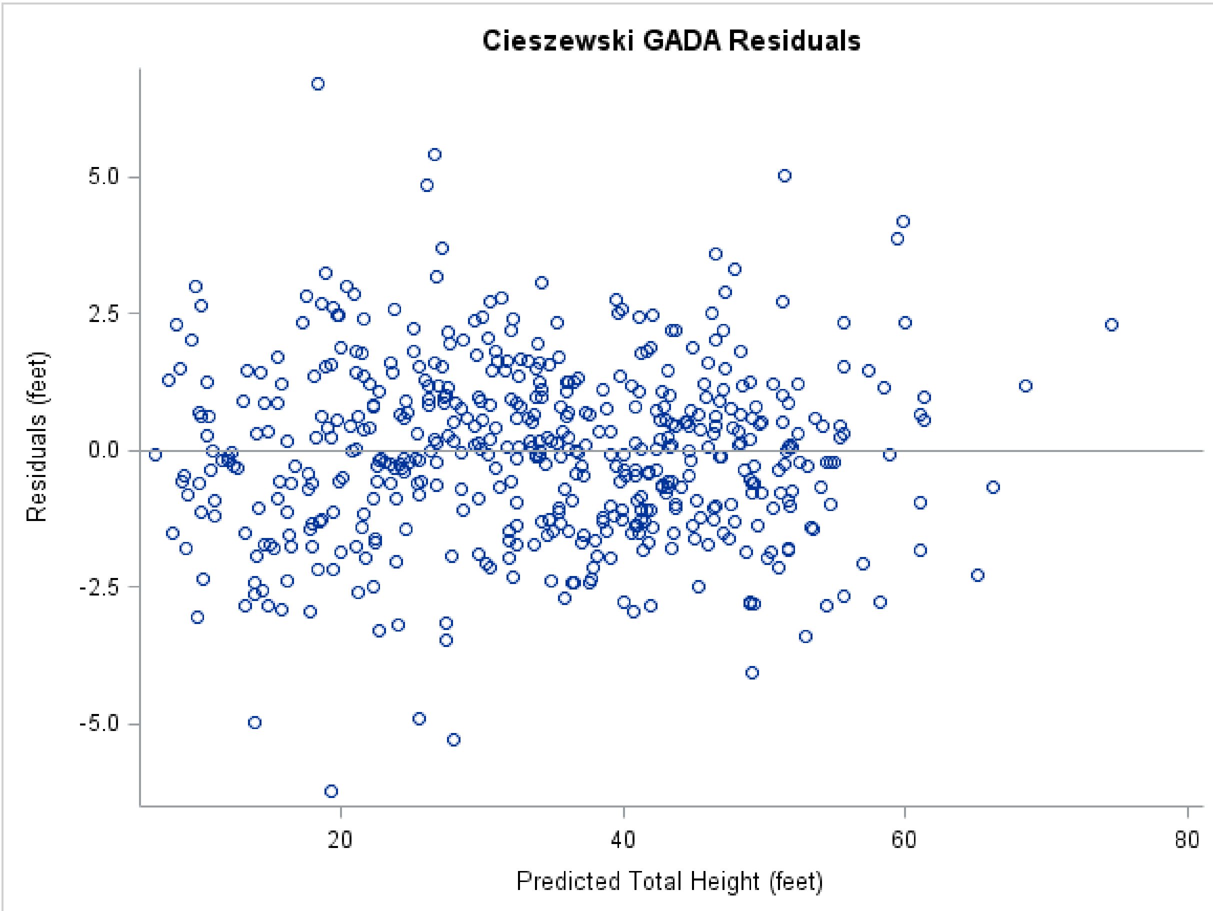

Figure 7. Plot of residuals against predicted total tree height for Cieszewski GADA model Equation [59].

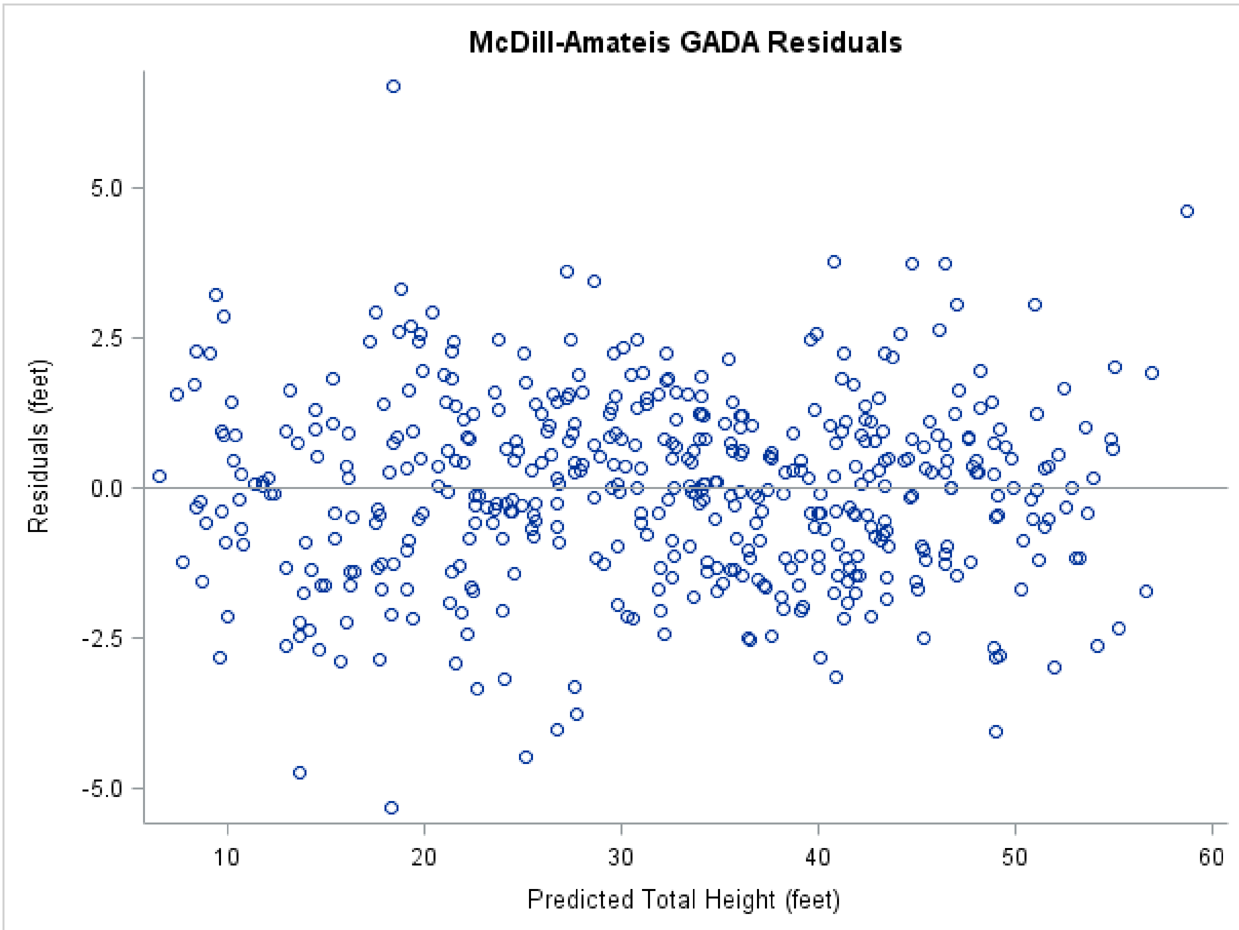

Figure 8. Plot of residuals against predicted total tree height for McDill-Amateis GADA model Equation [77].

In the process of estimating unobserved site index values to fit the models, final CSI values represented local or site-specific estimates of the total height at the index age (25 years in this study) for each ETPPRP Phase 2 plot. Since site index was not observed for the plots (maximum age = 19 years), these estimates were considered the best estimate of site index for the plots. As such, they were useful for any application that required an estimate of site index of each plot. These values were included in the Appendix.

The site index curves for all models show an appropriate curvilinear height-age relationship over time at different levels of site index (Figures 9-14). All models performed equally well in terms of R-square and RMSE. Both Schumacher models exhibit visibly obvious autocorrelation trends in their residual plots (Figures 3 and 5), while all other residual plots showed no trends (Figures 4, 6-8). The Chapman-Richards GADA, McDill-Amateis GADA, and Cieszewski GADA models performed best in terms of AIC, but the Cieszewski GADA model had a non-significant parameter estimate, which means it is not ideally formulated for this data set. Since the Chapman-Richards GADA model (Equation 41, Figure 12) and the McDill-Amateis GADA model (Equation 77, Figure 14) performed equally well without any demerits, both of these two models were chosen to further compare to the existing models of Diéguez-Aranda et al. (2006) and Coble and Lee (2010).

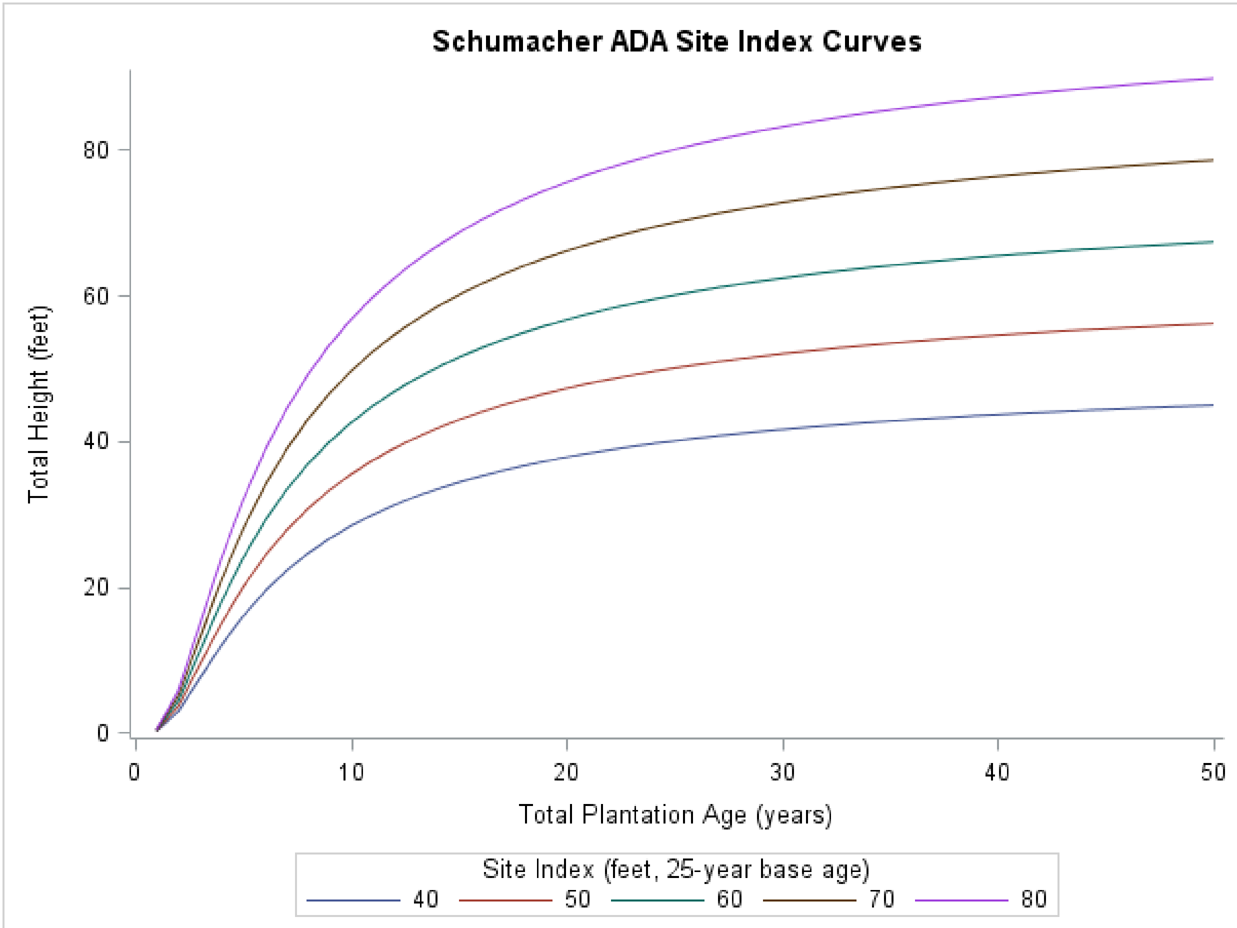

Figure 9. Site index curves for Schumacher ADA model Equation [6].

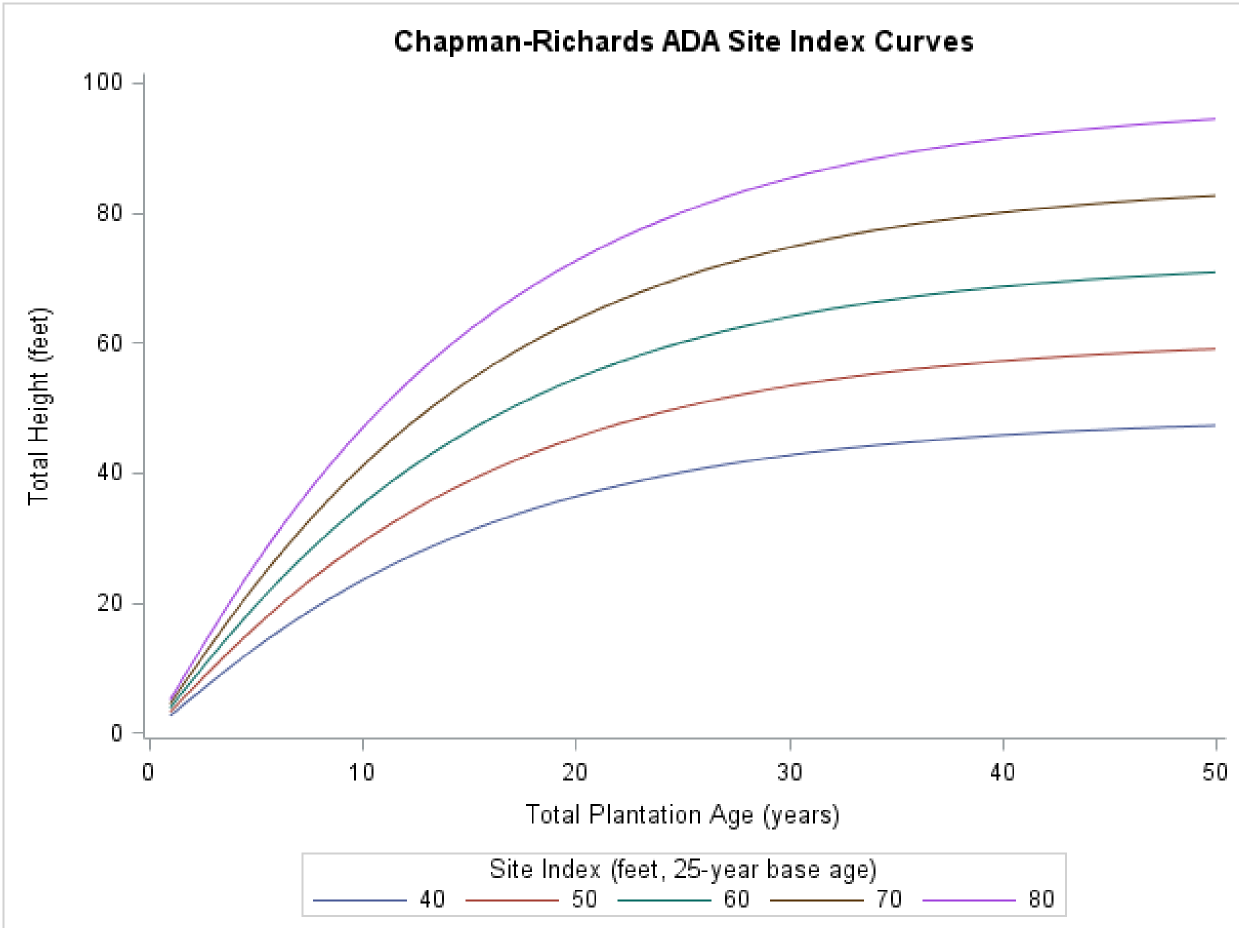

Figure 10. Site index curves for Chapman-Richards ADA model Equation [12].

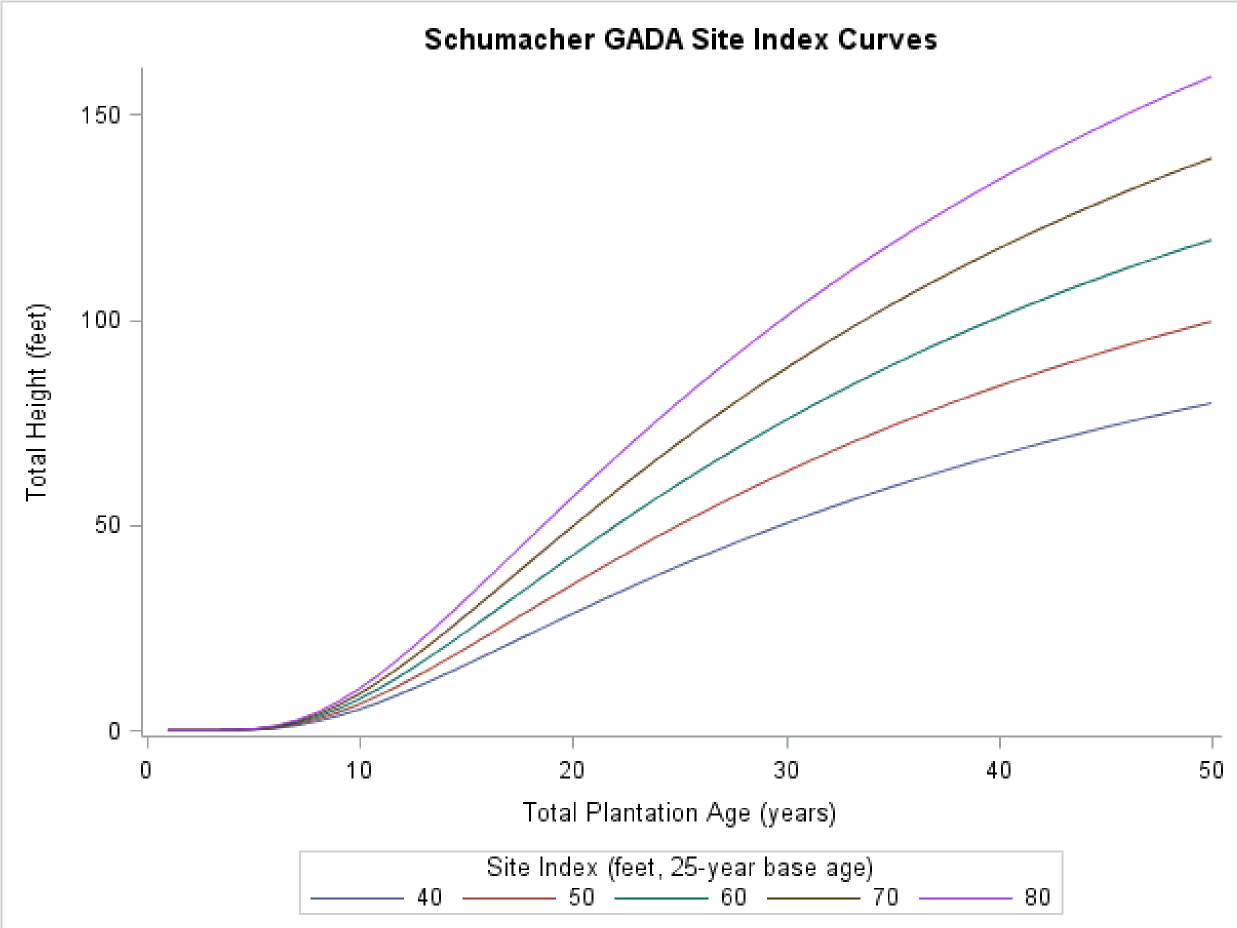

Figure 11. Site index curves for Schumacher GADA model Equation [20].

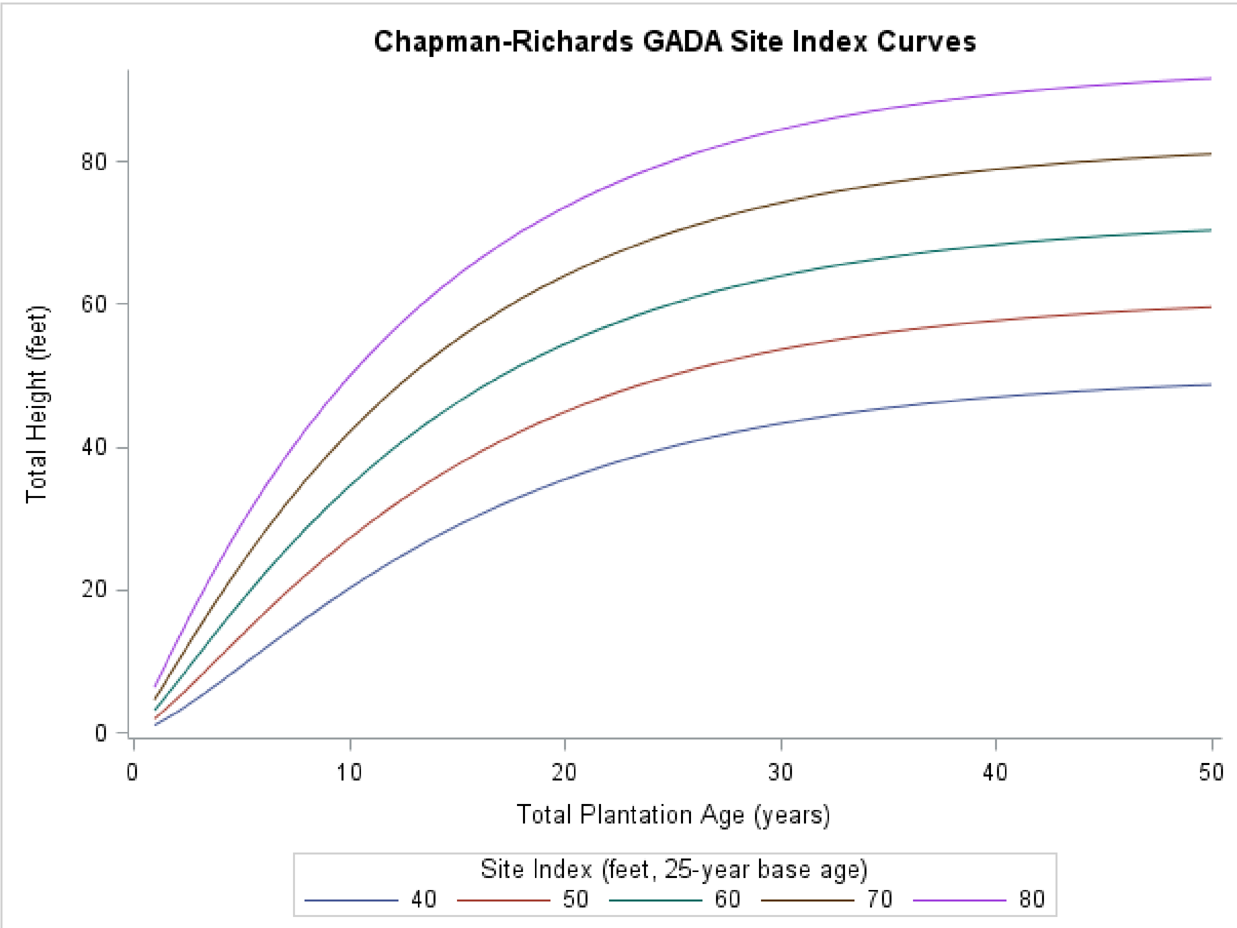

Figure 12. Site index curves for Chapman-Richards GADA model Equation [41].

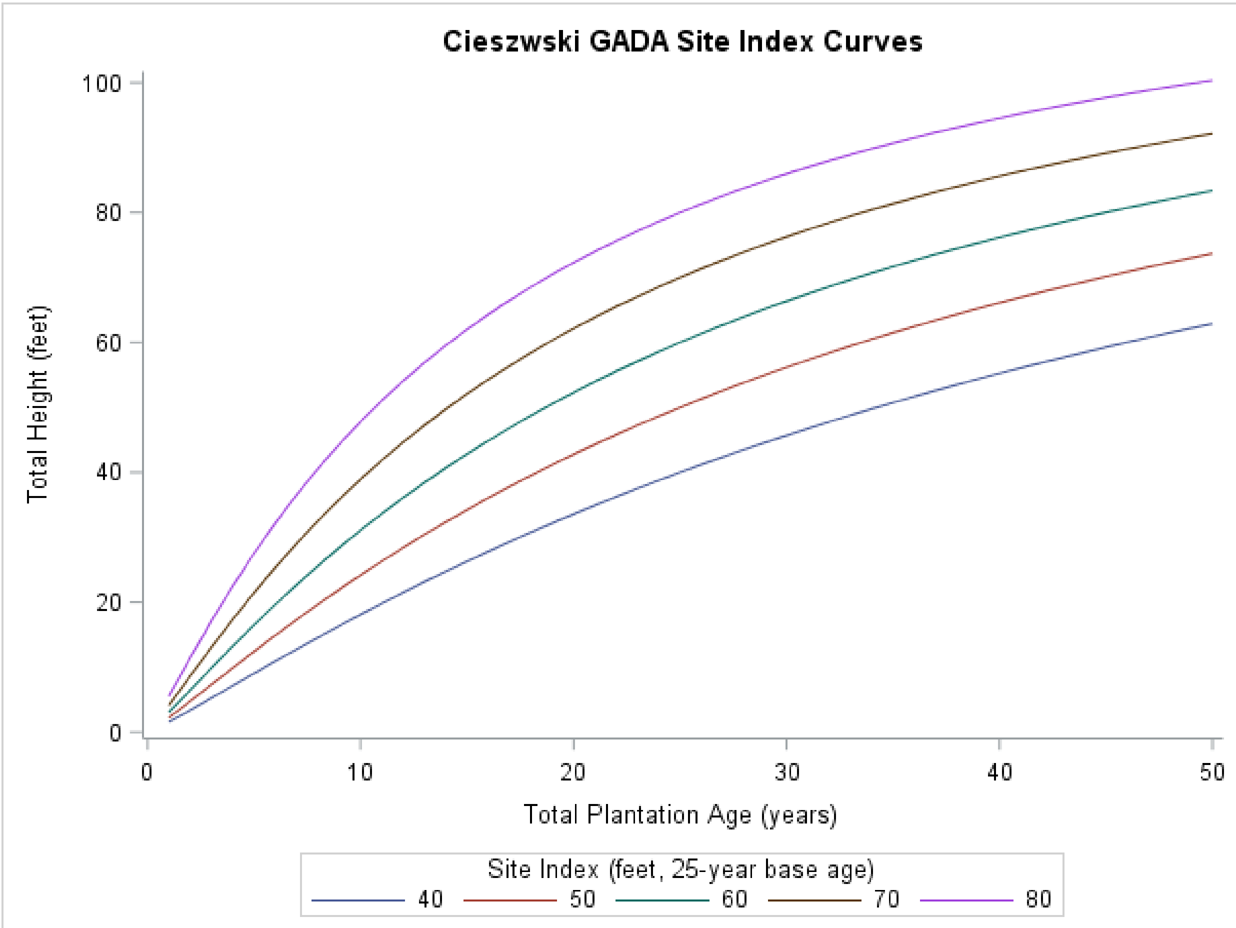

Figure 13. Site index curves for Cieszewski GADA model Equation [59].

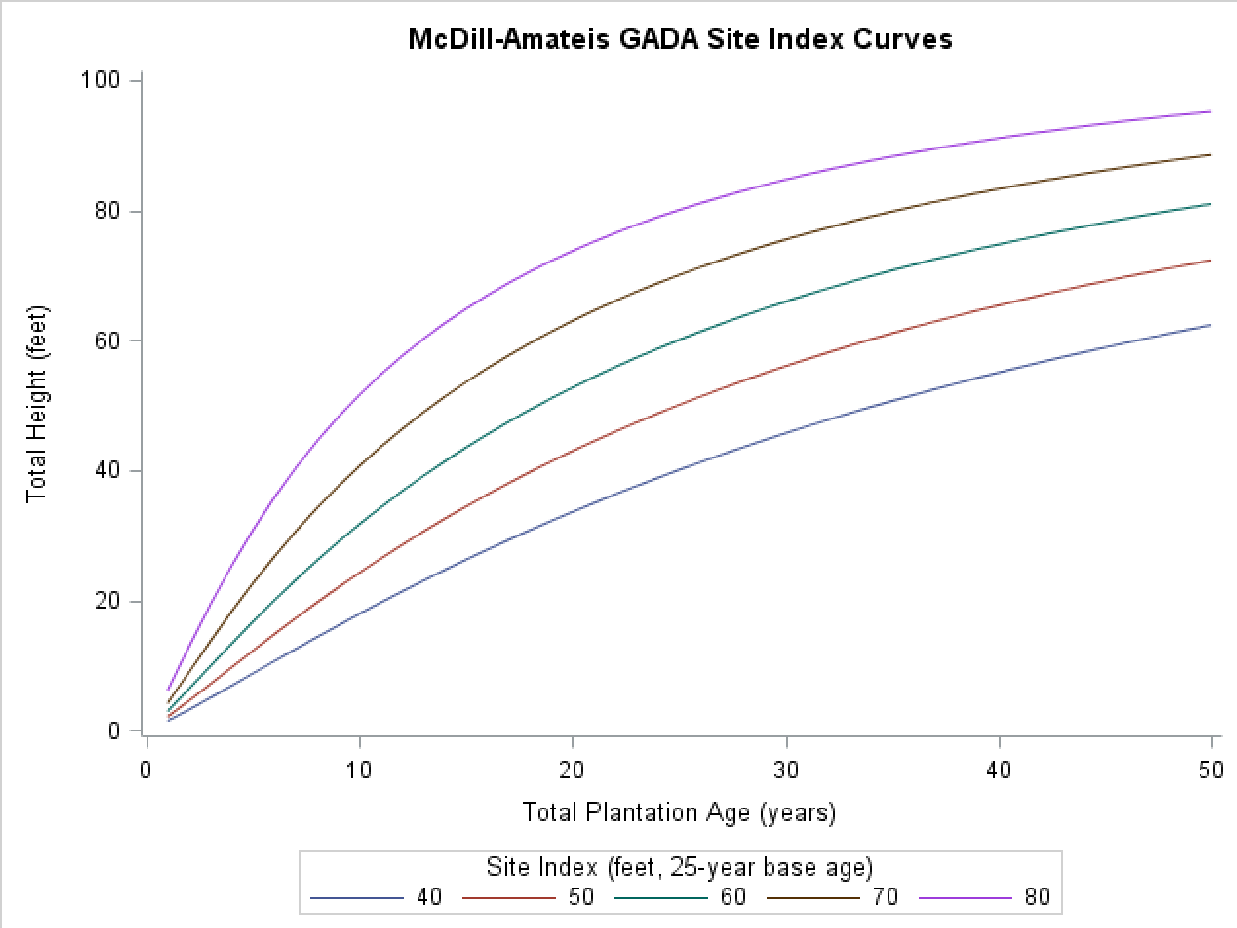

Figure 14. Site index curves for McDill-Amateis GADA model Equation [77].

The comparison to the existing curves was performed at four site index values that represented a range of poor to excellent site quality: 50, 60, 70, and 80 feet. Total height in feet was plotted over plantation age in years. Even though maximum age for the data used in this study was only 19 years, plantation age was plotted up to 50 years to examine the extrapolative behavior of all the models.

The Diéguez-Aranda et al. (2006) model (DA 2006) and the Coble and Lee (2010) model (CL 2010) predicted similar heights at ages less than 25 years. However, after age 25, DA 2006 consistently predicted greater heights than CL 2010 (Figures 15-18). Across the range of site indices examined, the Chapman-Richards GADA (Equation 41) and McDill-Amateis GADA (Equation 77) models predicted greater heights at ages less than 25 years than either the DA 2006 or CL 2010 models. As site index increased, the difference in heights also increased (Figures 15-18). After age 25, the greatest heights were predicted by DA 2006 for all site indices, followed by Equation 6, CL 2010, and Equation 41 for SI 50, 60, and 70. At SI 80, CL 2010 predicted greater heights than Equation 77, but still shorter heights than DA 2006 while Equation 41 still predicted the shortest heights.

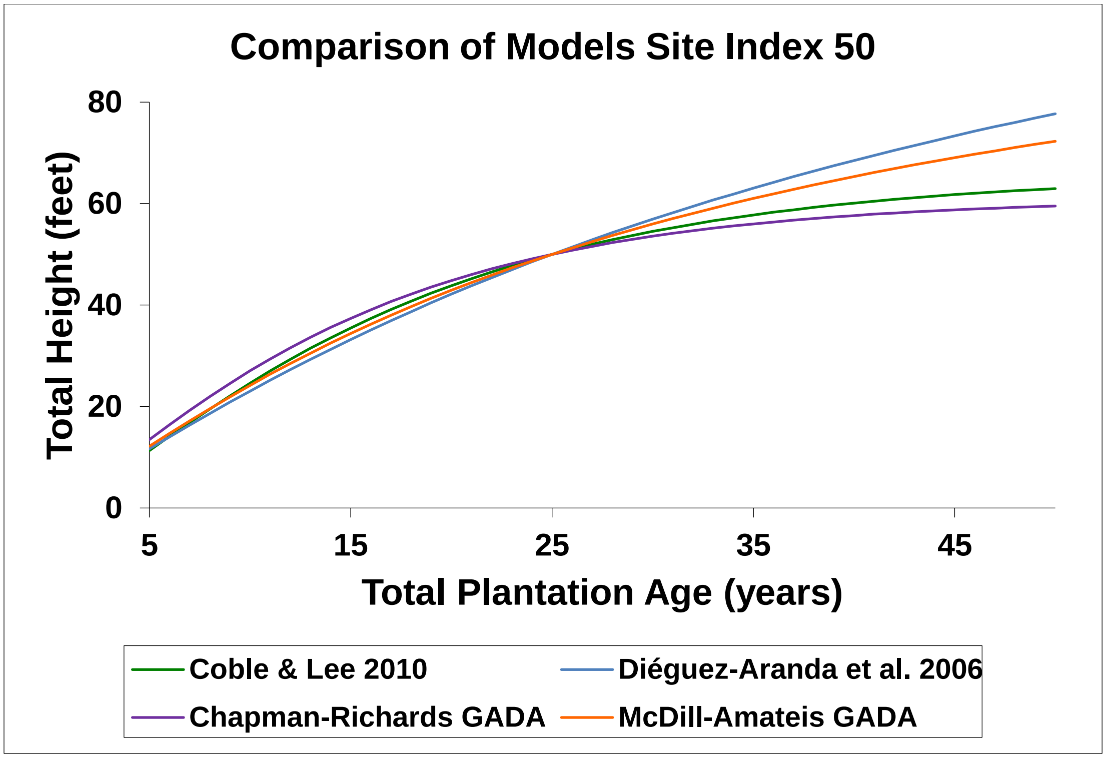

Figure 15. Comparison of models for site index 50.

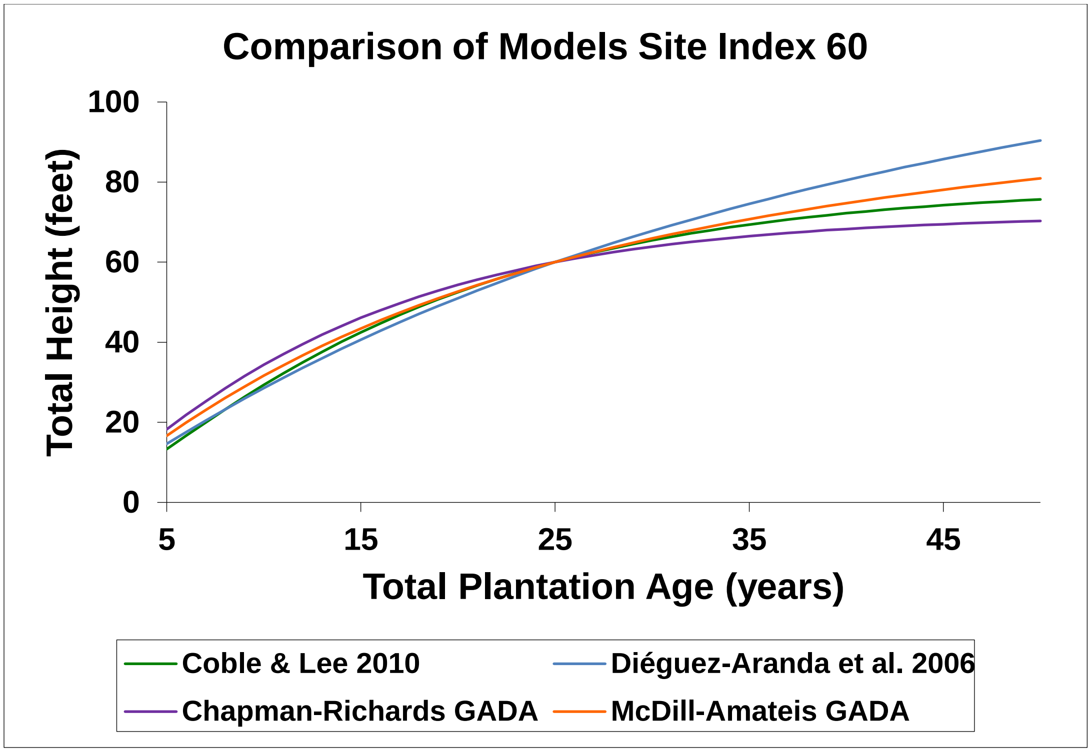

Figure 16. Comparison of models for site index 60.

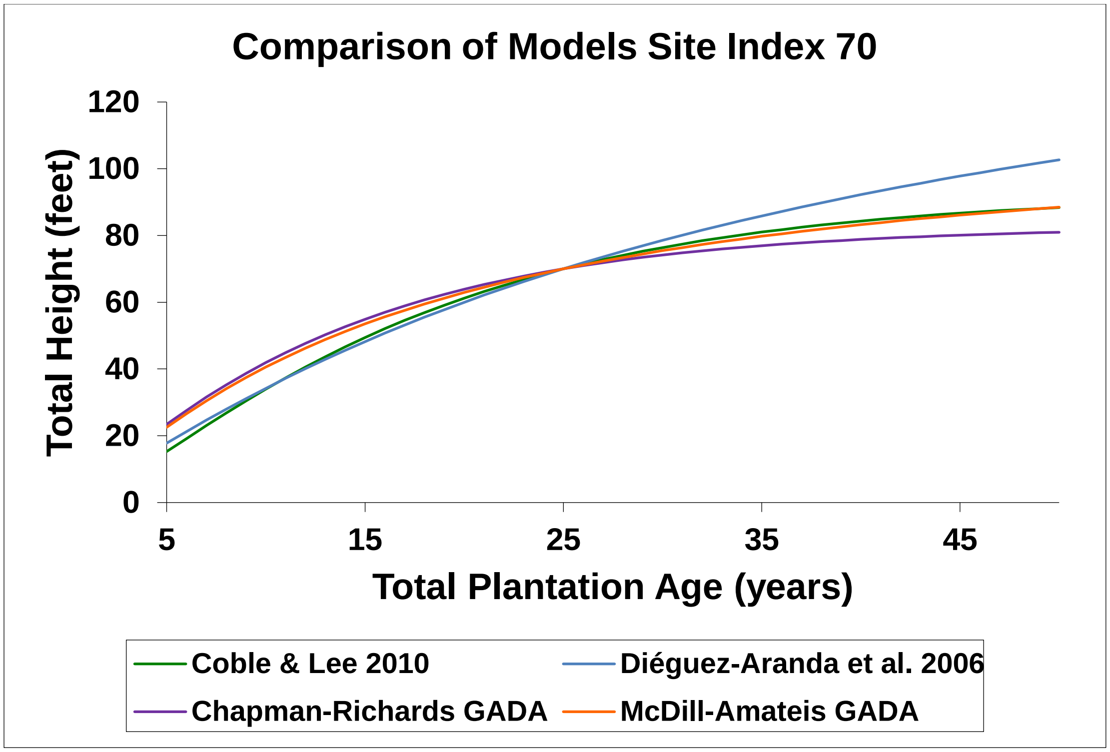

Figure 17. Comparison of models for site index 70.

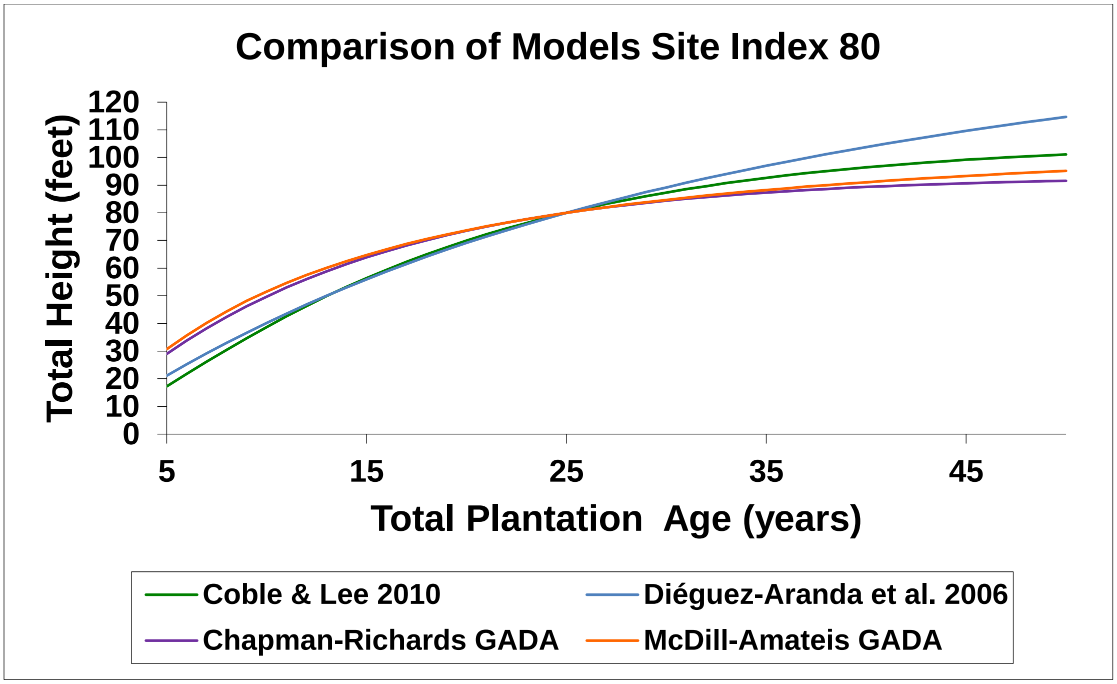

Figure 18. Comparison of models for site index 80.

Site index shows a steady decline over age for both the Diéguez-Aranda et al. 2006 and the Coble and Lee 2010 models (Figure 19). However, both the Chapman-Richards GADA (Equation 41) and McDill-Amateis GADA (Equation 77) models maintain a steady site index.

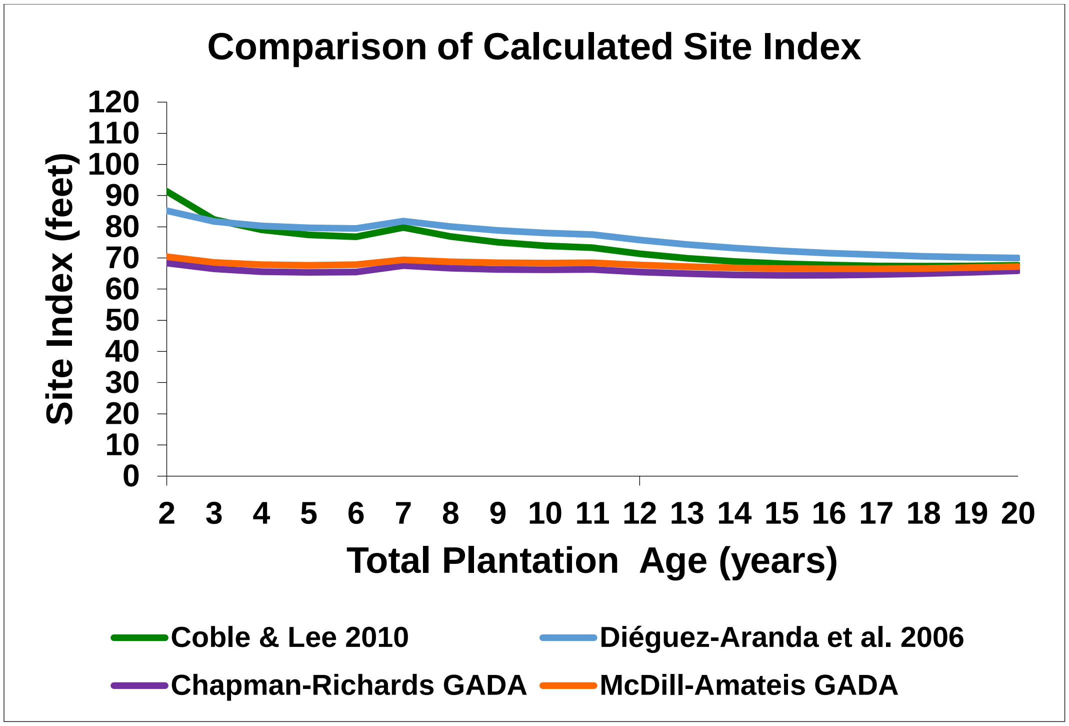

Figure 19. Comparison of calculated site index.

After having compared the four models to four different SI values (50, 60, 70, and 80 feet), we decided that the McDill-Amateis GADA model (Equation 77) was the best fit site index model because it was more parsimonious compared to the Chapman-Richards GADA model (Equation 41), otherwise theMcDill-Amateis GADA and Chapman-Richards GADA models perform equally well. The McDill-Amateis GADA model can be found in the following format:

$$H = \frac{112.1}{1 - \left(1 - (112.1) H_0^{-1}\right) \left(\frac{A}{A_0} \right)^{1.1729}} . \tag{78}$$

DISCUSSION

In this study both anamorphic and polymorphic site index models were developed. Results of this study based on fit statistics suggest that the polymorphic forms, probably due to their flexibility in allowing more than one parameter to be local, outperformed the anamorphic models (Table 2). Comparisons in performance using various models to model height and age curves have been done, and various conclusions were achieved in terms of producing adequate site curves (Burkhart and Tomé 2012; Weiskittel et al. 2011). Furnival et al. (1990) realized that there were biases present when certain types of data were used to formulate anamorphic site index curves. For example, stem analysis data presented a selectivity bias since stands on higher quality sites tended to be harvested at younger ages skewing the trajectory of the curves. The Algebraic Difference Approach (ADA) developed by Bailey and Clutter (1974) was originally used to generate polymorphic site index curves that were more flexible than anamorphic curves, but still had only single asymptotes (Cieszewski et al. 2006; Furnival et al. 1990). Cieszewski and Bailey (2000) compared GADA and ADA and formulated GADA derived equations that were polymorphic with multiple asymptotes and were base-age invariant which could not be achieved with ADA. Cieszewski et al.(2006) compared ADA and GADA methods for developing equations for site index curves for Scots pine and found GADA was superior. These advanced polymorphic models (multiple asymptotes) described growth patterns better than anamorphic or simple polymorphic (single asymptote) models for various species (Cieszewski et al. 2006; Krumland and Eng 2005). ADA was able to derive anamorphic and polymorphic curves with single asymptotes while GADA developed polymorphic curves with multiple asymptotes as well as being base-age invariant—all desirable characteristics (Cieszewski and Bailey 2000).

Statistically the Chapman-Richards GADA model (Equation 41, Figure 9) and the McDill-Amateis GADA model (Equation 77, Figure 11) provided the best SI predictions for intensively managed loblolly pine plantations in the western portion of the Western Gulf Coastal Plain (i.e., east Texas and western Louisiana). It was equally important to see their performances by comparing them to the SI models currently being utilized (CL 2010 and DA 2006). Both CL 2010 and DA 2006 were developed for extensively (non-intensively) managed loblolly pine plantations, with the former being specifically for east Texas and the latter targeting the southeastern United States. It was expected that differences existed between these two models due to regional differences in climate, genetics, and other dynamics. Our results showed that they were similar in heights below 25 years, but DA 2006 heights exceeded CL 2010 at ages greater than 25 years. One explanation for this, although speculative, was that loblolly pine trees growing further east, which are represented by DA 2006's equations, reached greater heights than loblolly pine trees growing in the West Gulf Coastal Plain, which were represented by CL 2010's equations, either due to unspecified genetic differences or environmental differences. Trees growing in east Texas have harsher conditions to survive, mainly greater evapotranspirative demand in August/September. In either case, DA 2006 did not represent east Texas well.

Data for CL 2010, the Chapman-Richards GADA model, and the McDill-Amateis GADA model were collected from the east Texas region. As expected, predicted heights of the Chapman-Richards GADA model (Equation 41) or the McDill-Amateis GADA model (Equation 77) were greater than those of CL 2010 at young ages (i.e., <=25 years), suggesting the intensive silvicultural activities and planting genetically improved seedlings greatly improved early height growth. However, after age 25, greater heights for CL 2010 versus the Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) were observed (Figure 17), and this underestimate was especially true for the Chapman-Richards GADA model (Equation 41). This was conjectured to have resulted from a lack of data at older ages from the intensively managed plantations. The CL 2010 model was created using data available for unthinned loblolly pine plots that ranged in age from 14 to 37 years creating a data set that, like DA 2006, comprised a full rotation (Coble and Lee 2006; 2010; Lenhart et al. 1986). The Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) did not reach the greater heights at older ages like the CL 2010 and DA 2006 models. The DA 2006 model was created using long-term data available for unthinned, extensively managed loblolly pine plots across the southeastern United States up to an age class of 45 years (43-47 years) which created a data set that encompassed a full rotation (Diéguez-Aranda et al. 2006). Similarly, the Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) had significantly greater heights than DA 2006 at younger ages. Many factors including climate, silviculture, and genetics may have contributed to these differences. The greater heights for the Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) mostly reflected the improved height growth due to intensive management. However, the faster height growth by the Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) were not evident at older ages because available data for this study was less than 19 years old. The height differences became more pronounced as site index increased, suggesting that genetic effects and effects of intensive silvicultural activities were not constant across site quality. In essence, there were more benefits from using improved genetics and intensive silvicultural practices on better sites as compared to poor sites.

The data for this study was limited by the fact that the oldest Phase 2 plantations were only 19 years. Therefore, data older than 19 years had to be extrapolated for the models and data are lacking to definitively establish the asymptote of the model. Despite this data limitation, this study is still worthwhile because many intensive silvicultural practices are applied before age 25.

Effects of silvicultural activities and genetics on height growth have been studied extensively. Zhao et al. (2016) found that the intensity of silvicultural treatments strongly interacted with site quality in that lower quality sites responded more to higher intensity silvicultural treatments than higher quality sites. Therefore, silvicultural treatment intensity should be based on site quality to optimize silvicultural prescriptions (Zhao et al. 2016). For a given stand, site index often varied significantly due to changes in genetics, silvicultural activities, and other environmental factors. Genetic variation explained more than 40% of the original variation in site index in Douglas-fir (Monserud and Rehfeldt 1990). Assessments of the effects of seed source on the variation of site index for loblolly pine have been presented by Nance and Wells (1981) and Buford and Burkhart (1987). Compared to unimproved seedlots, the improved seedlots changed the asymptotic coefficient of the height-age relationship significantly (Buford and Burkhart 1987). Site index has proven to be extremely sensitive to many silvicultural activities currently being used such as soil bedding, vegetation control, or fertilization (Weiskittel et al. 2011; Zhao et al. 2016). Therefore, the change in SI could be used to measure the response of stands to various management regimes (Zhao et al. 2016). Without doubt, the differences between the CL 2010 model and our SI model were a result of a complex byproduct of genetics, climate, applied silvicultural treatments, and even their interactions. The differences between DA 2006 and our SI model may have been due more to population differences in nature and climate. Without doubt, applying both the Chapman-Richards GADA (Equation 41) and the McDill-Amateis GADA models (Equation 77) developed in this study could greatly improve height prediction at ages 25 years and younger for intensively managed loblolly pine plantations across the Western Gulf Coastal Plain region than the models currently being used.

While the Chapman-Richards GADA model (Equation 41) and the McDill-Amateis GADA model (Equation 77) predicted early height growth well, the McDill-Amateis GADA model (Equation 77) obtained more realistic heights than the Chapman-Richards GADA model (Equation 41) after ages 25 years. We selected the McDill-Amateis GADA model (Equation 77) as the best model for predicting SI for intensively managed loblolly pine plantations. This model could be used in conjunction with growth and yield models for the Western Gulf Coastal Plain region (Coble et al. 2016).

CONCLUSION

Due to the similarities in the fits statistics of both models, the Chapman-Richards GADA model (Equation 41, Figure 12) and the McDill-Amateis GADA model (Equation 77, Figure 14) were both chosen to compare to the existing models of Diéguez-Aranda et al. (2006) and Coble and Lee (2010). The McDill-Amateis GADA model (Equation 77) was ultimately chosen as the best model having an R^2 value of 0.984, RMSE of 1.54 feet, AIC value of 410, and a strong Δ_i of 1.7 (Table 2). The McDill-Amateis GADA model (Equation 77) reliably predicted greater heights at younger ages on higher quality sites than the models of DA 2006 and CL 2010, but predicted shorter heights at older ages due to a lack of data for older ages. Although the data set was young and will require continued research through an entire stand rotation, this study was still valuable. Foresters now have a set of site index curves developed specifically for the West Gulf Coastal Plain. These site index curves will aid foresters in making more appropriate silvicultural prescriptions for intensively managed loblolly pine plantations in this unique region. Better management regimes can potentially increase the value of the stands and in turn benefit the economy. Several hypotheses for future research can be developed from this study. These hypotheses could include:

1. Height-age/Site index models developed with data from the southeastern United States (SE) will show greater heights at older ages compared to West Gulf models (WG).
2. Height-age/Site index models developed with data from improved plantations will show greater heights at all ages compared to unimproved plantations.
3. Heights at all ages will be greater in improved plantations versus unimproved plantations in any region.

This can be depicted visually with four curves of heights plotted over age. Two curves will represent improved/unimproved from the West Gulf Coastal Plain region, and the other two curves will represent improved/unimproved for the southeastern United States. The two improved curves (one from WG and the other from SE) will be greater at all ages as opposed to the two unimproved curves from the two regions. However, the SE curves will be greater at older ages and the same at younger ages versus the WG curves in a similar way (shape) for both improved and unimproved plantations.

In the Piedmont and Upper Coastal Plain of the southern United States, long-term productivity of loblolly pine has shown to increase with intensive site preparation treatments including herbaceous and woody competition control and mechanical treatments (Lauer and Zutter 2001; Zhao et al. 2009; Zutter and Miller 1998). A study by Zhao et al. (2009a) found that genetically improved slash pine seedlings planted in the Flatwoods area of the Lower Coastal Plain benefitted from various silvicultural treatments overall (Zutter and Miller 1998). Varied responses were possible with different combinations of silvicultural treatments applied to a range of sites under diverse environmental conditions, but the general conclusion was that more intensive methods provided increased growth over lower intensity treatments (Shiver et al. 1990; Zhao et al. 2009b).

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Table 3. Site index estimates for the Schumacher ADA equation (Equation 1).

Plot Number	Site Index Estimate (ft.)	Plot Number	Site Index Estimate (ft.)	Plot Number	Site Index Estimate (ft.)
200	50.1	249	49.1	298	61.9
201	48.1	250	62.1	299	50.7
202	52.8	251	61.1	300	61.4
203	54.6	252	53.9	301	57.1
204	47.0	253	65.5	302	48.1
205	50.0	254	63.4	303	59.3
206	53.1	255	59.5	304	58.1
207	69.0	256	58.9	305	59.5
208	54.5	257	55.3	306	62.9
209	54.5	258	58.3	307	60.5
210	58.1	259	76.6	308	62.2
211	56.1	260	59.2	309	62.2
212	43.8	261	69.4	310	52.1
213	60.1	262	55.5	311	66.3
214	56.0	263	51.2	312	47.0
215	61.0	264	51.4	313	45.5
216	43.9	265	56.6	314	53.3
217	58.6	266	67.4	315	51.6
218	58.8	267	54.2	316	45.4
219	59.3	268	70.5	317	54.7
220	56.2	269	58.7	318	65.5
221	55.1	270	57.7	319	59.1
222	58.6	271	57.2	320	65.7
223	61.3	272	48.2	321	59.3
224	57.6	273	61.2	322	59.9
225	60.6	274	65.4	323	63.3
226	61.0	275	57.7	324	59.1
227	58.7	276	55.4	325	46.4
228	52.6	277	44.6	326	60.9
229	57.7	278	49.2	327	51.3
230	56.0	279	47.1	328	50.7
231	54.6	280	57.1	329	51.1
232	48.8	281	56.4	330	46.8
233	58.7	282	51.3	331	55.3
234	61.7	283	61.6	332	54.9
235	67.0	284	63.9		
236	63.6	285	50.1		
237	62.1	286	56.5		
238	50.6	287	60.9		
239	64.3	288	59.4		
240	59.6	289	56.0		
241	61.9	290	49.8		
242	65.7	291	52.1		
243	64.9	292	66.4		
244	70.5	293	55.8		
245	48.4	294	50.3		
246	38.6	295	58.2		
247	50.4	296	56.9		
248	56.3	297	57.8		

Table 4. Site index estimates for the Chapman-Richards ADA equation (Equation 2).

Plot Number	Site Index Estimate (ft.)	Plot Number	Site Index Estimate (ft.)	Plot Number	Site Index Estimate (ft.)
200	60.7	248	67.9	296	67.5
201	55.5	249	58.3	297	70.5
202	63.0	250	73.8	298	73.6
203	66.2	251	71.9	299	61.9
204	54.7	252	63.9	300	74.4
205	57.1	253	77.7	301	69.3
206	63.0	254	78.1	302	57.5
207	83.7	255	69.4	303	72.5
208	66.7	256	69.9	304	70.9
209	64.0	257	66.7	305	72.7
210	68.0	258	69.1	306	75.8
211	64.2	259	94.4	307	72.4
212	49.0	260	72.9	308	75.1
213	70.8	261	82.0	309	76.2
214	68.2	262	64.8	310	62.2
215	70.4	263	59.8	311	81.0
216	51.6	264	60.3	312	55.9
217	69.0	265	66.8	313	54.9
218	66.0	266	80.0	314	63.8
219	70.06	267	64.9	315	63.1
220	67.1	268	85.6	316	55.4
221	66.5	269	70.2	317	66.3
222	69.9	270	68.6	318	78.3
223	71.7	271	69.5	319	72.1
224	68.4	272	59.8	320	78.5
225	73.6	273	74.3	321	72.0
226	74.3	274	79.0	322	73.0
227	69.2	275	68.5	323	77.4
228	64.1	276	68.9	324	70.1
229	69.7	277	53.3	325	56.6
230	66.9	278	60.0	326	74.3
231	66.2	279	56.4	327	63.0
232	57.5	280	68.9	328	62.2
233	69.3	281	69.0	329	62.8
234	73.4	282	62.6	330	57.3
235	79.5	283	74.4	331	67.9
236	76.0	284	77.3	332	67.5
237	73.0	285	60.7		
238	60.5	286	67.7		
239	76.2	287	71.7		
240	72.7	288	71.0		
241	73.9	289	66.0		
242	80.1	290	60.4		
243	77.4	291	62.9		
244	85.3	292	78.2		
245	57.8	293	65.8		
246	45.9	294	59.3		
247	59.9	295	69.1		

Table 5. Site index estimates for the Schumacher GADA equation (Equation 3).

Plot Number	Site Index Estimate	Plot Number	Site Index Estimate	Plot Number	Site Index Estimate
200	52.1	249	51.6	298	61.0
201	49.7	250	61.2	299	53.5
202	54.0	251	60.4	300	61.1
203	56.3	252	55.1	301	57.5
204	49.6	253	63.4	302	51.1
205	51.7	254	61.6	303	59.7
206	54.6	255	59.1	304	58.7
207	66.1	256	58.6	305	59.4
208	56.3	257	56.6	306	61.6
209	55.1	258	58.5	307	60.1
210	57.9	259	68.8	308	61.3
211	56.5	260	58.8	309	61.5
212	45.3	261	65.8	310	54.0
213	59.7	262	55.9	311	63.9
214	57.2	263	52.6	312	49.4
215	60.5	264	52.5	313	49.3
216	46.3	265	56.8	314	54.8
217	58.5	266	64.4	315	54.0
218	58.8	267	55.3	316	49.1
219	58.7	268	66.5	317	56.3
220	56.7	269	58.3	318	63.6
221	55.5	270	57.7	319	59.1
222	58.4	271	57.6	320	63.7
223	60.2	272	51.5	321	59.9
224	57.5	273	60.4	322	59.8
225	59.6	274	63.6	323	61.7
226	60.2	275	57.7	324	58.7
227	58.4	276	56.4	325	49.8
228	54.4	277	48.4	326	60.1
229	58.1	278	52.3	327	54.1
230	56.7	279	50.3	328	53.6
231	55.5	280	57.6	329	54.0
232	50.8	281	57.7	330	51.0
233	58.3	282	53.8	331	56.5
234	60.3	283	60.7	332	56.4
235	64.3	284	62.6		
236	62.4	285	52.9		
237	61.1	286	57.3		
238	52.8	287	60.1		
239	62.7	288	59.1		
240	59.2	289	56.6		
241	60.8	290	52.1		
242	63.0	291	53.9		
243	62.9	292	64.1		
244	67.4	293	56.4		
245	51.1	294	52.0		
246	42.9	295	59.0		
247	52.2	296	57.3		
248	57.2	297	58.4		

Table 6. Site index estimates for the Chapman-Richards GADA equation (Equation 4).

Plot Number	Site Index Estimate	Plot Number	Site Index Estimate	Plot Number	Site Index Estimate
200	61.7	248	67.2	296	66.9
201	56.8	249	59.9	297	69.2
202	63.3	250	71.7	298	71.6
203	66.2	251	70.5	299	62.9
204	56.7	252	64.2	300	72.1
205	58.6	253	74.5	301	68.2
206	63.6	254	73.7	302	59.5
207	78.9	255	68.5	303	70.6
208	66.4	256	68.6	304	69.6
209	64.0	257	66.4	305	70.6
210	67.2	258	68.3	306	72.5
211	64.4	259	83.6	307	70.5
212	50.4	260	70.6	308	72.2
213	69.4	261	77.6	309	73.0
214	67.7	262	64.7	310	63.0
215	69.4	263	60.7	311	76.4
216	53.5	264	60.8	312	57.5
217	68.1	265	66.2	313	57.7
218	65.7	266	76.0	314	64.1
219	68.6	267	64.7	315	63.7
220	66.3	268	79.8	316	57.9
221	65.7	269	68.5	317	66.2
222	68.5	270	67.5	318	73.6
223	70.0	271	68.3	319	69.9
224	67.3	272	61.5	320	74.9
225	71.0	273	71.9	321	70.6
226	71.0	274	75.1	322	71.0
227	68.1	275	67.5	323	73.4
228	64.3	276	67.7	324	68.8
229	68.3	277	56.3	325	58.7
230	66.3	278	61.6	326	71.6
231	65.8	279	58.5	327	63.8
232	58.9	280	67.7	328	63.2
233	68.1	281	68.1	329	63.7
234	70.9	282	63.5	330	59.9
235	75.8	283	71.6	331	67.1
236	73.2	284	73.9	332	66.9
237	71.1	285	62.1		
238	61.6	286	67.1		
239	73.6	287	70.2		
240	70.4	288	69.4		
241	71.4	289	65.8		
242	75.4	290	61.5		
243	74.0	291	63.3		
244	80.2	292	75.1		
245	59.5	293	65.6		
246	49.8	294	60.4		
247	60.8	295	68.7		

Table 7. Site index estimates for the Cieszewski GADA equation (Equation 5).

Plot Number	Site Index Estimate	Plot Number	Site Index Estimate	Plot Number	Site Index Estimate
200	63.7	249	62.0	297	70.4
201	59.2	250	72.6	298	72.4
202	65.2	251	71.5	299	64.8
203	67.7	252	65.9	300	72.9
204	59.0	253	75.0	301	69.6
205	60.7	254	74.3	302	61.6
206	65.4	255	69.8	303	71.6
207	78.7	256	69.9	304	70.7
208	67.9	257	68.0	305	71.6
209	65.7	258	69.6	306	73.4
210	68.6	259	82.6	307	71.6
211	66.1	260	71.6	308	73.1
212	52.8	261	77.6	309	73.7
213	70.6	262	66.4	310	64.9
214	69.0	263	62.8	311	76.5
215	70.6	264	62.8	312	59.9
216	56.1	265	67.7	313	59.9
217	69.5	266	76.3	314	65.8
218	67.2	267	66.4	315	65.5
219	69.9	268	79.4	316	60.2
220	67.9	269	69.8	317	67.7
221	67.4	270	68.9	318	74.4
222	69.8	271	69.6	319	71.1
223	71.1	272	63.5	320	75.4
224	68.8	273	72.7	321	71.5
225	72.0	274	75.5	322	71.9
226	72.2	275	68.9	323	74.1
227	69.5	276	69.1	324	70.0
228	66.1	277	58.6	325	61.0
229	69.7	278	63.6	326	72.5
230	67.9	279	60.7	327	65.6
231	67.4	280	69.1	328	65.1
232	61.2	281	69.4	329	65.5
233	69.4	282	65.3	330	62.0
234	72.0	283	72.5	331	68.6
235	76.1	284	74.5	332	68.3
236	74.0	285	64.0		
237	72.1	286	68.5		
238	63.6	287	71.3		
239	74.2	288	70.6		
240	71.5	289	67.4		
241	72.4	290	63.6		
242	75.8	291	65.1		
243	74.6	292	75.5		
244	79.7	293	67.2		
245	61.7	294	62.5		
246	52.4	295	69.9		
247	62.9	296	68.4		
248	68.7	297	70.4		

Table 8. Site index estimates for the McDill-Amateis GADA equation (Equation 6).

Plot Number	Site Index Estimate	Plot Number	Site Index Estimate	Plot Number	Site Index Estimate
200	64.6	248	69.2	296	68.9
201	60.1	249	63.0	297	70.7
202	66.0	250	72.6	298	72.5
203	68.3	251	71.7	299	65.7
204	60.1	252	66.7	300	72.9
205	61.6	253	74.7	301	70.0
206	66.2	254	74.0	302	62.7
207	77.8	255	70.2	303	71.8
208	68.5	256	70.3	304	71.0
209	66.4	257	68.6	305	71.7
210	69.0	258	70.0	306	73.3
211	66.7	259	80.4	307	71.7
212	53.6	260	71.8	308	73.0
213	70.8	261	76.8	309	73.5
214	69.5	262	67.1	310	65.7
215	70.9	263	63.7	311	75.9
216	57.2	264	63.7	312	61.0
217	69.9	265	68.3	313	61.0
218	67.8	266	75.7	314	66.6
219	70.2	267	67.1	315	66.3
220	68.5	268	78.3	316	61.3
221	68.0	269	70.1	317	68.3
222	70.2	270	69.4	318	74.0
223	71.3	271	70.0	319	71.3
224	69.2	272	64.5	320	75.0
225	72.1	273	72.8	321	71.7
226	72.2	274	75.1	322	72.1
227	69.9	275	69.4	323	73.8
228	66.8	276	69.6	324	70.4
229	70.1	277	59.7	325	62.1
230	68.5	278	64.6	326	72.5
231	68.1	279	61.8	327	66.5
232	62.1	280	69.6	328	66.0
233	69.8	281	69.9	329	66.3
234	72.0	282	66.1	330	63.1
235	75.6	283	72.5	331	69.1
236	73.8	284	74.2	332	68.9
237	72.2	285	64.9		
238	64.5	286	69.0		
239	74.0	287	71.5		
240	71.6	288	70.8		
241	72.4	289	68.0		
242	75.2	290	64.5		
243	74.3	291	66.0		
244	78.6	292	75.2		
245	62.7	293	67.8		
246	53.6	294	63.4		
247	63.9	295	70.3		

VITA

Kynda Annettee Reed Trim graduated from Stephen F. Austin State University in 2006 with a Bachelor of Science degree in Interdisciplinary Studies. For the next 5 years, she taught 5th grade mathematics and science. She then returned to Angelina College and graduated in 2013 with an Associate of Applied Sciences in Radiation Technology. In 2015, she returned to Stephen F. Austin State University to pursue a Master of Science degree in Forestry while working as a radiation and CT technologist. Kynda and her husband, Dwain, own and operate their own reforestation business as well as an 8-house broiler farm and raise whitetail deer. She received her Master of Science degree in Forestry in 2018.

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