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ABSTRACT: *A parameter recovery procedure for the Weibull distribution function based on four percentile equations was used to develop a diameter distribution yield prediction model for unmanaged loblolly pine (*Pinus taeda *L.) plantations in East Texas. This model was compared with the diameter distribution models of Lenhart and Knowe, which have been used in East Texas. All three models were evaluated with independent observed data. The model developed in this study performed better than the other two models in prediction of trees per acre and cubic-foot volume per acre (wood and bark, excluding stump) across diameter classes. Lenhart's model consistently underestimated the larger-diameter classes because it was developed originally with data mostly collected in young plantations. Knowe's model overestimated volume in sawtimber-sized trees, which could lead to overestimations of volume in older loblolly pine plantations found in East Texas. An example also is provided to show users how to use this new yield prediction system. These results support the recommendation that forest managers should use growth and yield models designed and/or calibrated for the region in which they are implemented.* South. J. Appl. For. 30(1):13–20.

Key Words: *Pinus taeda*, growth and yield models, Weibull distribution, parameter recovery.

Forestland in East Texas occupies about 12.1 million ac (Miles 2005). Of this area, 2.9 million ac are classified as pine plantations, with about 2.7 million ac (90%) of this total on private land. Many acres of natural forest in East Texas were converted to loblolly pine plantations during the 1970s and early 1980s to provide raw material for the growing local forest products industry. A variety of site preparation techniques were used to establish these plantations, such as shearing, chopping, windrowing, and burning. However, no other treatments except possibly a prescribed burn were applied during the life of these plantations. Although many of these unmanaged plantations have been converted to intensively managed plantations in recent years, unmanaged pine plantations still can be found on many sites across East Texas. Both industrial and nonindustrial forest managers still need growth and yield information for these types of plantations growing on the western extreme of the southern pine range.

Several southwide or West Gulf Region studies provided yield estimates by diameter class that are applicable to unmanaged loblolly pine plantations in East Texas (Feduccia et al. 1979, Amateis et al. 1984, Burkhart et al. 1987). However, only 1–3% of the observations used in these studies were located in East Texas. Lenhart (1987, 1988) developed a diameter distribution yield prediction system based on 234 observations across East Texas. His data set represented relatively young plantations (mean age, 9 years; range, 3–19 years) commonly found at that time. Today, these unmanaged plantations are older and are not well represented by Lenhart's yield prediction system anymore.

The objective of this study was to develop a new diameter distribution yield prediction system for unmanaged loblolly pine plantations in East Texas. We hoped to improve on Lenhart (1988) in two ways: (1) use data from

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older plantations in model development and (2) build a new yield prediction system based on different underlying equations rather than update his yield prediction equations. We also wanted to compare our new model with that of Knowe (1992). Although not developed with East Texas data, Knowe's model often is used in East Texas for older, unmanaged plantations (up to 30 years).

Study Site Description

This study used 1,111 observations for loblolly pine from 269 remeasured permanent plots located in East Texas pine plantations (Table 1). From the total 1,111 observations, approximately 10% ($n = 104$ observations from 82 permanent plots) were randomly selected and removed from the data set used for model fitting. They were reserved for model evaluation. Thus, a total of 1,007 observations from 187 permanent plots were used for model fitting.

The 269 permanent plots are part of the East Texas Pine Plantation Research Project (ETPPRP; Lenhart et al. 1985), which covers 22 counties across East Texas. Generally, the counties are located within the rectangle from 30 to 35° north latitude and 93 to 96° west longitude. Each plot consists of two adjacent subplots separated by a 60-ft buffer. Within a subplot, diameter at breast height (dbh; measured at 4.5 ft above the groundline), total height, and the survival status (live or dead) were monitored for each planted loblolly pine tree over a 20-year period. Plots were remeasured on fixed, 3-year intervals. Data from only one subplot (the development subplot) were used in this study.

Diameter Distribution Model

Bailey and Dell (1973) first introduced the Weibull cumulative distribution function to model diameter distributions in single-species, single-cohort stands:

$$F(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \quad (1)$$

where ($a \leq x < \infty$), 0 otherwise,

a = location parameter (minimum diameter),
b = scale parameter, and
c = shape parameter.

To compute relative proportions of trees per acre (TPA) by diameter class, substitute the upper and lower limits of the class into Equation 1. Next, subtract the cumulative distribution up to the lower limit of the class from the upper limit to find the proportion of TPA in that class (Avery and Burkhart 1994):

$$P_i = F(U_i) - F(U_{i-1}),$$

or

$$P_i = \left(1 - \exp\left[-\left(\frac{U_i - a}{b}\right)^c\right]\right) - \left(1 - \exp\left[-\left(\frac{U_{i-1} - a}{b}\right)^c\right]\right) \quad (2)$$

where,

P_i = proportion of trees in diameter class i,
U_i = upper limit of diameter class i,

and other variables are defined as aforementioned.

Da Silva (1986) introduced a parameter recovery method subsequently used by Bailey et al. (1989) and Brooks et al. (1992) that is based on the 0th, 25th, 50th, and 95th diameter percentiles. The parameter recovery procedure uses the expected value of the minimum observation from a sample size n from the Weibull distribution, the four percentiles (D_0, D_{25}, D_{50}, and D_{95}, respectively), and the second moment of the Weibull distribution to estimate the a, b, and c parameters.

Da Silva's procedure first determines the predicted location parameter (a) using the predicted values for D_0 and D_{50}, sample size (n = TPA * (100 * 100)/43,560 = TPA * 0.229568411), and an initial assumption that the shape parameter (c) is 3.0:

Table 1. Observed stand characteristics for East Texas unthinned loblolly pine plantation datasets for percentile diameter prediction model (Equations 6–10).

	Model development dataset (n = 1,007 observations from 187 plots)				Model evaluation dataset (n = 104 observations from 82 plots)			
Variables	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
A	14.2	6.7	3.0	37.0	14.3	6.7	3.0	29.0
Hd	45.3	19.8	3.0	94.0	44.5	20.8	3.0	86.0
SI	69.4	12.2	25.0	117.0	68.0	13.4	27.0	102.0
ITPA	703.7	142.2	363.0	1,361.0	717.8	165.2	363.0	1,361.0
TPA	450.4	145.4	30.0	1,002.0	437.6	158.2	87.0	958.0
D_0	2.2	1.6	0.0	7.7	2.2	1.7	0.0	7.4
D_{25}	4.8	2.2	0.0	10.3	4.7	2.4	0.0	12.2
D_{50}	5.8	2.5	0.0	11.7	5.7	2.8	0.0	13.1
D_{95}	7.9	3.2	0.0	16.3	7.9	3.6	0.0	16.4
Dq	5.9	2.5	0.0	11.5	5.8	2.8	0.0	12.9
RNTB	0.1	0.1	0.0	0.9	0.1	0.1	0.0	0.6

Note: A = plantation age (total years), Hd = average height of dominant and codominant trees (feet), SI = site index (base age = 25 yr), ITPA = trees per acre at planting, TPA = trees per acre, PBA = loblolly pine basal area per acre (BAPA, ft^2), D_i = diameter (in.) at the i = 0, 25, 50, and 95th percentiles, Dq = quadratic mean diameter (in.), RNTB = ratio of nonplanted BAPA (volunteer pines, hardwoods, and large shrubs) to the total BAPA (nonplanted plus planted pine).

$$a = (n^{1/3}D_0 - D_{50})/(n^{1/3} - 1), \text{ if } a < 0.0 \text{ then } a = 0. \quad (3)$$

The shape parameter (c) was estimated by using the value from Equation 3, $D_{95,}$ and D_{25}:

$$c = 2.343088/ln\left[\frac{D_{95} - a}{D_{25} - a}\right]. \quad (4)$$

The scale parameter (b) was obtained by solving the second moment of the Weibull distribution for the positive root with the estimates for a, c, and Dq^2:

$$b = -\frac{a\Gamma_i}{\Gamma_2} + \sqrt{\left(\frac{a}{\Gamma_2}\right)^2(\Gamma_1{}^2 - \Gamma_2) + \frac{Dq^2}{\Gamma_2}}, \quad (5)$$

where,

Γ = the gamma function,
Γ_1 = $\Gamma(1 + 1/c)$,
Γ_2 = $\Gamma(1 + 2/c)$, and
Dq = quadratic mean diameter.

Quadratic Mean Diameter Model

Quadratic mean diameter (Dq) is the most important independent variable in predicting the percentile-based diameter prediction equations. The following model was used in this study (Table 2):

$$Dq = \exp\left(\beta_0 + \beta_1\left(\frac{1}{Hd}\right) + \beta_2 ln(A) + \beta_3 ln(A * TPA)\right) + \varepsilon \quad (6)$$

where

Hd = average height of dominant and codominant trees,
A = plantation age,
TPA = trees per acre,
β_i = coefficients to be estimated,
ε = random error

and all other variables are defined as aforementioned.

Percentile Prediction Equations

The prediction equations for the 0th, 25th, 50th, and 95th percentiles were fitted simultaneously for this study (Table 2):

$$D_0 = \beta_0 + \beta_1 * Dq + \beta_2 * A + \varepsilon \quad (7)$$

$$D_{25} = \beta_0 + \beta_1 * Dq + \beta_2 * A + \varepsilon \quad (8)$$

$$D_{50} = \beta_0 + \beta_1 * Dq + \beta_2 * A + \varepsilon \quad (9)$$

$$D_{95} = \beta_0 + \beta_1 * Dq + \beta_2 * A + \varepsilon \quad (10)$$

where all variables are defined as before. Because Dq appears in Equations 6–10, seemingly unrelated regression (SUR) was used to account for correlation across the equations (Borders 1989, Robinson 2004). SUR also reduced serious autocorrelation associated with fitting Equations 6–10 to the repeated measurement data from the permanent plots. (Using the Durbin-Watson test statistic, we failed to reject the null hypothesis, H0; the error terms are not autocorrelated, at both the 95% and the 99% confidence levels for negative autocorrelation [all Durbin-Watson statistics ranged from 2.1 to 2.4]. For positive autocorrelation,

Table 2. Parameter estimates and fit statistics of East Texas loblolly pine plantation predictive equations for quadratic mean diameter (Dq), percentiles of the diameter distribution (D_0, D_{25}, D_{50}, and D_{95}), DI guide curve (Hd), and individual loblolly pine tree height (h_i).

Equation	Parameter	Parameter Estimate	Standard Error	$Pr(\beta_i = 0)$	R^2	RMSE
6	β_0	3.35089	0.07879	<0.0001	0.975	0.109
(Dq)	β_1	−24.29940	0.26272	<0.0001		
	β_2	0.35747	0.01306	<0.0001		
	β_3	−0.22096	0.00968	<0.0001		
7	β_0	−1.19901	0.05819	<0.0001	0.803	0.695
(D_0)	β_1	0.47833	0.01753	<0.0001		
	β_2	0.03834	0.00650	<0.0001		
8	β_0	−0.32783	0.02346	<0.0001	0.983	0.280
(D_{25})	β_1	0.97042	0.00707	<0.0001		
	β_2	−0.04100	0.00262	<0.0001		
9	β_0	−0.11524	0.01283	<0.0001	0.996	0.153
(D_{50})	β_1	1.04189	0.00387	<0.0001		
	β_2	−0.01489	0.00143	<0.0001		
10	β_0	0.41539	0.03326	<0.0001	0.984	0.397
(D_{95})	β_1	1.17813	0.01002	<0.0001		
	β_2	0.04418	0.00372	<0.0001		
12	β_0	86.4424	0.75190	<0.0001	0.874	7.319
(Hd)	β_1	0.08480	0.00191	<0.0001		
	β_2	1.60620	0.02670	<0.0001		
15	β_0	0.02806	0.00111	<0.0001	0.626	0.116
(h_i)	β_1	0.36027	0.00183	<0.0001		
	β_2	0.03718	0.00113	<0.0001		

RMSE = root mean square errors.

we failed to reject H0 at both the 95% and the 99% confidence levels for Equations 8–10 [all Durbin-Watson statistics = 1.9], and the 99% confidence level for Equations 6 and 7 [both Durbin-Watson statistics = 1.6].)

Survival Model

Adams et al. (1996) developed a survival model that incorporates site quality and the incidence of fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *fusiforme*). The model consists of a system of two equations, one of them represents the number of surviving trees infected by fusiform rust and the other represents the number of trees not infected by fusiform rust. The Adams survival model was refitted using the most current loblolly development subplot data set:

$$N_{i2} = (N_{i1} - 0.120333N_{u1}) \exp(-0.00096 * \mathrm{SI} * (A_2 - A_1)) + 0.120333N_{u1} \exp(-0.00022 * \mathrm{SI} * (A_2 - A_1))$$

$R^2 = 0.572$ RMSE $= 25.250$ $n = 949$

$$N_{u2} = N_{u1} \exp(-0.00022 * \mathrm{SI} * (A_2 - A_1))$$

$$R^2 = 0.942 \quad \mathrm{RMSE} = 34.120 \quad n = 949 \tag{11}$$

where

A_2 = projection age (years),
A_1 = initial age (years),
N_{i2} = number of surviving infected TPA at A_2,
N_{i1} = number of surviving infected TPA at A_1,
N_{u2} = number of surviving uninfected TPA at A_2,
N_{u1} = number of surviving uninfected TPA at A_1,
SI = site index in feet (index age, 25 years),

and all other variables are defined as aforementioned.

SI and Individual Tree Height Prediction Equations

The Chapman-Richards growth function was used as a guide curve to develop an anamorphic SI prediction equation to estimate SI for a given index age (base age, 25 years). A total of 11,367 age-height pairs of the ten tallest trees in the loblolly development subplots were used to fit the guide curve equation (Table 2):

$$Hd = \beta_0[1 - \exp(-\beta_1 * A)]^{\beta_2} + \varepsilon \tag{12}$$

where all variables are defined as aforementioned.

If SI is unknown, but A and Hd are known, then SI is estimated by the following equation:

$$SI = Hd\left[\frac{0.87997}{1 - \exp(-0.0848 * A)}\right]^{1.6062} \tag{13}$$

Equation 13 can be algebraically rearranged to predict Hd from the SI and A.

$$Hd = SI\left[\frac{1 - \exp(-0.0848 * A)}{0.87997}\right]^{1.6062} \tag{14}$$

The height-diameter model developed by Lenhart (1968) was used for prediction of individual loblolly pine tree height. A total of 36,995 individual loblolly pine tree height-diameter observations from the most current loblolly development subplot data set were used to fit Lenhart's model (Table 2):

$$ln(h_i) = ln(Hd) + \beta_0 + (\ln(d_i) - \ln(D_{\max})) * (\beta_1 + \beta_2 \ln(Dq)) + \varepsilon, \tag{15}$$

where

h_i = predicted height of the ith tree,
d_i = dbh of the ith tree,
$D_{\max}$ = midpoint value of the largest diameter class,

and all other variables are defined as aforementioned.

A property of Equation 15 is that as d approaches $D_{\max}$, h approaches $e\beta_0 * Hd$. Other variants of this type of tree height prediction model for even-aged stands have been developed, which relate tree height to dbh and variety of stand attributes (Clutter et al. 1983, Amateis et al. 1984, Zhang et al. 1997).

Individual Tree Volume Model

The individual tree cubic-foot volume equation from Lenhart et al. (1987) was used in this study:

$$V = 0.002103D^{1.958489}H^{1.062348} - 0.0020323(d^{3.187878}/D^{1.187878})(H - 4.5), \tag{16}$$

where

V = cubic-foot volume for wood and bark, excluding stump,
D = dbh (in inches),
H = total height of the tree (in feet), and
d = merchantable top diameter (in inches).

Equation 16 can be used to estimate total stem content as well as stem content to any upper-stem diameter. This equation also can be converted into taper function to estimate upper-stem diameter outside bark (d) and height position (h) on the stem where d occurs.

Comparison and Evaluation Criteria

The diameter distribution growth model developed in this study was compared with the models of Lenhart (1988) and Knowe (1992). All three models were evaluated with the independent 10% evaluation data set (n = 104). To provide fair comparisons, all three models used the same volume equation, height-diameter equation, and survival equations; only the core diameter distribution prediction equations differed. Furthermore, because Knowe's model requires an estimate of competing vegetation, the ratio of nonplanted woody vegetation (volunteer pine, hardwoods, and large shrubs) basal area per acre to total (planted pine plus nonplanted vegetation) basal area per acre this ratio was set to 0.1, which is the mean value observed in the data set (Table 1). If this ratio was set to zero, Knowe's model would grow pine as if no competition existed, which would overpredict growth compared with the other two models.

The three models were evaluated at two levels: (1) *Dq*, overall volume (volume = cubic-foot volume per acre for total tree including wood and bark, but excluding stump) and TPA and (2) volume and TPA by 1-in.-diameter classes. To compare *Dq*, overall volume, and TPA, this study used an estimation procedure developed by Reynolds (1984). His procedures test both bias and precision rather than overall prediction accuracy. These procedures were converted to a BASIC program (Rauscher 1986), and then later to an SAS program (SASATEST; Gribko and Wiant [1992]). SASATEST examines both bias and precision on an absolute or percentage basis, but this study used percent bias only. In SASATEST, percent bias is calculated as a percentage of the observed value:

$$BIAS = 100\left(\frac{(\hat{Y} - Y)}{Y}\right)$$

where $\hat{Y}$ is predicted *Dq*, volume, or TPA, and Y is observed *Dq*, volume, or TPA. In this study, precision is expressed as the standard deviation (SD) of percent bias, which also is calculated by SASATEST. SASATEST then uses the mean percent bias (measure of bias) and the SD (measure of precision) to calculate a 95% confidence interval. If this confidence interval does not contain zero, then the bias is significant at the $\alpha = 0.05$ level. SASATEST also checks the errors between predicted and observed values for departures from normality. If nonnormality is detected (as was the case for this study), a 10% trimmed mean and jackknife SD were used to provide more robust confidence intervals. The mean squared error (MSE = bias^2 + σ^2) also was used to evaluate *Dq*, volume, and TPA because a biased estimator with a small variance may be preferable to an unbiased estimator with a large variance (Devore 1982).

To compare volume and TPA at the diameter class level, mean volume and TPA were calculated for each 1-in.-diameter class. Then, for each of the three models, mean difference (MD = predicted [− observed) for both volume and TPA were calculated for each 1-in.-diameter class. Then, MD was plotted over diameter class to further examine model prediction trends across the range of diameter classes, where negative values represent underpredictions and positive values represent overpredictions.

Evaluation Results

Bias was not significant ($P > 0.05$) for our model's prediction of overall TPA and *Dq* (Table 3). TPA precision was highest for our model, and then Knowe (1992), and, last, Lenhart (1988). *Dq* precision was highest for Lenhart (1988), followed by our model, and then Knowe (1992). Bias was significant ($P < 0.05$) for predictions of overall volume by our model and Lenhart (1988), but not for Knowe (1992; Table 3). Volume precision was about equal for our model and Knowe (1992), although it was much lower for Lenhart (1988). We were puzzled at the overall

Table 3. RMSE and 95% CI for observed and predicted TPA, volume, and quadratic mean diameter (*Dq*) from three loblolly pine growth and yield models (this study, Knowe [1992], and Lenhart [1988]).

Source		95% CI	RMSE
This study	TPA	−2.52 ± 3.05	15.34
	Volume	−6.15 ± 5.45[a]	27.72
	Dq	−0.23 ± 2.32	11.51
Knowe (1992)	TPA	−9.08 ± 4.15[a]	22.49
	Volume	2.71 ± 6.10	30.40
	Dq	6.20 ± 2.65[a]	14.55
Lenhart (1988)	TPA	−27.82 ± 6.59[a]	42.94
	Volume	−25.14 ± 6.24[a]	39.86
	Dq	−2.32 ± 2.02[a]	10.31

Note: All values are expressed as a percent of observed.
[a]Significant ($P < 0.05$).
CI = confidence interval, RMSE = root means square errors.

bias results for Knowe (1992) because the sign for TPA bias was opposite those of *Dq* and volume bias. However, the diameter class results, discussed next, showed that Knowe's model greatly underpredicted TPA in the 3-in.-diameter class, which contributed to the overall negative bias for TPA. Similarly, Lenhart's model greatly underpredicted TPA in both the 3- and the 4-in.-diameter classes, although the signs for all measures of overall bias were all negative. Lenhart's model also showed much higher bias for overall TPA and volume compared with the other two models (Table 3).

At the diameter class level, TPA and volume MD values for our model usually fell closest to the zero line compared with the models of Knowe and Lenhart (Figures 1 and 2). In particular, our model predicted TPA and volume better than the other models for diameter classes greater than 9 in. Knowe's model overpredicted TPA and volume for diameter classes greater than 9 in. Although Knowe's model overpredicted TPA for diameter classes between 4 and 9 in., it underpredicted volume. This seems to indicate that more trees were predicted than observed, but they were not of sufficient size to have predicted more volume than observed. Although all models overpredicted TPA for diameter classes around 7 in. (Figure 1) and overpredicted volume

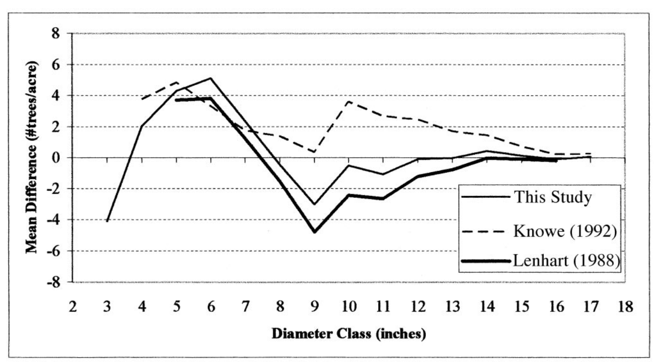

Figure 1. Mean difference (predicted − observed) of TPA from three loblolly pine growth and yield models (this study, Knowe [1992]), and Lenhart [1988]). Note, diameter classes 3 and 4 are not shown for Lenhart or for Knowe's diameter class 3 because the mean differences exceeded −8.0 (mean differences were −57, −61, and −57, respectively).

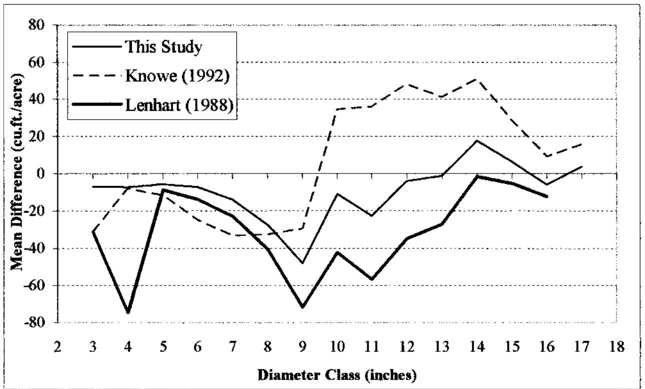

Figure 2. Mean difference (predicted − observed) of cubic-foot volume wood and bark excluding stump (volume) from three loblolly pine growth and yield models (this study, Knowe [1992], and Lenhart [1988]).

for diameter classes below 9 in. (Figure 2), Lenhart's model underpredicted TPA for diameter classes above 7 in. (Figure 1) and underpredicted volume for all diameter classes (Figure 2). Although not shown in Figure 1, Knowe's model greatly underpredicted TPA in the 3-in.-diameter class (MD = −57), while Lenhart's model greatly underpredicted TPA in both the 3-in. and the 4-in.-diameter classes (MD = −61 and −57, respectively). This large underprediction occurred because these models failed to predict any trees in these diameter classes, although trees were observed. Our model did not suffer from this problem because it predicted trees in these diameter classes.

Comparison of Models with Examples of Yield Prediction

The predicted cubic-foot volume per acre for wood and bark excluding stump (volume) by 1-in.-diameter classes for the model developed in this study was compared with the models of Knowe (1992) and Lenhart (1988) (Figure 3, a–c). The comparison used an example of 25-year-old loblolly pine plantation initially planted at 600 TPA for three different SI values: 50, 70, and 90 ft. Fusiform rust infection rate was set to 10%. These conditions are representative of loblolly pine plantations at rotation age in East Texas.

For an SI of 50, both Knowe's and Lenhart's models predicted lower volume compared with our model, although the range of diameter classes was similar (Figure 3a). For an SI of 70, Lenhart's model continued to predict lower volume compared with our model, but Knowe's model predicted volume similar to our model (Figure 3b). The range of diameter classes was similar between the three models, except that Lenhart's model predicted a smaller maximum-diameter class than the other two models. We are not surprised by this result considering Lenhart's model was developed mostly with younger plantation data. For an SI of 90, Lenhart's model again predicted less volume, but Knowe's model predicted more volume compared with our model (Figure 3c). Differences in the range of diameter classes were more pronounced between the three models,

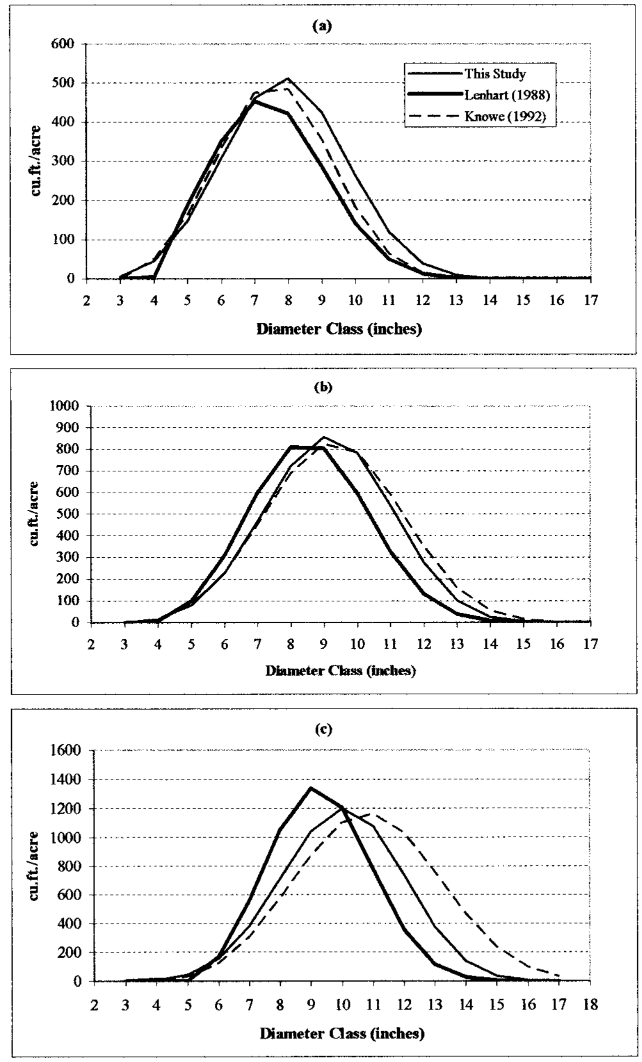

Figure 3. Predictions of volume by diameter class from three loblolly pine growth models (this study, Lenhart [1988], Knowe [1992]) for a 25-year-old loblolly pine plantation in East Texas with 600 initial TPA at each of the following site indices: (a) 50 ft, (b) 70 ft, and (c) 90 ft.

but only for the larger-diameter classes. Lenhart's model predicted more volume in the 9-in.-diameter class compared with the other two models. We again attribute this to Lenhart's use of younger and therefore smaller trees to develop his model. Because his model estimated fewer trees in the larger-diameter classes compared with the other two models, most of his volume was concentrated in the lower-diameter classes, which is evident for the 9-in. class. Lenhart's model has increasingly less volume for diameter classes greater than 10 in. compared with the other two models for this same reason. For the entire range of SI, Lenhart's model predicted less volume compared with our model. Knowe's model predicted less volume at lower SI, more volume at higher SI, and similar volume at medium SI compared with our model. This supports the notion that Knowe's model predicts volume less reliably compared with our model at the extremes of SI typically found in East Texas. This notion is further supported from the earlier result that showed Knowe's model significantly overpredicted volume for larger-diameter classes.

Applications

To show how to use the new model, we will illustrate the procedure used to produce Figure 3b.

1. Calculate the average height of the dominant and codominant trees (Equation 14; recall SI = 70 ft; age = 25 years):

$$Hd = 70 * \left[\frac{1 - \exp(-0.0848 * 25)}{0.87997}\right]^{1.6062} = 70 \text{ feet}$$

2. Calculate surviving TPA (Equation 11; recall initial TPA = 600; fusiform infection rate = 10%):

$$N_{i2} = (60 - 0.120333 * 540)\exp(-0.00096 * 70 * (25 - 0)) + 0.120333 * 540 * \exp(-0.00022 * 70 * (25 - 0)) = 43.3$$

$$N_{u2} = 540 * \exp(-0.00022 * 70 * (25 - 0)) = 367.4$$

total TPA = 43.3 + 367.4 = 410.7

3. Calculate Dq (Equation 6):

$$Dq = \exp\left(3.35089 - 24.29940\left(\frac{1}{70}\right) + 0.35747 ln(25) - 0.22096 ln(25 * 410.7)\right) = 8.3 \text{ in.}$$

4. Calculate percentiles (Equations 7–10):

$$D_0 = -1.19901 + 0.47833 * 8.3 + 0.03834 * 25 = 3.7 \text{ in.}$$

$$D_{25} = -0.32783 + 0.97042 * 8.3 - 0.04100 * 25 = 6.7 \text{ in.}$$

$$D_{50} = -0.11524 + 1.04189 * 8.3 - 0.01489 * 25 = 8.2 \text{ in.}$$

$$D_{95} = 0.41539 + 1.17813 * 8.3 + 0.04418 * 25 = 11.3 \text{ in.}$$

5. Calculate the Weibull parameters (Equations 3–5):

$$n = 410.7 * 0.229568411 = 94.2837$$

$$a = (94.2837^{1/3} * 3.7 - 8.2)/(94.2837^{1/3} - 1) = 2.4$$

$$c = 2.343088/ln\left[\frac{11.3 - 2.4}{6.7 - 2.4}\right] = 3.2$$

$$b = -\frac{2.4 * \Gamma(1 + 1/3.2)}{\gamma(1 + 2/3.2)} + \sqrt{\left(\frac{2.4}{\Gamma(1 + 2/3.2)}\right)^2 (\Gamma(1 + 1/3.2)^2 - \Gamma(1 + 2/3.2)) + \frac{8.3^2}{\Gamma(1 + 2/3.2)}} = 6.2$$

6. Calculate the TPA in each diameter class (Equation 2). For example, use diameter class = 10 in.:

$$P_{10} = \left(1 - \exp\left[-\left(\frac{10.5 - 2.4}{6.2}\right)^{3.2}\right]\right) - \left(1 - \exp\left[-\left(\frac{9.5 - 2.4^{3.2}}{6.2}\right]\right) = 0.12$$

$$TPA_{10} = 0.12 * 410.7 = 49 \text{ TPA in 10-in. class}$$

This process is repeated for all diameter classes.

7. Calculate the average height for each diameter class (Equation 15). Again, use the 10-in.-diameter class as an example:

$$ln(h_{10}) = ln(70) + 0.02806 + (ln(10) - ln(14)) * (0.36027 + 0.03718 ln(8.3)) = 62 \text{ feet}$$

This process is repeated for all diameter classes.

8. Calculate the volume for each diameter class (Equation 16). Again, use the 10-in.-diameter class as an example (note that $d = 0$ because total tree volume is desired):

$$CFV_{tree} = 0.002103 * 10^{1.958489} 62^{1.062348} = 15.3 \text{ ft}^3$$

CFV for the 10-in.-diameter class = 15.3 ft^3 * 49 TPA = 750 ft^3 per ac. This process is repeated for all diameter classes.

9. Once all diameter classes are calculated, sum volume across diameter classes to find the total volume for the plantation. This process can be repeated for different ages and SI to create stand-level yield curves (Figure 4). Note, rounding errors are present in this example; the answers will differ if all decimal places are carried throughout the calculations.

Conclusions

The model developed in this study represents an improvement over the model of Lenhart (1988). Lenhart's model was developed with data predominantly collected from young plantations. The data used to develop the new

Figure 4. Predictions of cubic-foot volume wood and bark excluding stump per acre (volume) over time from the new East Texas loblolly pine growth-and-yield model with 600 initial TPA for different levels of SI (SI in feet, base age, 5 years).

model in this study incorporate more observations in older plantations. The model developed in this study is also superior to the model of Knowe (1992) for unmanaged loblolly pine plantations in East Texas. Knowe's model consistently overpredicted volume in diameter classes greater than 9 in. It also predicted greater volume for higher SIs and lower volume for lower SIs, which could lead to incorrect estimations of volumes if it was used for yield forecasting in East Texas. Based on the results of this study, we recommend that growth models be developed and/or calibrated for the regions in which they are used.

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