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A New Diameter Distribution Model for Unmanaged Slash Pine Plantations in East Texas

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ABSTRACT

A parameter recovery procedure for the Weibull distribution function based on four percentile equations was used to develop a new diameter distribution yield prediction model for unmanaged slash pine (*Pinus elliottii* Engelm.) plantations in East Texas. This new model was similar in structure to the model of Lee and Coble (Lee, Y.J., and D.W. Coble. 2006. A new diameter distribution model for unmanaged loblolly pine plantations in east Texas. *South. J. Appl. For.* 30(1):13–20) in their work with East Texas loblolly pine plantations. The new model was compared with the diameter distribution model of Lenhart (Lenhart, J.D. 1988. Diameter distribution yield prediction system for unthinned loblolly and slash pine plantations on non-old-fields in east Texas. *South. J. Appl. For.* 12(4):239–242. 1988), which was developed for slash pine plantations in East Texas, as well as to two other models developed using iterative techniques suggested and inspired by Cao (Cao, Q. 2004. Predicting parameters of a Weibull function for modeling diameter distribution. *For. Sci.* 50(5):682–685). The model developed in this study was preferred over Lenhart (Lenhart 1988) and the other two models in prediction of total trees per acre, basal area per acre, quadratic mean diameter, and cubic-foot volume per acre (wood and bark, excluding stump). An example also is provided to show users how to use this new yield prediction system. We recommend that the model developed in this study be used to estimate growth and yield of East Texas slash pine plantations.

Keywords: *Pinus elliottii*, growth and yield models, Weibull distribution, parameter recovery

Slash pine was planted extensively in East Texas during the 1970s and early 1980s to provide raw material for the growing local forest products industry. However, these plantations suffered up to 50% infection rates from fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *fusiforme*); Coble and Lee 2004). This caused interest to wane for establishing additional slash pine plantations. With the promise of rust-resistant seedlings, interest has grown to again plant slash pine. At the same time, many of the earlier slash pine plantations have been converted to loblolly pine plantations, although some slash pine plantations still can be found on many sites throughout East Texas. Therefore, industrial and nonindustrial forest managers still need growth and yield information for slash pine plantations growing on the western extreme of the southern pine range.

Dell et al. (1979) first provided yield estimates by diameter classes that were applicable to slash pine plantations in the West Gulf region, although East Texas was underrepresented in their study. Lenhart (1988) developed a diameter distribution yield prediction system for unmanaged slash pine plantations based on 124 observations across East Texas. His data set represented relatively young plantations (mean age, 8 years; range, 3–18 years) commonly found at that time. A variety of site preparation techniques were used to establish these plantations, such as shearing, chopping, windrowing, and burning. However, no other treatments were applied after establishment, except possibly a prescribed burn in older plantations. Today, these unmanaged plantations are older and are no longer well represented by Lenhart's data set.

The objective of this study was to develop a new diameter distribution yield prediction system for unmanaged slash pine plantations in East Texas. Three new models were developed using the methodologies of Lee and Coble (2006), an iterative modification of Lee and Coble (2006), and an iterative procedure introduced by Cao (2004). These three new models also were compared to the slash pine diameter distribution model of Lenhart (1988) for East Texas.

Methods

Data Description

This study used 484 observations from 84 remeasured permanent plots located in East Texas slash pine plantations (Table 1). These plots are part of the East Texas Pine Plantation Research Project (Lenhart et al. 1985). From the total 484 observations, approximately 10% ($n = 45$ observations from 36 permanent plots) were randomly selected and removed from the data set used for model fitting. They were reserved for model evaluation. Thus, a total of 439 observations from 84 permanent plots were used for model fitting. After validation was complete, the 90 and 10% data sets were combined into a 100% data set (i.e., all observations) to refit the final prediction equations. Refer to Lee and Coble (2006) for a description of the study sites.

Statistical Analysis

Lee and Coble (2006) developed a new diameter distribution model for loblolly pine in East Texas. The functional forms of their prediction equations for diameter percentiles, quadratic mean diameter, surviving trees, dominant height, and individual tree height

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Table 1. Observed stand characteristics for East Texas unmanaged slash pine plantation data sets.

Variables	Model development data set (n = 439 observations from 84 plots)				Model evaluation data set (n = 45 observations from 36 plots)			
	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
A	14.1	6.3	3.0	33.0	13.5	6.7	2.0	29.0
Hd	45.2	19.4	4.0	91.0	41.9	20.8	6.0	87.0
SI	73.7	12.2	15.0	117.0	71.0	13.1	35.0	96.0
ITPA	723.6	185.5	453.0	1,361.0	708.5	163.2	453.0	1,089.0
TPA	360.1	174.9	61.0	1,002.0	353.0	162.3	70.0	741.0
PBA	66.9	40.7	0.5	168.3	58.8	39.5	0.7	148.5
D_0	2.4	1.6	0.0	10.2	2.1	1.9	0.0	9.5
D_{25}	5.0	2.2	0.0	11.3	4.6	2.5	0.0	10.7
D_{50}	6.0	2.5	0.0	12.3	5.6	2.8	0.0	11.6
D_{95}	8.0	3.2	0.0	15.8	7.5	3.6	0.6	15.1
Dq	6.0	2.4	0.1	12.5	5.6	2.8	0.3	11.7

A, plantation age (total yr); Hd, average height of dominant and codominant trees (ft); SI = site index (base age, 25 yr); ITPA = TPA at planting; PBA, slash pine basal area per acre (BAPA, ft^2); D_i, diameter (in.) at the i = 0th, 25th, 50th, and 95th percentiles; Dq, quadratic mean diameter (in.); SD, standard deviation.

were applied to the slash pine data of this study to develop the first new model (hereafter called LEE). These equations are presented:

$$Dq = \exp\left(\begin{array}{l}\alpha_0 + \alpha_1(1/Hd) + \alpha_2\ln(A)\\ \quad + \alpha_3\ln(A * \mathrm{TPA})\end{array}\right) + \varepsilon, \quad (1)$$

$$D_0 = \beta_0 + \beta_1 * Dq + \beta_2 * A + \varepsilon, \quad (2)$$

$$D_{25} = \chi_0 + \chi_1 * Dq + \chi_2 * A + \varepsilon, \quad (3)$$

$$D_{50} = \delta_0 + \delta_1 * Dq + \delta_2 * A + \varepsilon, \quad (4)$$

$$D_{95} = \phi_0 + \phi_1 * Dq + \phi_2 * A + \varepsilon, \quad (5)$$

$$N_{i2} = (N_{i1} - \psi_0 N_{u1})\exp(\psi_1 * \mathrm{SI} * (A_2 - A_1)) + \psi_0 N_{u1}\exp(\psi_2 * \mathrm{SI} * (A_2 - A_1)),$$

$$N_{u2} = N_{u1}\exp(-\psi_2 * \mathrm{SI} * (A_2 - A_1)), \quad (6)$$

$$Hd = \gamma_0[1 - \exp(-\gamma_1 * A)]^{\gamma_2} + \varepsilon, \quad (7)$$

$$\mathrm{SI} = Hd\left[\frac{1 - \exp(-\gamma_1 * 25)}{1 - \exp(-\gamma_1 * A)}\right]^{\gamma_2}, \quad (8)$$

$$Hd = \mathrm{SI}\left[\frac{1 - \exp(-\gamma_1 * A)}{1 - \exp(-\gamma_1 * 25)}\right]^{\gamma_2}, \quad (9)$$

$$\ln(H) = \ln(Hd) + \tau_0 + (\ln(D) - \ln(D_{\max}) * (\tau_1 + \tau_2\ln(Dq)) + \varepsilon, \quad (10)$$

$$V = 0.002021D^{1.790506}H^{1.183087} - 0.0024438(d^{3.62363}/D^{1.62363})(H - 4.5) \quad (11)$$

where TPA = trees per acre, Dq = quadratic mean diameter (in.), D_0 = 0th diameter percentile (in.), D_{25} = 25th diameter percentile (in.), D_{50} =50th diameter percentile (in.), D_{95} = 95th diameter percentile (in.), Hd = average height (ft) of dominant and codominant trees, A = plantation age (years), A_2 = projection age (years), A_1 = initial age (years), N_{12} = number of surviving infected TPA at A_2, N_{i1} = number of surviving infected TPA at A_1, N_{u2} = number of surviving uninfected TPA at A_2, N_{u1} = number of surviving uninfected TPA at A_1, SI = site index in feet (index age = 25 years), H = total height of the tree (ft), D = dbh (in.), $D_{\max}$ = midpoint value of the largest diameter class, V = cubic-foot volume for wood and bark, excluding stump (Lenhart et al. 1987), d = merchantable top diameter (in.) is zero for this study, ln = natural logarithm, exp = exponential function, α_i, β_i, χ_i, δ_i, ϕ_i, ψ_i, γ_i, τ_i = coefficients to be estimated, and ε = random error.

The LEE model used the same procedure (Da Silva 1986) as Lee and Coble (2006) to estimate the three parameters of the Weibull cdf. See Lee and Coble (2006) for further details.

In keeping with Da Silva (1986), Lee and Coble (2006) specified that the shape parameter c be set equal to 3.0 to initially determine the location parameter a for the Weibull distribution function. For the second new model of this study (hereafter called ModLEE), c was determined iteratively; otherwise, the procedure was identical to that of Lee and Coble (2006). The final value for c was selected when the change in total cubic-foot volume did not differ between iterations by more than 0.1 ft^3. This procedural modification from Lee and Coble (2006) refined the estimate for c, rather than relying on a fixed value of 3.0.

As in Lee and Coble (2006), seemingly unrelated regression was used in the MODEL procedure of SAS to account for correlation across the equations (Robinson 2004, Borders 1989). Seemingly unrelated regression also reduced serious autocorrelation associated with fitting the system of equations with serially correlated data (i.e., remeasured plot data). The Durbin-Watson test statistic was not significant at both the 95 and 99% confidence levels for negative and positive autocorrelation (all DW statistics ranged from 0.8 to 2.1).

Cao (2004) compared six methods to determine the three parameters of the Weibull cdf. He recommended his method 6, cdf regression, as the superior choice; so it was implemented in this study (hereafter called CAO). This method predicts the a, b, and c parameters of the Weibull cdf directly using the equations

$$a = 0.5 * D_0', \quad (12)$$

$$b = \exp(\xi_1 + \xi_2\mathrm{RS} + \xi_3\ln(\mathrm{TPA}) + \xi_4\ln(Hd) + \xi_5A^{-1} + \varepsilon), \quad (13)$$

$$c = \exp(\varphi_1 + \varphi_2\mathrm{RS} + \varphi_3\ln(\mathrm{TPA}) + \varphi_4\ln(Hd) + \varphi_5A^{-1} + \varepsilon), \quad (14)$$

$$D_0' = \exp(\theta_1 + \theta_2\mathrm{RS} + \theta_3\ln(\mathrm{TPA}) + \theta_4\ln(Hd) + \theta_5A^{-1} + \varepsilon), \quad (15)$$

Table 2. Parameter estimates and fit statistics of East Texas slash pine plantation predictive equations for quadratic mean diameter (Dq), percentiles of the diameter distribution (D_0, D_{25}, D_{50}, and D_{95}), survival (N), SI guide curve (Hd), and individual slash pine tree height (h_i) for the LEE model developed in this study.

Equation	Parameter	Parameter estimate	Standard error	$Pr(\beta_i = 0)$	R^2	RMSE
1 (Dq)	α_0	2.36552	0.10506	<0.0001	0.963	0.122
	α_1	−18.31000	0.33774	<0.0001		
	α_2	0.41638	0.01664	<0.0001		
	α_3	−0.14639	0.01205	<0.0001		
2 (D_0)	β_0	−1.09330	0.09127	<0.0001	0.783	0.774
	β_1	0.62666	0.03305	<0.0001		
	β_2	−0.01682	0.01291	<0.0001		
3 (D_{25})	χ_0	−0.31598	0.03254	<0.0001	0.985	0.277
	χ_1	0.93690	0.01167	<0.0001		
	χ_2	−0.02009	0.00456	<0.0001		
4 (D_{50})	δ_0	−0.11710	0.02035	<0.0001	0.995	0.173
	δ_1	1.03594	0.00737	<0.0001		
	δ_2	−0.00831	0.00288	<0.0001		
5 (D_{95})	ε_0	0.29507	0.04838	<0.0001	0.984	0.411
	ε_1	1.20065	0.01732	<0.0001		
	ε_2	0.03680	0.00677	<0.0001		
6 (N)	ψ_0	1.237712	0.2399	<0.0001	$N_{i2} = 0.485$ $N_{u2} = 0.846$	$N_{i2} = 52.229$ $N_{u2} = 56.686$
	ψ_1	−0.00108	0.000084	<0.0001		
	ψ_2	−0.00058	0.000044	<0.0001		
7 (Hd)	γ_0	100.00000	2.02060	<0.0001	0.894	6.738
	γ_1	0.06450	0.00278	<0.0001		
	γ_2	1.44520	0.03390	<0.0001		
10 (h_i)	τ_0	0.03559	0.00109	<0.0001	0.637	0.119
	τ_1	0.40604	0.00208	<0.0001		
	τ_2	0.05222	0.00133	<0.0001		

where RS = relative spacing = (43,560/TPA)0.5/Hd, D'_0 = 0th diameter percentile (in.) for CAO method, ξ_i, φ_i, θ_i = coefficients to be estimated, and all other variables are defined as before.

The coefficients b (Equation 13) and c (Equation 14) were iteratively determined by minimizing the function,

$$\sum_{i=1}^{p}\sum_{j=1}^{n_i}\frac{(F_{ij}\hat{F}_{ij})^2}{n_i},$$

where F_{ij} = observed cumulative probability of tree j in the ith plot–age combination, $\hat{F}_{ij} = 1 - \exp\{-[(x_{ij} - a)/b]^c\}$ or the value of the Weibull cdf evaluated at x_{ij}, x_{ij} = dbh of tree j in the ith plot–age combination, n_i = number of trees in the ith plot–age combination, p = number of plot–age combinations, and all other variables are defined as before.

This approach is similar to simultaneously fitting two regression equations for b and c, but the objective was to minimize the sums of squares of error with respect to the cdf rather than to b and c (Cao 2004). Cao (2004) provides detailed information about this method, including PROC NLIN SAS code in an appendix, which was modified for use in this study.

Comparison and Evaluation Criteria

Three new diameter distribution growth models developed in this study and the model of Lenhart (1988) were compared with the independent 10% evaluation data set ($n = 45$). This study used the same evaluation procedures as Lee and Coble (2006). Percent bias (%Bias = 100 * [(predicted − observed)/observed]) and RMSE were used to evaluate whole-stand predictions of volume (cubic feet per acre), basal area (square feet per acre), and quadratic mean diameter. Mean difference (MD; predicted − observed) of volume and TPA by 1-in. diameter classes were graphically analyzed for trends across diameter class. The reader is referred to Lee and Coble (2006) for additional details.

Results and Discussion

The three models in this study (LEE, ModLEE, and CAO) were fit to the 90% model fitting data set. All coefficients for LEE were significantly different from zero ($P < 0.0001$). All coefficients for CAO were also significantly different from zero ($P < 0.05$), except for the θ_5 coefficient in Equation 15. No obvious trends were evident in any residual plots (figures not shown). No parameter estimates, variances, or fit statistics are shown for the analysis of the 90% data set, although they are presented for the final LEE model fit to the 100% combined data set (Table 2).

Percent bias was lowest for ModLEE (−0.85) and LEE (−1.20) in terms of whole-stand volume, followed by CAO (−1.45) and

Table 3. Root mean square errors (RMSE) and 95% confidence intervals (CI) for percent bias of volume (ft^3/ac), basal area per acre (BAPA, ft^2), and quadratic mean diameter (Dq, in.) from four slash pine growth and yield models.

Model	Stand parameter	95% CI	RMSE
LEE	Volume	−1.20 ± 11.20	36.83
	BAPA	0.81 ± 11.01	36.37
	Dq	−0.08 ± 4.83	15.88
ModLEE	Volume	−0.85 ± 11.16	36.72
	BAPA	0.82 ± 11.06	36.38
	Dq	−0.08 ± 4.83	15.88
CAO	Volume	−1.45 ± 26.65	81.06
	BAPA	6.44 ± 24.74	81.60
	Dq	3.28 ± 9.70	32.08
Lenhart (1988)	Volume	−11.23 ± 11.44	37.62
	BAPA	−7.15 ± 10.01	33.94
	Dq	−5.95 ± 4.20	15.10

Note that percent bias = (predicted − observed/observed) * 100.

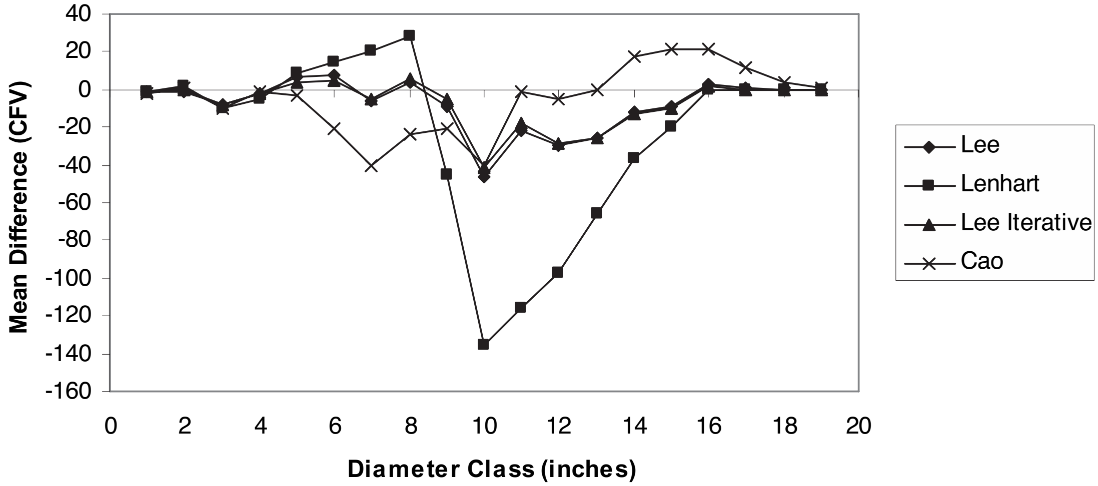

Figure 1. MD (predicted − observed) of cubic-foot volume wood and bark excluding stump (volume) from four slash pine growth and yield models.

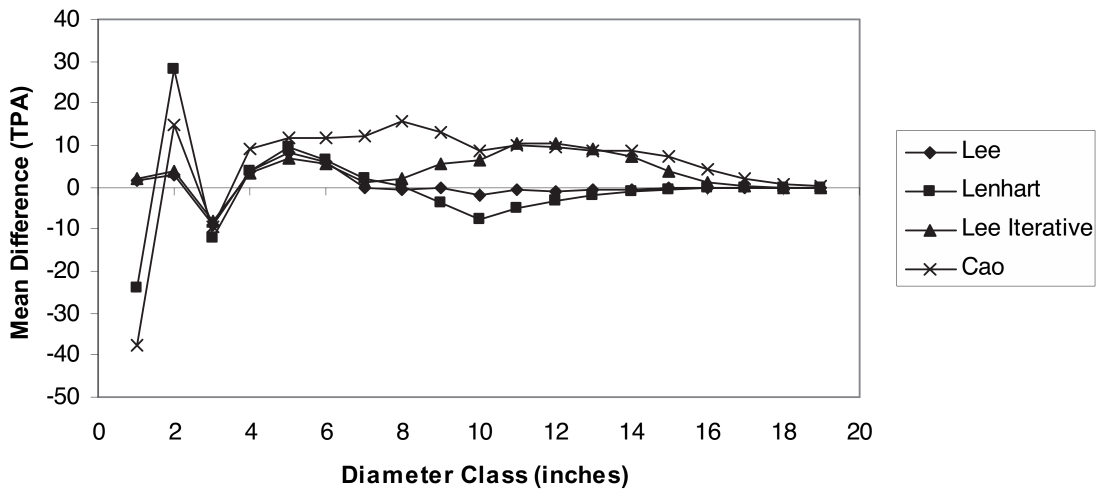

Figure 2. MD (predicted − observed) of TPA from four slash pine growth and yield models.

then Lenhart (−11.23; Table 3). Percent bias for whole-stand basal area per acre was lowest for LEE (0.81) and ModLEE (0.82), followed by CAO (6.44) and Lenhart (−7.15; Table 3). Percent bias for whole-stand quadratic mean diameter also was lowest for LEE (−0.08) and ModLEE (−0.08), followed by CAO (3.28) and Lenhart (−5.95; Table 3). RMSE values followed a similar trend as percent bias for whole-stand volume, basal area per acre, and quadratic mean diameter. In this study, the iterative methods used in ModLEE and CAO did not provide better predictions of whole-stand volume, basal area, or quadratic mean diameter in terms of percent bias and RMSE. This may seem counterintuitive to the findings of Cao (2004), but it may simply be that Cao (2004) used a single loblolly pine plantation in Louisiana while this study used 84 slash pine plantations scattered across East Texas, or another reason could be that the model structures are different. LEE and ModLEE are more specific in prediction of the percentile equations (i.e., D_0, D_{25}, D_{50}, and D_{95}), but Cao (2004) used only one general form for all regression equations to compare several different methodologies. In any case, we saw no reason to prefer the more complicated iterative methods used in ModLEE and CAO over the traditional parameter recovery technique used in LEE to predict whole-stand volume, basal area, and quadratic mean diameter. So, we prefer the LEE model over the other three models in terms of the whole-stand predictions as well as simplicity of use and fitting.

At the diameter class level, the MD in volume was similar for LEE and ModLEE (Figure 1). This is not surprising considering that the only difference between the two methods is a fine-tuning of the Weibull c parameter. CAO also is similar to LEE and ModLEE, while the MD values for Lenhart are quite different from the other three models. Across the range of diameters, volumes are predicted similarly for LEE, ModLEE, and CAO; no particular diameter class appears to be poorly predicted. This is not the case for Lenhart in the 10- to 15-in. diameter class range. Thus, any of these three models are preferred over Lenhart in terms of volume prediction by diameter class. In terms of TPA prediction by diameter class, LEE, ModLEE, and Lenhart have the lowest MD values above the 4-in. diameter class (Figure 2). CAO appears to have the highest MD values above the 4-in. diameter class compared with the other three models. Below the 4-in. diameter class, only LEE and ModLEE have low MD values; the MD values for CAO and Lenhart greatly exceed those for LEE and ModLEE in the 1- to 3-in. diameter classes. Again, these results may seem to contradict those of Cao (2004), but we attribute this to differences in data sets between the studies (our study used 84 slash pine plantations in East Texas and Cao's study

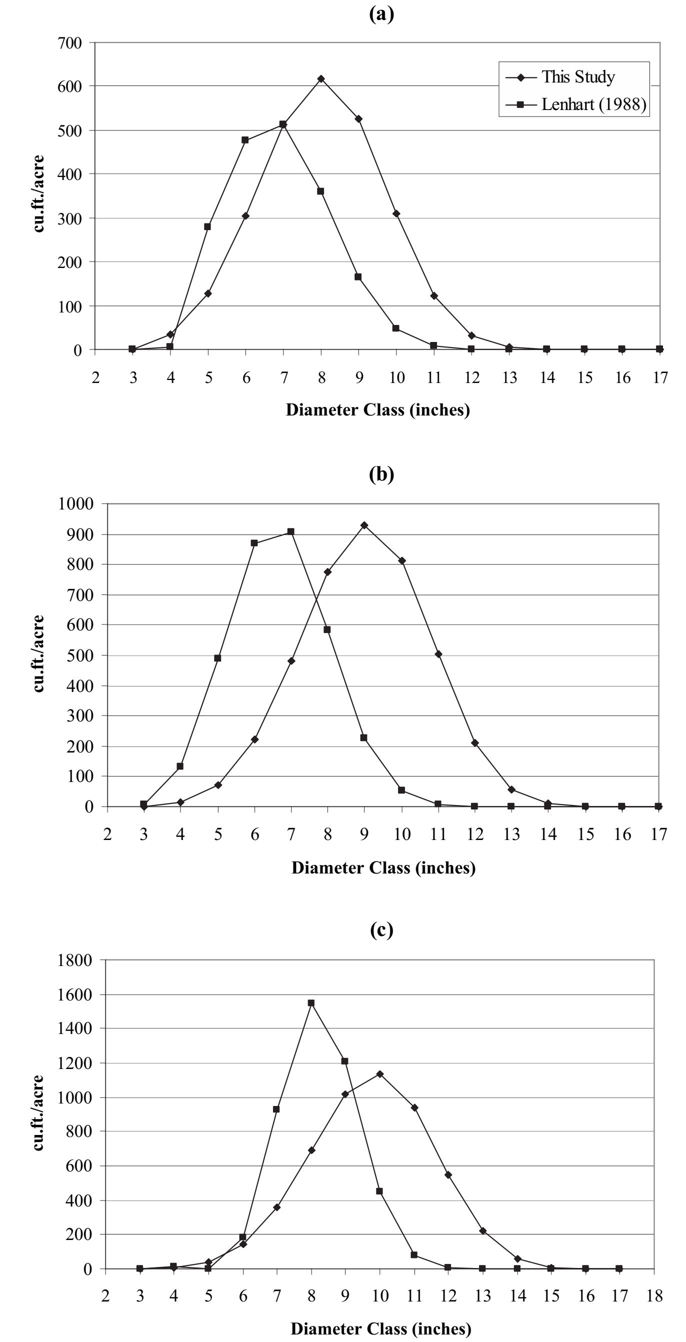

Figure 3. Predictions of volume by diameter class from two slash pine growth models for a 25-year-old slash pine plantation in East Texas with 600 initial TPA at each of the following site indices: (a) 50 ft, (b) 70 ft, and (c) 90 ft.

used one loblolly pine plantation in Louisiana) and/or model structure. Thus, based on the MD values by diameter class, LEE or ModLEE are preferred over CAO or Lenhart. In terms of volume by diameter class, LEE, ModLEE, and CAO provide similar predictions. However, CAO does not predict TPA by diameter class as well as the other three models. Therefore, our final preference in terms of volume and TPA predictions by diameter class is LEE because it performs as well as the iterative models (ModLEE and CAO), but it is easier to implement.

At this point, we will no longer consider ModLEE or CAO because LEE is the preferred model. Equations 1–11 were refit to the 100% combined data set to obtain final parameter estimates for LEE (Table 2). All coefficients were significantly different from zero ($P <$ 0.0001) and there were no obvious trends in the residual plots (figures not shown).

These final parameter estimates (Table 2) were used to further compare predicted cubic-foot volume per acre for wood and bark excluding stump (volume) by 1-in. diameter classes for LEE to volume predicted by Lenhart (1988; Figure 3, a–c), because Lenhart (1988) is the model commonly used for growth and yield estimation of East Texas slash pine plantations. The comparison used an example 25-year-old slash pine plantation initially planted at 600 TPA for three different SI values: 50, 70, and 90 ft. Fusiform rust infection rate was set to 10%. These conditions are representative of slash pine plantations at rotation age in East Texas.

For SI = 50 ft, Lenhart predicted lower volumes spread across smaller trees compared with LEE (Figure 3a). For SI =70 ft, Lenhart predicted similar volumes to LEE (in terms of maximum values), but for dramatically smaller trees (Figure 3b). His maximum volumes fell in the 6- to 7-in. diameter class range, whereas LEE's maximum volumes occurred in the 9- to 10-in. diameter class range. This result comes as no surprise, because Lenhart was developed mostly with younger plantation data. For SI = 90 ft, LEE produced a smaller maximum volume (1,150 ft^3/ac) than Lenhart (1,350 ft^3/ac) (Figure 3c). However, Lenhart's maximum volume occurs at a smaller diameter class (8 in.) than for LEE (10 in.). Furthermore, LEE spreads volumes across a much wider range of diameter classes. For the entire range of SI, Lenhart predicted less volume compared with LEE. However, for all SI values, Lenhart consistently predicted a greater frequency of smaller diameter trees. Again, Lenhart's use of younger and therefore smaller trees to build his model most likely caused this result. In summary, we recommend LEE over ModLEE, CAO, and Lenhart to estimate growth and yield of East Texas slash pine plantations.

Applications

To show how to use the new LEE model, we will illustrate the procedure used to produce Figure 3b.

1. Calculate the average height of the dominant and codominant trees (Equation 9; recall SI =7 0 ft; age = 25 years):

$$Hd = 70 * \left[\frac{1 - \exp(-0.0645 * 25)}{0.8006}\right]^{1.4452} = 70 \text{ ft.}$$

2. Calculate surviving TPA (Equation 6; recall initial TPA = 600, fusiform infection rate = 10%):

$$N_{i2} = (60 - 1.237712 * 540)\exp(-0.00108 * 70 * (25-0)) + 1.237712 * 540 * \exp(-0.00058 * 70 * (25-0)) = 150.3,$$

$$N_{u2} = 540 * \exp(-0.00058 * 70 * (25-0)) = 195.7,$$

Total TPA = 150.3 + 195.7 = 346.0.

3. Calculate Dq (Equation 1):

$$Dq = \exp\begin{pmatrix} 2.36552 - 18.31000(1/70) \\ + 0.41638\ln(25) \\ - 0.14639\ln(25 * 346) \end{pmatrix} = 8.3 \text{ in.}$$

4. Calculate percentiles (Equations 2–5):

$$D_0 = -1.09330 + 0.62666 * 8.3 - 0.01682 * 25$$
$$= 3.7 \text{ in.}$$
$$D_{25} = -0.31598 + 0.9369 * 8.3 - 0.02009 * 25$$
$$= 7.0 \text{ in.}$$
$$D_{50} = -0.1171 + 1.03594 * 8.3 - 0.00831 * 25$$
$$= 8.3 \text{ in.}$$
$$D_{95} = 0.29507 + 1.20065 * 8.3 + 0.03680 * 25$$
$$= 11.2 \text{ in.}$$

5. Calculate the Weibull parameters (see Equations 3–5 of Lee and Coble 2006):

$$n = 346 * 0.229568411 = 79.4307,$$

$$a = (79.4307^{1/3} * 3.7 - 8.3)/(79.4307^{1/3} - 1) = 2.3,$$

$$c = 2.343088 \Big/ \ln\left[\frac{11.2 - 2.3}{7.0 - 2.3}\right] = 3.7,$$

$$b = -\frac{2.3 * \Gamma(1 + 1/3.7)}{\Gamma(1 + 2/3.7)} + \sqrt{\left(\frac{2.3}{\Gamma(1 + 2/3.7)}\right)^2 ((\Gamma(1 + 1/3.7))^2 - \Gamma(1 + 2/3.7) + \frac{8.3^2}{\Gamma(1 + 2/3.7)}}$$
$$= 6.4$$

Note: Use the GAMMALN function in Excel to calculate $\Gamma(x)$.

6. Calculate the TPA in each diameter class (see Equation 2 of Lee and Coble 2006). For example, use diameter class = 10 in.:

$$P_{10} = \left(1 - \exp\left[-\left(\frac{10.5-2.3}{6.4}\right)^{3.7}\right]\right) - \left(1 - \exp\left[-\left(\frac{9.5-2.3}{6.4}\right)^{3.7}\right]\right)$$
$$= 0.13$$

$$\text{TPA}_{10} = 0.13 * 346 = 45 \text{ trees per acre in 10-in. class.}$$

This process is repeated for all diameter classes.

7. Calculate the average height for each diameter class (Equation 10). Again, use the 10-in. diameter class as an example:

$$\ln(h_{10}) = \ln(70) + 0.03559 + (\ln(10) - \ln(13)) * (0.40604 + 0.05222 \ln(89.3))$$
$$= 63 \text{ ft.}$$

This process is repeated for all diameter classes.

8. Calculate the cubic-foot volume (CFV) for each diameter class (Equation 11). Again, use the 10-in. diameter class as an example (note that $d = 0$ since total tree volume is desired):

$$\text{CFV}_{\text{tree}} = 0.002021 * 10^{1.790506} 63^{1.183087} = 16.8 \text{ ft}^3,$$

CFV for the 10-in. diameter class = 16.8 ft^3 * 45 TPA = 756 ft^3/ac. This process is repeated for all diameter classes.

9. Once all diameter classes are calculated, sum the volume across diameter classes to find the total volume for the plantation. Note that rounding errors are present in this example; the answers will differ if all decimal places are carried throughout the calculations.

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