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A New Whole-Stand Model for Unmanaged Loblolly and Slash Pine Plantations in East Texas

Dean W. Coble

ABSTRACT

A new compatible whole-stand growth-and-yield model to predict total tree cubic-foot volume per acre yield (outside and inside bark) was developed for unmanaged loblolly pine (*Pinus taeda*) and slash pine (*Pinus elliottii*) plantations in East Texas. This model was compared with the noncompatible whole-stand model of Lenhart (Lenhart, 1996, Total and partial stand-level yield prediction for loblolly and slash pine plantations in east Texas, *South. J. Appl. For.* 20(1):36–41) and the Lenhart (1996) model refit to current data. For the two species, all three models were evaluated with independent observed data. The model developed in this study outperformed both Lenhart models in prediction of future yield and basal area per acre for all age classes combined and by 5-year age classes. The Lenhart models consistently overestimated yield and basal area per acre. All three models predicted surviving trees per acre similarly. An example is also provided to show users how to use the new whole-stand model.

Keywords: *Pinus taeda*, *Pinus elliotti*, compatible growth-and-yield models, Schumacher yield model

Whole-stand growth-and-yield models predict future yields as a function of stand-level attributes, such as site index (SI), age, and stand density (Avery and Burkhart 2002). MacKinney et al. (1937), MacKinney and Chaiten (1939), and Schumacher (1939) developed the earliest whole-stand growth-and-yield models. Clutter (1963) and Sullivan and Clutter (1972) used a Schumacher yield function as the basis for their compatible growth-and-yield model for loblolly pine stands. Buckman (1962) also developed a compatible growth-and-yield model for red pine stands in the Lake States. In a compatible growth-and-yield model, the yield function can be found by mathematically integrating the growth function. Subsequently, others used this concept to develop whole-stand growth-and-yield models for loblolly (Burkhart et al. 1972, 1985, Amateis et al. 1986, Ledbetter et al. 1986, Borders et al. 1990, Harrison and Borders 1996, Ochi and Cao 2003, Borders et al. 2004) and slash (Pienaar and Harrison 1989, Martin et al. 1999) pine plantations.

Lenhart (1996) developed a noncompatible whole-stand growth-and-yield model exclusively with data from unmanaged loblolly and slash pine plantations in East Texas. He used a Schumacher-type function to predict future yields for different merchantability standards (Amateis et al. 1986). However, his data represented relatively young plantations (average age, 10 years; range, 5–24 years) commonly established in the early 1980s. These plantations are still found in East Texas, but they are currently older than those represented by Lenhart (1996).

The objective of this study was to develop new whole-stand growth-and-yield models for unmanaged loblolly and slash pine plantations in East Texas. The intent was to improve on Lenhart (1996) by using new equations and his original data set with an additional 15 years of new measurements from older plantations to forecast future yields.

Data Description

This study used 987 observations from 173 remeasured permanent plots located in East Texas loblolly pine plantations (Table 1). From the total 987 observations, approximately 10% ($n = 103$ observations from 18 permanent plots) were randomly selected and removed from the data set used for model fitting and reserved for model evaluation. Thus, a total of 884 loblolly pine observations from 155 permanent plots were used for model fitting. For slash pine, a total of 430 observations from 78 permanent plots were used (Table 2). Ten percent ($n = 40$ observations from 8 permanent plots) were removed from the model fitting data set and reserved for model evaluation, leaving 390 observations from 70 plots for model fitting.

The 173 loblolly pine and 78 slash pine permanent plots are part of the East Texas Pine Plantation Research Project (ETPPRP; Lenhart et al. 1985), which covers 22 counties across East Texas. Generally, the counties are located within the rectangle from 30 to 35° north latitude and 93 to 96° west longitude. Each plot consists of two adjacent subplots approximately 0.25 ac in size (100 × 100-ft) separated by a 60-ft buffer. Within a subplot, dbh (measured at 4.5 ft above the groundline), total height, and the survival status (live or dead) were monitored for each planted loblolly pine tree over a 27-year period (note: Lenhart (1996)) used the same data set, but the plots had only been monitored over a 12-year period). Plots were remeasured on fixed, 3-year intervals. Data from only one subplot (the development subplot) were used in this study. For each tree, dbh and total height were used to estimate total tree (from the stump to the tree tip) cubic-foot volume (outside bark (ob) and inside bark

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Table 1. Observed stand characteristics for East Texas unmanaged loblolly pine plantation data sets for whole-stand model.

	Model development data set (n = 884 observations from 155 plots)				Model evaluation data set (n = 103 observations from 18 plots)			
Variables	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
A	14.0	6.6	2.0	37.0	14.2	6.9	3.0	32.0
H	44.7	19.3	5.0	93.0	45.1	20.8	7.0	93.0
SI	72.1	10.0	33.0	96.0	72.5	9.9	53.0	88.0
TPA	455.5	147.7	83.0	1,002.0	437.8	147.5	170.0	858.0
BA	89.7	52.2	0.5	220.7	85.9	54.6	0.7	215.3
Dq	5.8	2.4	0.1	12.9	5.7	2.5	0.3	11.5
V (ob)	1,870.6	1,523.7	0.1	6,607.7	1,833.3	1,596.7	0.4	6,135.2
V (ib)	1,545.8	1,257.4	0.1	5,450.0	1,514.8	1,317.6	0.3	5,063.7

A, plantation age (total yr); H, average height of dominant and codominant trees (ft); SI, site index (base age, 25 yr); TPA, trees per acre; BA, loblolly pine basal area per acre (BAPA, ft^2); Dq = quadratic mean diameter (inches); V = total tree cubic-foot volume per acre, SD, standard deviation.

Table 2. Observed stand characteristics for East Texas unmanaged slash pine plantation data sets for whole-stand model.

	Model development data set (n = 390 observations from 70 plots)				Model evaluation data set (n = 40 observations from 8 plots)			
Variables	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
A	13.4	6.2	2.0	31.0	13.2	6.1	3.0	27.0
H	42.7	19.3	4.0	91.0	41.6	19.8	8.0	87.0
SI	74.7	10.0	36.0	101.0	72.8	5.0	65.0	81.0
TPA	374.9	173.4	61.0	1,002.0	369.0	182.8	70.0	671.0
BA	64.7	40.7	0.5	162.3	54.4	37.6	0.8	120.5
Dq	5.7	2.5	0.1	11.5	5.5	2.8	0.3	11.7
V (ob)	1,500.6	1,267.7	0.1	5,430.1	1,260.6	1,198.9	1.3	4,453.8
V (ib)	1,139.0	1,000.0	0.1	4,408.3	958.2	957.4	0.7	3,627.1

A, plantation age (total yr); H, average height of dominant and codominant trees (ft); SI, site index (base age, 25 yr); TPA, trees per acre; BA, loblolly pine basal area per acre (BAPA, ft^2); Dq, quadratic mean diameter (inches); V. total tree cubic-foot volume per acre; SD, standard deviation.

(ib)). The cubic-foot volume equations of Coble and Hilpp (2006) were used for loblolly pine, and the cubic-foot volume equations of Lenhart et al. (1987) were used for slash pine. Dominant height was determined by averaging the heights of the tallest 10 trees on a subplot (which approximately represents the tallest 40 trees per acre) that were free of damage, forks, and stem fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *Fusiforme*).

The New Whole-stand Model

The Schumacher (1939) model was used to predict future total tree cubic-foot volume per acre (V_2) as a function of future dominant height, plantation age, and basal area per acre:

$$\ln V_2 = a_0 + a_1 \ln H_2 + a_2 \frac{1}{A_2} + a_3 \ln B_2, \tag{1}$$

where H_2 = future average height (feet) of dominant and codominant trees; A_2 = future plantation age (years); B_2 = future basal area (squared feet) per acre; a_i = parameter estimates; ln = natural logarithm function.

Future basal area per acre (B_2) and future dominant height (H_2) must be known before Equation 1 can be used. The model of Borders et al. (2004) was used to predict current basal area per acre (squared feet, B_1):

$$\ln B_1 = b_0 + b_1 \frac{1}{A_1} + b_2 \ln N_1 + b_3 \ln H_1 + b_4 \ln N_1 \frac{1}{A_1} + b_5 \ln H_1 \frac{1}{A_1}, \tag{2}$$

where A_1 = current plantation age (years); N_1 = current trees per acre; H_1 = current average height (feet) of dominant and codominant trees; b_i = parameter estimates.

To predict future basal area per acre (B_2), Borders et al. (2004) derived a basal area projection equation from Equation 2 by isolating the b_1 term in Equation 2:

$$\ln B_2 = c_0 + \frac{A_1}{A_2}\left(\ln B_1 - c_0 - c_1 \ln N_1 - c_2 \ln H_1 - c_3 \frac{\ln N_1}{A_1} - c_4 \frac{\ln H_1}{A_1}\right) + c_1 \ln N_2 + c_2 \ln H_2 + c_3 \frac{\ln N_2}{A_2} + c_4 \frac{\ln H_2}{A_2}, \tag{3}$$

where N_2 = future trees per acre; c_i = parameter estimates; and all other variables are as defined previously.

A Chapman-Richards model (Coble and Lee 2006) was used to predict future dominant height (H_2):

$$H_2 = H_1 \left(\frac{1 - e^{-d_1 A_2}}{1 - e^{-d_1 A_1}}\right)^{d_2}, \tag{4}$$

where d_i = parameter estimates; e = exponential function; and all other variables are as defined previously.

Finally, a negative-exponential survival model (Zhao et al. 2007, Equation 32) was used to predict future surviving trees per acre (N_2), which is an independent variable in Equation 3:

$$N_2 = N_1 e^{k \cdot \mathrm{SI}(A_2 - A_1)}, \tag{5}$$

Table 3. Parameter estimates and fit statistics of East Texas loblolly pine plantation predictive equations for ob future total tree cubic-foot volume per acre (ln V_2), current basal area per acre (ft^2, ln B_1), future basal area per acre (ft^2, ln B_2), future dominant height (ft, H_2), and future surviving trees per acre (N_2).

Equation	Parameter	Parameter estimate	Standard error	Pr(parameter = 0)	R^2	RMSE
1 ($\ln V_2$)	a_0	−1.53354445	0.0394	<0.0001	0.998	0.0456
	a_1	1.16418329	0.0116	<0.0001		
	a_2	0.26335955	0.0946	0.0055		
	a_3	0.97534638	0.0049	<0.0001		
2 ($\ln B_1$)	b_0	−5.12109968	0.2527	<0.0001	0.968	0.2169
	b_1	−14.89666626	1.9164	<0.0001		
	b_2	0.55230267	0.0317	<0.0001		
	b_3	1.55304841	0.0310	<0.0001		
	b_4	−0.14111514	0.2859	0.6217		
	b_5	5.27278019	0.1659	<0.0001		
3 ($\ln B_2$)	c_0	−3.19205665	0.3718	<0.0001	0.947	0.1425
	c_1	0.58186175	0.0485	<0.0001		
	c_2	1.09433140	0.0436	<0.0001		
	c_3	0.74064246	1.1107	0.5050		
	c_4	5.17072095	0.1886	<0.0001		
4 (H_2)	d_1	0.07811158	0.0039	<0.0001	0.943	4.1515
	d_2	1.48389930	0.0411	<0.0001		
5 (N_2)	k	−0.00021308	7.495E-6	<0.0001	0.973	23.5832

RMSE, root mean square error.

Table 4. Parameter estimates and fit statistics of East Texas loblolly pine plantation predictive equations for ib future total tree cubic-foot volume per acre (ln V_2), current basal area per acre (ft^2, ln B_1), future basal area per acre (ft^2, ln B_2), future dominant height (ft, H_2), and future surviving trees per acre (N_2).

Equation	Parameter	Parameter estimate	Standard error	Pr(parameter = 0)	R^2	RMSE
1 ($\ln V_2$)	a_0	−1.70171933	0.0394	<0.0001	0.998	0.0454
	a_1	1.15943023	0.0116	<0.0001		
	a_2	0.27127090	0.0942	0.0041		
	a_3	0.97456387	0.0049	<0.0001		
2 ($\ln B_1$)	b_0	−5.11826231	0.2527	<0.0001	0.968	0.2169
	b_1	−14.91102867	1.9167	<0.0001		
	b_2	0.55188370	0.0317	<0.0001		
	b_3	1.55275343	0.0310	<0.0001		
	b_4	−0.14150303	0.2860	0.6208		
	b_5	5.28023138	0.1659	<0.0001		
3 ($\ln B_2$)	c_0	−3.19383800	0.3718	<0.0001	0.946	0.1425
	c_1	0.58176619	0.0485	<0.0001		
	c_2	1.09492902	0.0436	<0.0001		
	c_3	0.73979043	1.1105	0.5055		
	c_4	5.16835658	0.1886	<0.0001		
4 (H_2)	d_1	0.07811170	0.0039	<0.0001	0.943	4.1515
	d_2	1.48386038	0.0411	<0.0001		
5 (N_2)	k	−0.00021310	7.495E-6	<0.0001	0.973	23.5832

where SI = site index in feet (index age, 25 years); k = parameter estimate; and all other variables are as defined previously.

Equations 1–5 were simultaneously fit to the fitting data set using the MODEL Procedure in SAS/ETS (SAS Institute, Inc., 2004) to obtain parameter estimates for model evaluation. After model evaluation was complete, the fitting and evaluation data sets were combined into one data set. Equations 1–5 were then simultaneously fit to the combined data to obtain final parameter estimates (Tables 3–6). In all cases, seemingly unrelated regression was used to account for correlation across the equations (Borders 1989, Robinson 2004).

Model Evaluation

The new whole-stand model's equations for future yield (Equation 1), basal area per acre (Equation 3), and surviving trees per acre (Equation 5) were evaluated with the 10% evaluation data set. Lenhart's (1996) equations for $\ln V_2$, $\ln B_2$, and N_2 were also evaluated with the 10% evaluation data set to determine if the new model was an improvement over Lenhart (1996). Lenhart's equations for $\ln V_2$, $\ln B_2$, and N_2 were also fit to the current development data set to provide a current comparison with the new model (hereafter called modified Lenhart). This third comparison allows for a separate evaluation of model form and data age in Lenhart (1996). In Lenhart's model, $\ln B_2$ is not predicted directly, but rather natural logarithm of future quadratic mean diameter ($\ln Dq_2$). However, $\ln B_2$ was derived from his N_2 and $\ln Dq_2$ equations for purposes of comparison.

All comparisons were made in original units rather than natural logarithm units thereby making the comparisons more meaningful to the reader. Thus, V_2, B_2, and N_2 for all three models (new model; Lenhart 1996, modified Lenhart) were evaluated with the 10% evaluation data set.

Evaluation of all three models was performed at two levels of resolution: (1) all age classes and (2) 5-year age classes (from 5 to 30

Table 5. **Parameter estimates and fit statistics of East Texas slash pine plantation predictive equations for ob future total tree cubic-foot volume per acre (ln V_2), current basal area per acre (ft^2, ln B_1), future basal area per acre (ft^2, ln B_2), future dominant height (ft, H_2), and future surviving trees per acre (N_2).**

Equation	Parameter	Parameter estimate	Standard error	Pr(parameter = 0)	R^2	RMSE
1 (ln V_2)	a_0	−1.23873624	0.0483	<0.0001	0.999	0.0387
	a_1	1.13402680	0.0132	<0.0001		
	a_2	0.39260901	0.1199	0.0011		
	a_3	0.96958127	0.0048	<0.0001		
2 (ln B_1)	b_0	−6.23132969	0.3001	<0.0001	0.966	0.2115
	b_1	−10.93367473	2.5930	<0.0001		
	b_2	0.64518403	0.0361	<0.0001		
	b_3	1.64301972	0.0363	<0.0001		
	b_4	−0.08801420	0.3733	0.8137		
	b_5	4.55471494	0.2623	<0.0001		
3 (ln B_2)	c_0	−2.80921145	0.4280	<0.0001	0.971	0.1106
	c_1	0.63062849	0.0463	<0.0001		
	c_2	0.97003672	0.0553	<0.0001		
	c_3	2.46131790	1.1700	0.0360		
	c_4	2.44542174	0.2660	<0.0001		
4 (H_2)	d_1	0.05225618	0.0051	<0.0001	0.955	3.7808
	d_2	1.25939432	0.0468	<0.0001		
5 (N_2)	k	−0.00030932	0.000013	<0.0001	0.979	24.7091

Table 6. **Parameter estimates and fit statistics of East Texas slash pine plantation predictive equations for ib future total tree cubic-foot volume per acre (ln V_2), current basal area per acre (ft^2, ln B_1), future basal area per acre (ft^2, ln B_2), future dominant height (ft, H_2), and future surviving trees per acre (N_2).**

Equation	Parameter	Parameter estimate	Standard error	Pr (parameter = 0)	R^2	RMSE
1 (ln V_2)	a_0	−2.24692686	0.0546	<0.0001	0.998	0.0450
	a_1	1.33647003	0.0149	<0.0001		
	a_2	0.41097117	0.1359	0.0026		
	a_3	0.95130442	0.0054	<0.0001		
2 (ln B_1)	b_0	−6.22771268	0.2936	<0.0001	0.965	0.2115
	b_1	−11.29231165	2.5142	<0.0001		
	b_2	0.64797833	0.0352	<0.0001		
	b_3	1.64255519	0.0359	<0.0001		
	b_4	−0.00158467	0.3618	0.9965		
	b_5	4.45116276	0.2549	<0.0001		
3 (ln B_2)	c_0	−2.90167368	0.4240	<0.0001	0.971	0.1107
	c_1	0.64055826	0.0460	<0.0001		
	c_2	0.97614432	0.0546	<0.0001		
	c_3	2.21070791	1.1552	0.0563		
	c_4	2.46616364	0.2641	<0.0001		
4 (H_2)	d_1	0.05232592	0.0051	<0.0001	0.955	3.7808
	d_2	1.26007790	0.0468	<0.0001		
5 (N_2)	k	−0.00030950	0.000013	<0.0001	0.979	24.7091

years). For the all age class evaluation, four criteria (Kozak and Smith 1993) were used:

- Mean Bias = Bias = $\dfrac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)}{n}$,

- Mean Percent bias = %Bias = $\dfrac{\sum_{i=1}^{n}\left[100\left(\dfrac{(Y_i - \hat{Y}_i)}{Y_i}\right)\right]}{n}$,

- Standard error of the estimate = SEE = $\sqrt{\dfrac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n - k}}$,

- Percent SEE = %SEE = $\left(\dfrac{SEE}{\bar{Y}}\right)100$,

where Y_i = observed V_2, B_2, and N_2 for observation i; $\hat{Y}_i$ = predicted V_2, B_2, and N_2 for observation i; $\bar{Y}$ = mean V_2, B_2, and N_2; n = number of observations; k = number of estimated parameters in equation (k = 4 for Equation 1, k = 5 for Equation 3, k = 1 for Equation 5, and k = 4 for all equations of Lenhart 1996, and modified Lenhart). Note that a negative mean bias value corresponds to an overprediction while a positive mean bias value corresponds to an underprediction.

For the 5-year age class evaluation, mean predicted and observed V_2, B_2, and N_2 were calculated for each 5-year age class. For all three models, mean bias (defined previously) was then calculated for V_2, B_2, and N_2 by 5-year age classes. Mean bias was next plotted over age class to further examine model prediction trends across the range of ages, where negative values represent overpredictions and positive values represent underpredictions.

Table 7. Loblolly pine mean bias, mean %bias, SEE, %SEE, and number of samples (*n*) for both ob and ib predicted future yield (V_2), future basal area per acre (B_2), and future surviving trees per acre (N_2) from three growth and yield models: this study (ETPPRP), Lenhart (1996), and Lenhart (1996) refit with current data (modified Lenhart).

Equation, Component, and Model	*n*	Mean bias	Mean %bias	SEE	%SEE
1 V_2					
Wood and bark (ob)					
ETPPRP	103	−26.97	−0.92	87.26	3.7
Lenhart (1996)	103	−800.29	−47.30	1,373.89	58.21
Modified Lenhart	103	−180.90	−24.23	603.55	25.57
Wood only (ib)					
ETPPRP	103	−22.00	−0.92	71.36	3.66
Lenhart (1996)	103	−671.20	−40.96	1,231.94	63.18
Modified Lenhart	103	−148.98	−24.17	497.06	25.49
3 B_2					
ob and ib					
ETPPRP	103	−0.87	−2.87	10.73	10.41
Lenhart (1996)	103	−25.93	−29.73	44.18	42.89
Modified Lenhart	103	−5.98	−16.33	20.59	19.99
5 N_2					
ob and ib					
ETPPRP	103	−2.08	−0.98	24.74	5.94
Lenhart (1996)	103	−6.31	−2.04	25.76	6.19
Modified Lenhart	103	−0.80	−0.73	25.20	6.05

Results and Discussion

The parameter estimates for loblolly and slash pine (ob and ib) using the combined data set (fitting plus evaluation data) were reported for Equations 1–5 (Tables 3–6). All parameter estimates were significantly different from zero at the 0.01 significance level, except for the $\ln N_1/A_1$ term in Equation 2 as well as the $\ln N_2/A_2$ term in Equation 3. Although these two interaction terms were not significant, they were not removed from the final equations so not to alter the mathematical compatibility between the basal area prediction (Equation 2) and projection (Equation 3) equations. All R^2 values for each individual equation exceeded 94%, and the R^2 value for the yield model (Equation 1) exceeded 99%. Not surprisingly, the intercept term of the yield model (Equation 1) was the only parameter value that differed dramatically between ob and ib measurements within a species.

Predicted values for V_2 and N_2 from the new model were less than 1% of observed values for loblolly pine, while predicted values for B_2 were less than 3% (Table 7). For slash pine, predicted values for V_2 were also less than 1% and B_2 were less than 3% of observed values (Table 8). However, the new model overpredicted N_2 by about 9% (mean %bias = 8.97, Table 8). Lenhart (1996) overpredicted V_2 by about 41–47%, B_2 by about 30%, and N_2 by about 2% for loblolly pine (Table 7). For slash pine, Lenhart (1996) overpredicted V_2 by about 12%, B_2 by about 14%, and N_2 by about 7% (Table 8). Modified Lenhart overpredicted V_2 by about 24% and B_2 by about 16% for loblolly pine (Table 7). However, modified Lenhart predicted N_2 within 1% of observed values for loblolly pine (Table 7). For slash pine, modified Lenhart overpredicted V_2 by about 18%, B_2 by about 15%, and N_2 by about 7% (Table 8). For Lenhart's model, predictions of V_2, B_2, and N_2 for loblolly pine improved with the addition of the older plantation data; all mean percent bias values were lower for modified Lenhart compared with Lenhart (1996; Table 7). This trend did not hold for slash pine (Table 8), although both versions of Lenhart better predicted V_2 and B_2 (but not N_2) for slash pine than loblolly pine (Tables 7 and 8). However, even using the current data set (thereby eliminating age differences the data), modified Lenhart was unable to predict V_2 and B_2 for loblolly and slash pine as well as the new model in terms of mean percent bias. However, all three models were similar in predicting N_2. Furthermore, all values of %SEE for the new model were lower than those of Lenhart of either model (Tables 7 and 8). Thus, based on this, all age class evaluation (mean %bias and %SEE), the new model better predicts future yield and basal area per acre than Lenhart (1996), or modified Lenhart, while prediction of survival is similar between the three models.

For 5-year age classes, loblolly and slash pine (ob) mean bias values for V_2 and B_2 for the new model fell closest to the zero line compared with Lenhart (1996) or modified Lenhart, (Figures 1 and 2). In fact, all values for V_2 and B_2 were near zero, which means that the new model was not over- or underpredicting to any appreciable degree. Modified Lenhart increasingly overpredicted V_2 and B_2 for loblolly pine as plantation age class increases. Lenhart (1996) also overpredicted V_2 and B_2 for slash pine as plantation age class increases, although not as dramatically as for loblolly pine. The only exception was for the 25-year age class; Lenhart (1996) underpredicted this age class, and then dropped near or below the zero line for the 30-year age class. Lenhart (2008) performed similarly to Lenhart (1996), although its trend line was closer to the zero line than Lenhart (1996). This is indicative of the improvement from using current data; predictions were improved using data from older plantations, although they never surpassed those of the new model. Mean bias values for N_2 were similar between the three models for loblolly and slash pine (Figure 3). For both species and all three models, survival for age classes less than 25 years old was predicted best, with the worst predictions for the oldest two age classes. Although the models predicted N_2 similarly, the superior performance by the new model for the V_2 and B_2 predictions represents an improvement over Lenhart (1996, 2008). Note that the mean bias trends by age class for ib V_2, B_2, and N_2 were similar to those for ob (figures not presented).

Table 8. Slash pine mean bias, mean %bias, SEE, %SEE, and number of samples (*n*) for both ob and ib predicted future yield (V_2), future basal area per acre (B_2), and future surviving trees per acre (N_2) from three growth-and-yield models: this study (ETPPRP), Lenhart (1996), and Lenhart refit with current data (modified Lenhart).

Equation, Component, and Model	*n*	Mean bias	Mean %Bias	SEE	%SEE
1 V_2					
Wood and bark (ob)					
ETPPRP	40	8.56	−0.38	105.67	6.70
Lenhart (1996)	40	53.27	−12.98	519.46	32.95
Modified Lenhart	40	−92.17	−18.51	479.92	30.45
Wood only (ib)					
ETPPRP	40	5.09	−0.81	93.95	7.77
Lenhart (1996)	40	72.37	−12.09	431.39	35.66
Modified Lenhart	40	−65.63	−19.67	386.87	31.98
3 B_2					
ob and ib					
ETPPRP	40	−1.52	−2.57	6.34	9.96
Lenhart (1996)	40	−3.90	−13.62	17.40	27.35
Modified Lenhart	40	−4.54	−15.28	17.26	27.12
5 N_2					
ob and ib					
ETPPRP	40	−14.56	−8.97	33.90	10.23
Lenhart (1996)	40	−15.17	−7.47	38.05	11.49
Modified Lenhart	40	−13.55	−7.06	37.11	11.20

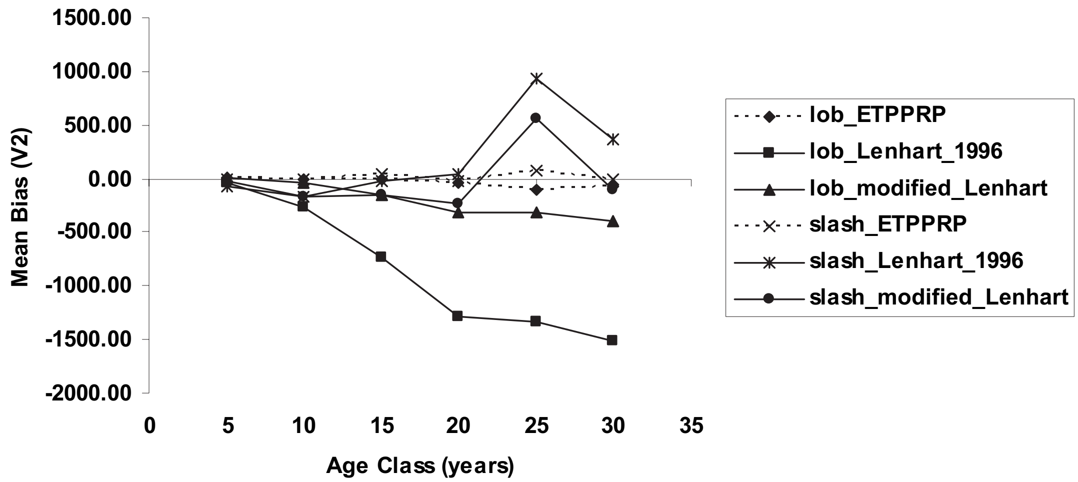

Figure 1. Mean bias of future yield (V_2) for loblolly and slash pine (ob) from three growth and yield models: this study (ETPPRP), Lenhart (1996), and Lenhart refit with current data (modified Lenhart).

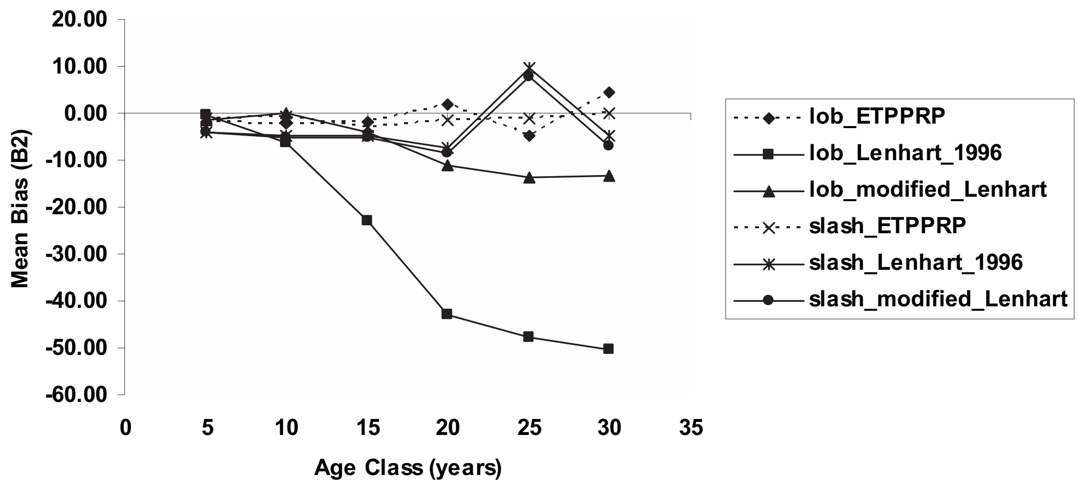

Figure 2. Mean bias of future basal area per acre (B_2) for loblolly and slash pine (ob) from three growth-and-yield models: this study (ETPPRP), Lenhart (1996), and Lenhart refit with current data (modified Lenhart).

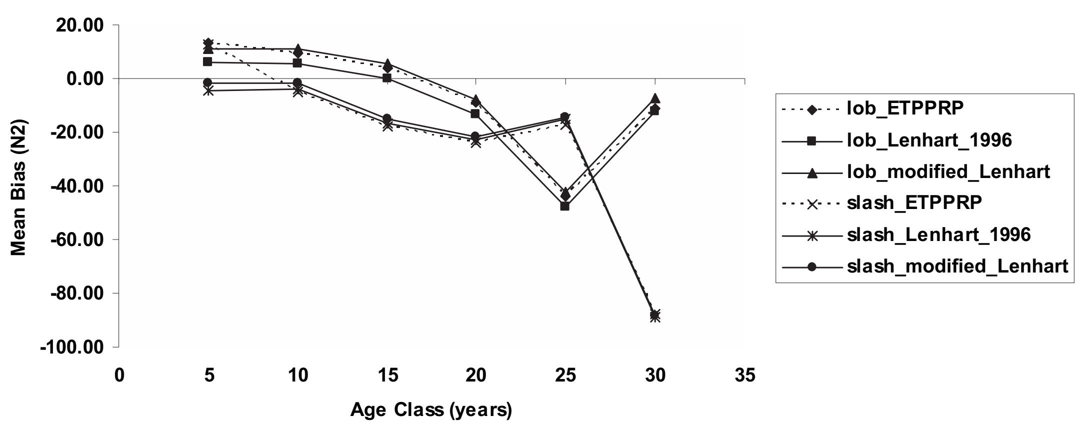

Figure 3. Mean bias of future surviving trees per acre (N_2) for loblolly and slash pine (ob) three growth-and-yield models: this study (ETPPRP), Lenhart (1996), and Lenhart refit with current data (modified Lenhart).

In conclusion, the new model better predicted V_2 and B_2 versus the two Lenhart models. All three models predicted N_2 similarly. Lenhart's model benefited from refitting to current data; modified Lenhart outperformed Lenhart (1996) in terms bias (mean %bias) and precision (%SEE). The plantation age in the data set used by Lenhart (1996) averaged 10–11 years old versus 13–14 years old in this study (also, maximum plantation age was 24 years old versus 30+ years, respectively). Clearly, additional data from older plantations improved performance in the Lenhart model. However, modified Lenhart still failed to outperform the new model in terms of bias and precision. I believe the new model in this study benefits from being compatible as well as fit with simultaneous estimation techniques, and these two characteristics make the new model preferable to Lenhart's model.

Application

To show how to use the new model, I will illustrate how to calculate the ob cubic-foot yield at rotation age = A_2 = 25 years for a loblolly pine plantation with SI = 70 ft (index age, 25 years). Let A_1 = 1 year and N_1 = initial planting density = 605 trees per acre.

1. Calculate H_1 from Equation 4 by setting H_1 = SI = 70, A_1 = Index age = 25, $H_2 = H_1$, and $A_2 = A_1 = 1$:

$$H_1 = 70\left(\frac{1 - e^{-0.07811158*1}}{1 - e^{-0.07811158*25}}\right)^{1.48389930} = 1.886 \text{ feet.}$$

Note that you can also obtain this value from field measurements.

2. Calculate H_2 from Equation 4 using the value for H_1 from the previous equation:

$$H_2 = 1.886\left(\frac{1 - e^{-0.07811158*25}}{1 - e^{-0.07811158*1}}\right)^{1.48389930} = 70.0 \text{ feet.}$$

3. Calculate N_2 from Equation 5:

$$N_2 = 605e^{-0.00021308*70(25-1)} = 422.95 \text{ tpa.}$$

4. Calculate B_1 using Equation 2:

$$\begin{aligned}\ln B_1 = &-5.12109968 - 14.89666626 * \frac{1}{1} \\ &+ 0.55230267 * \ln(605) \\ &+ 1.55304841 * \ln(1.886) - 0.14111514 * \frac{\ln(605)}{1} \\ &+ 5.27278019 * \frac{\ln(1.886)}{1} \\ = &-13.053313.\end{aligned}$$

So, $B_1 = e^{-13.053313} = 0.000002$ ft^2/ac.

5. Calculate B_2 using Equation 3:

$$\begin{aligned}\ln B_2 = &\ 3.19205665 \\ &+ \frac{1}{25}\left(\begin{array}{l}\ln(0.000002) + 3.19205665 - 0.58186175 * \ln(605) \\ \quad - 1.09433140 * \ln(1.886) - 0.74064246 * \frac{\ln(605)}{1} \\ \quad - 5.17072095 \frac{\ln(1.886)}{1}\end{array}\right) \\ &+ 0.58186175 * \ln(422.95) + 1.09433140 * \ln(70.0) \\ &+ 0.74064246 * \frac{\ln(422.95)}{25} \\ &+ 5.17072095 * \frac{\ln(70.0)}{25} = 5.13879.\end{aligned}$$

So, $B_2 = e^{5.13879} = 170.49225$ or 170.49 ft^2/ac.

6. Calculate V_2 using Equation 1:

$$\begin{aligned}\ln V_2 = &-1.53354445 + 1.16418329 * \ln(70.0) \\ &+ 0.2633595 * \frac{1}{25} \\ &+ 0.97534638 * \ln(170.49225) \\ = &\ 8.43502.\end{aligned}$$

So, $V_2 = e^{8.43502} = 4605.5604$ or 4,606 ft^3/ac.

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