2011

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A Modified Stand Table Projection Growth Model for Unmanaged Loblolly and Slash Pine Plantations in East Texas

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ABSTRACT

Four methodologies to project future trees per acre by diameter class were compared to develop a new modified stand table projection growth model for unmanaged loblolly pine (*Pinus taeda* L.) and slash pine (*Pinus elliottii* Engelm.) plantations in East Texas. The new models were fit to 92,882 observations from 153 permanent plots located in loblolly pine plantations and 33,792 observations from 71 permanent plots located in slash pine plantations throughout East Texas. The new models were validated with 12,750 observations from 22 permanent plots and 3,724 observations from 9 permanent plots located in loblolly and slash pine plantations, respectively. The validation data were used to select between the four methodologies. The results indicated which of the new models produced the best results, based on error indexes calculated for trees per acre and basal area per acre at the stand table and diameter class levels across a range of projection lengths. We recommend that this new model be used by forest managers for projecting stand tables in East Texas loblolly and slash pine plantations. An example is also provided to show users how to use the new modified stand table projection model.

Keywords: growth and yield models, West Gulf Coastal Plain, empirical models, southern pine plantation management

A frequency table of trees arranged by diameter classes, known as a stand table, is used routinely to make forest management decisions. Stand tables are derived from current inventories of existing stands, and through the use of projection models, a future stand table can be obtained. Early stand table projection models used diameter growth rates, based on increment core samples, and estimates of mortality to project a stand table into the future (Chapman and Meyer 1949, Avery and Burkhart 1983). More recent stand table projection models are based on empirical measurements and are modified (hence the name modified stand table projection models) by constraining the future stand table to exhibit stand characteristics, such as basal area and density, as estimated by growth and yield models. Pienaar and Harrison (1988) calculated survival and projected basal area growth on the basis of relative size. The resulting stand table was constrained to be consistent with whole-stand estimates of basal area per acre and trees per acre. Borders et al. (2004) followed the methodology of Pienaar and Harrison (1988) but used a Bayesian algorithm to allocate mortality to the diameter classes. Nepal and Somers (1992) projected the stand table using an implied diameter growth equation developed from the Weibull distribution (Bailey 1980). The stand table was then adjusted using an algorithm that assumed that trees in each diameter class followed a doubly truncated Weibull distribution. Cao and Baldwin (1999) modified the Nepal and Somers (1992) model by calculating survival at the beginning of the growing period and using a constrained least-squares procedure (Matney et al. 1990) to adjust the future stand table. This procedure was later modified by Cao (2007) to incorporate predictions from individual-tree diameter growth and survival models.

The objective of this study was to develop a modified stand table projection growth model for unmanaged loblolly pine (*Pinus taeda* L.) and slash pine (*Pinus elliottii* Engelm.) plantations in East Texas. Four new models were developed using the methodologies of Nepal and Somers (1992), Cao and Baldwin (1999), Cao (2007), and a combination of Pienaar and Harrison (1988) and Cao and Baldwin (1999). These four models were validated with East Texas pine plantation data not used for model fitting based on an error index developed by Reynolds et al. (1988).

Methods

Data Description

This study used 105,632 observations from 175 permanent plots and 37,516 observations from 80 permanent plots located in unmanaged loblolly and slash pine plantations, respectively (Table 1). An observation is defined as a single diameter class from a particular plot at a specific time. These plantations are characterized by some form of mechanical site preparation (e.g., shear, rake, pile, possibly burn), woods-run seedlings, but no midrotation activity such as a thinning or fertilization. The research plots are part of the East Texas Pine Plantation Research Project (ETPPRP) (Lenhart et al. 1985), which covers 22 counties across East Texas. Generally, the counties are located within the geographic area from 30° to 35°N latitude and 93° to 96°W longitude. Each plot consists of two adjacent subplots approximately 0.25 ac in size (100 × 100 ft) separated by a 60-ft buffer. Within a subplot, dbh (dbh to nearest 0.1 in.,

Manuscript received March 30, 2010, accepted August 23, 2010.

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Table 1. Observed stand characteristics for East Texas unmanaged loblolly and slash pine plantation data sets.

Variables[c]	Model development data set[a]				Model validation data set[b]			
	Mean	SD	Minimum	Maximum	Mean	SD	Minimum	Maximum
Loblolly pine								
A	14.0	6.7	2.0	36.0	14.2	7.0	3.0	37.0
Hd	44.9	19.2	5.0	95.0	45.0	19.8	5.0	93.0
SI	70.5	10.2	31.0	105.0	69.9	10.5	30.0	108.0
TPA	499.6	146.4	83.0	1002.0	460.9	145.7	187.0	758.0
BA	94.1	53.1	0.5	220.7	87.0	49.0	0.5	178.7
Slash pine								
A	12.3	5.9	2.0	33.0	15.2	7.4	3.0	35.0
Hd	39.9	18.5	3.0	91.0	46.6	19.8	10.0	88.0
SI	80.4	14.2	26.0	138.0	78.4	12.6	57.0	124.0
TPA	444.6	181.7	57.0	1002.0	461.1	177.9	105.0	706.0
BA	66.6	43.1	0.5	162.3	86.1	48.2	2.7	184.6

[a] Loblolly pine: $n = 92{,}882$ observations from 153 plots; slash pine: $n = 33{,}792$ observations from 71 plots.
[b] Loblolly pine: $n = 12{,}750$ observations from 22 plots; slash pine: $n = 3724$ observations from 9 plots.
[c] A, stand age (years); Hd, dominant height (feet); SI, site index (base age = 25 years); TPA, trees per acre; BA, basal area per acre (square feet); SD = standard deviation.

measured at 4.5 ft above the groundline), total height (feet), and the survival status (live or dead) were monitored for each planted tree over a 28-year period. Plots were remeasured every 3 years as long as they physically existed. Data from only one subplot (the development subplot) were used in this study. Dominant height (feet) was determined by averaging the total heights of the tallest ten trees on a subplot (which approximates the average height of the tallest 40 trees per acre) that were free of damage, forks, and stem fusiform rust (*Cronartium quercuum* [Berk.] Miyabe ex Shirai f. sp. *Fusiforme*).

From the total 105,632 loblolly pine observations, approximately 10% ($n = 12{,}750$ observations from 22 plots) were randomly selected and removed from the complete data set and reserved for model validation. The remaining 92,882 observations from 153 permanent plots were placed in the data set used for model fitting. From the total 37,516 slash pine observations, approximately 10% ($n = 3{,}724$ observations from nine plots) were randomly selected and removed from the complete data set and reserved for model validation. The remaining 33,792 observations from 71 permanent plots were placed in the data set used for model fitting.

Statistical Analysis

A modified stand table projection growth model consists of three steps: (1) projecting whole-stand values of basal area and trees per acre, (2) computing survival and allocating mortality by diameter class, and (3) projecting growth while adjusting the future stand table to match projected whole-stand trees per acre and basal area per acre.

For step 1, this study used the whole-stand model developed by Allen et al. (2010), which improved on the model of Coble (2009) with a new survival function and added an equation to predict future arithmetic mean diameter (Allen 2010). This whole-stand model was used for all subsequent comparisons, which provided a common basis of comparison between all four new models. For steps 2 and 3, this study examined four methodologies to project stand tables: (1) Nepal and Somers (1992) (NS model); (2) Cao and Baldwin (1999) (CB model); (3) Cao (2007) (CAO model); (4) a new model that combines aspects of Cao and Baldwin (1999) and Pienaar and Harrison (1988) (ETPPRP model).

NS Model

The first step in the NS model recovers the parameters of the Weibull distribution from the average diameter and the basal area per acre to approximate the current diameter distribution from the current and future stand. The Weibull cumulative probability distribution is defined as follows:

$$F_j(x) = 1 - e^{-[(x-a_j)/b_j]^{c_j}} \tag{1}$$

where c_j = shape parameter at time j; b_j = scale parameter at time j; a_j = location parameter at time j; and $e(\cdot)$ = exponential function.

Equation 1 implies the following individual diameter growth equation for the diameter (in in.) class midpoints (D_j) at times $j = 1$ and 2 (Bailey 1980).

$$D_2 = a_2 + b_2\left(\frac{D_1 - a_1}{b_1}\right)^{c_1/c_2}. \tag{2}$$

The movement of the trees from one diameter class to another can be computed under the assumption that the trees in each diameter class follow a doubly truncated Weibull distribution. If the ith diameter class is specified by the lower and upper limits, l_i and u_i, respectively, and if d_1 and d_2 are the diameter values in the ith class, where $l_i < d_1 < d_2 < u_i$, then the number of trees in the interval, $n_{[d_1,d_2]}$ is given by

$$n_{[d_1,d_2]} = n_{1i}\left[\frac{F_1(d_2) - F_1(d_1)}{F_1(u_i) - F_1(l_i)}\right]. \tag{3}$$

The future stand table must then be adjusted by multiplying each diameter class by an appropriate proportion (P_i). The following equation is used to predict P_i:

$$P_i = \alpha_0 e^{\alpha_1 D_i}, \tag{4}$$

subject to the constraints

$$\alpha_0 = \frac{N_2}{\sum_{i=1}^{k}(e^{\alpha_1 D_i})h_i}, \tag{5}$$

$$B_2 = \sum_{i=1}^{k} \frac{g(e^{\alpha_1 D_i})h_i D_i^2 N_2}{\sum_{i=1}^{k}(e^{\alpha_1 D_i})h_i}, \tag{6}$$

where N_2 = total (whole-stand) number of surviving trees per acre at time 2; B_2 = total (whole-stand) basal area (square feet) per acre at time 2; h_i = projected number of trees in D_i before correction;

$g = 0.005454$; k = number of diameter classes in the future stand; α_0, α_1 = coefficients to be determined; and all other variables are as defined before.

The secant method (Press et al. 1996) can be used to solve for α_1 in Equation 6, which is then used in Equation 5 to solve for α_0. By applying this process to each diameter class, the future stand table can be adjusted.

CB Model

The first step in the CB model is to calculate mortality by use of a survival function for each diameter class. The surviving trees per acre in the ith diameter class at time 2 ($\hat{n}_{i2}$) is predicted by

$$\hat{n}_{i2} = n_{i1}(1 - e^{\beta_1 * (D_i - D\min_1 + 1)}), \tag{7}$$

where n_{i1} = surviving trees per acre in the ith diameter class at time 1; $D\min_1$ = midpoint of the minimum diameter at time 1; and β_1 = coefficient to be determined, and all other variables are as defined before.

The secant method (Press et al. 1996) can also be used on Equation 7 to solve for β_1. The coefficient β_1 is calculated such that the trees across all diameter classes will sum to N_2. Because the mortality rate will not be evenly distributed among the diameter classes, the diameter distribution will change after mortality. This will cause a change in the arithmetic mean diameter at time 1 (D_1) and basal area per acre at time 1 (B_1). These whole-stand attributes must be updated after mortality has been removed but prior to projection to time 2:

$$\bar{D}_1 = \frac{\sum_{i=1}^{k} \hat{n}_{i2} D_i}{N_2} \tag{8}$$

$$B_1 = g \sum_{i=1}^{k} \hat{n}_{i2} D_i^2. \tag{9}$$

If there was a sufficient shift in the diameter distribution based on arithmetic and quadratic mean diameters, then the minimum diameter was assumed to increase. The future minimum diameter ($D\min_2$) can be obtained from the current minimum diameter ($D\min_1$) using an equation derived from Tang et al. (1997):

$$D\min_2 = \beta_0 + \beta_1 D\min_1 + \varepsilon. \tag{10}$$

If the error term, ε, is ignored, the coefficients β_0 and β_1 of Equation 10 can be calculated from

$$\hat{\beta}_1^2 = \frac{Dq_2^2 - \bar{D}_2^2}{Dq_1^2 - \bar{D}_1^2} \tag{11}$$

and

$$\hat{\beta}_0 = \bar{D}_2 - b_1 \bar{D}_1, \tag{12}$$

where Dq_j = quadratic mean diameter at time j = 1 and 2; and D_j = arithmetic mean diameter at time j = 1 and 2.

Next, the unadjusted future stand table was determined as in Nepal and Somers (1992) using Equations 1–3. The future stand table was then adjusted using a constrained least-squares method similar to the method in Cao and Baldwin (1999), with the difference being that the second constraint, involving arithmetic mean diameter at time 2, was omitted (their Equation 12b). Our study used predicted values to evaluate the stand table adjustment procedures, and Cao (2007) has shown that the constraint involving average diameter causes difficulty in convergence when using predicted instead of observed values. Thus, the adjusted number of trees per acre in each diameter class (n_{i2}) is given as

$$n_{i2} = \hat{n}_{i2} + \lambda_1 + \lambda_2 D_i^2, \tag{13}$$

where λ_1 and λ_2 are Lagrangian multipliers, calculated as

$$\lambda_1 = \frac{N_2 - \sum_{i=1}^{k} \hat{n}_{i2} - \lambda_2 \sum_{i=1}^{k} D_i^2}{k}, \tag{14}$$

$$\lambda_2 = \frac{\left(\sum_{i=1}^{k} D_i^2\right)\left(\sum_{i=1}^{k} \hat{n}_{i2} - N_2\right) + k\left(\frac{B_2}{g} - \sum_{i=1}^{k} \hat{n}_{i2} D_i^2\right)}{k \sum_{i=1}^{k} D_i^4 - \left(\sum_{i=1}^{k} D_i^2\right)^2}, \tag{15}$$

and all other variables are as defined before. This adjustment sometimes computed negative values of n_{i2}, usually where gaps existed in the diameter distribution. To correct for negative n_{i2}, $\hat{n}_{i2}$ was set to zero, and the adjustment process was repeated.

CAO Model

As for the CB model, the first step in the CAO model is to calculate mortality at the beginning of the growth projection period by use of a survival function for each diameter class, except that an individual-tree survival equation is used instead of Equation 7:

$$p_{i,t+1} = \left[1 + \exp\left(b_1 + b_2 A_t + b_3 \mathrm{Hd}_t + b_4 B_t + b_5 \frac{D_{i,t}}{Dq_t} + b_6 \mathrm{Rust}_{i,t}\right)\right]^{-1}, \tag{16}$$

where $p_{i,t+1}$ = probability that ith tree survives at time $t + 1$, given that it was alive at time t; A_t = plantation age in years at time t; Hd_t = dominant height in feet at time t; B_t = basal area (square feet) per acre at time t; $D_{i,t}$ = diameter of the ith tree at time t; Dq_t = quadratic mean diameter at time t; $\mathrm{Rust}_{i,t}$ = presence (1) or absence (0) of stem fusiform rust for the ith tree at time t; and all other variables are as defined before.

During the growth period, all trees in each diameter class will increase in diameter. The next step is to project the lower and upper limits of each diameter class by using an individual tree diameter growth equation,

$$\hat{D}_{i,t+1} = D_{i,t} + \exp(b_1 + b_2 A_t^{-1} + b_3 \mathrm{Hd}_t + b_4 B_t + b_5 \ln(D_{i,t})), \tag{17}$$

where $D_{i,t+1}$ = diameter of the ith tree at time $t + 1$; ln(·) = natural logarithm function; and all other variables are as defined before.

After diameter growth, the trees are reclassified into new diameter classes assuming that the distribution follows the doubly truncated Weibull distribution of Equation 3, as in the NS and CB models. The future stand table will not be consistent with whole-stand projections of basal area per acre and trees per acre, so the number of trees in each diameter class must be adjusted to match the

Table 2. Means (and standard deviations) of the Reynolds et al. (1988) error indices based on trees per acre and basal area per acre for loblolly pine for four models. Numbers in bold denote the smallest mean error among the four models.

Models	Projection length (years)						
	3	6	9	12	15	18	21
Error index based on number of trees/acre							
ETPPRP[a]	**113 (55)**	134 (51)	163 (85)	182 (79)	194 (100)	165 (66)	164 (82)
Nepal and Somers (1992)	122 (73)	132 (72)	**134 (59)**	138 (54)	**140 (71)**	**108 (28)**	103 (8)
Cao and Baldwin (1999)	124 (77)	**131 (74)**	**134 (63)**	**136 (56)**	141 (70)	112 (33)	**87 (6)**
Cao (2007)	196 (97)	241 (106)	243 (95)	234 (90)	220 (91)	183 (76)	203 (59)
Error index based on basal area/acre							
ETPPRP	**31 (15)**	42 (18)	57 (26)	68 (18)	79 (25)	80 (23)	86 (39)
Nepal and Somers (1992)	34 (16)	40 (18)	**46 (17)**	**52 (20)**	**57 (25)**	**50 (18)**	52 (16)
Cao and Baldwin (1999)	33 (16)	**39 (18)**	**46 (19)**	**52 (20)**	58 (25)	57 (22)	**45 (12)**
Cao (2007)	52 (28)	70 (30)	77 (26)	78 (18)	85 (28)	81 (24)	103 (21)
n (number of plots)	84	66	50	34	21	10	3

[a] ETPPRP, East Texas Pine Plantation Research Project.

whole-stand values. This is accomplished by using the same constrained least-squares Equation 13 as in the CB model.

ETPPRP Model

As with the CB and CAO models, mortality was removed at the beginning of the projection period so that dead trees would not be included in the growth projections. The ETPPRP model uses Equation 7 to predict future surviving trees per acre in the ith diameter class. As with the CB model, Equation 7 provides a new stand table in which diameter class values for trees per acre are consistent with the whole-stand estimate.

The model developed by Pienaar and Harrison (1988) and later used by Borders et al. (2004) was used in the ETPPRP model to project the stand table:

$$b_{i2} = \frac{B_2\left(\frac{b_{i1}}{\bar{b}_1}\right)^{\alpha} n_{i2}}{\sum_{i=1}^{k}\left(\frac{b_{i1}}{\bar{b}_1}\right)^{\alpha} n_{i2}} + \varepsilon_i, \quad (18)$$

where b_{i1} = basal area corresponding to the midpoint of the ith diameter class at time 1; b_{i2} = basal area corresponding to the midpoint of the ith diameter class at time 2; $\bar{b}_1$ = average basal area of the n_{i2} trees per acre that survive from time 1 to time 2; $\alpha = (A_2/A_1)^{\gamma}$; γ = coefficient to be determined using the NLIN procedure of SAS (2004); A_1 = plantation total age in years at time 1; A_2 = plantation total age in years at time 2; and all other variables are as defined before.

Equation 18 allowed each diameter class midpoint to be converted into a basal area class midpoint. The stand table was then projected into the future while being constrained to match whole-stand estimates of basal area per acre (B_2). The values of b_{i2} were then converted to diameter classes using

$$D_{i2} = \sqrt{b_{i2}/g}. \quad (19)$$

The projected diameter class midpoints did not fit into traditional 1-in. diameter classes (e.g., the 6-in. diameter class ranges from 5.5 to 6.5 in.). Assuming that trees were uniformly distributed in each of the projected diameter classes, traditional 1-in. diameter classes were obtained by taking the proportion of the new class that fell in the traditional diameter class limits.

Model Validation

The four new stand table projection models were validated on their ability to project future stand tables from 3 to 21 (loblolly) or 18 (slash) years for all possible projection lengths using the 10% validation data set. For example, if a plot was measured at 10, 13, 16, 19, 22, 25, 28, and 31 years, then there will be seven 3-year projection periods, six 6-year periods, five 9-year periods, four 12-year periods, three 15-year periods, two 18-year periods, and one 21-year period. An error index proposed by Reynolds et al. (1988) was used to validate predicted trees per acre and basal area per acre for all possible projection lengths at the diameter-class level, as well as all diameter classes combined (Cao 2007):

$$\mathrm{EI}_{Nj} = \sum_{i=1}^{m_j} |n_{ij} - \hat{n}_{ij}| \quad \mathrm{EI}_{Bj} = \sum_{i=1}^{m_j} |b_{ij} - \hat{b}_{ij}|,$$

where EI_{Nj} = error indices based on number of trees per acre for the jth plot; EI_{Bj} = error indices based on basal area per acre for the jth plot; n_{ij} = observed number of trees per acre in the ith diameter class in the jth plot; $\hat{n}_{ij}$ = predicted number of trees per acre in the ith diameter class in the jth plot; b_{ij} = observed basal area per acre (square feet) in the ith diameter class in the jth plot; $\hat{b}_{ij}$ = predicted basal area per acre (square feet) in the ith diameter class in the jth plot; and m_j = the number of diameter classes in the jth plot.

Results and Discussion

For all diameter classes combined, the means and standard deviations of the Reynolds et al. (1988) error index for the four modified stand table projection models based on the 10% validation data set are shown in Tables 2 and 3 for loblolly and slash pine, respectively. For loblolly and slash pine, the CB and NS models consistently produced the lowest mean error index values, based on either trees per acre or basal area per acre, for all four models and all projection lengths. However, the ETPPRP model produced the best results of the four models for the 3-year projection length only. The ETPPRP model performed third best for loblolly pine, followed by the CAO model, but this was reversed for slash pine. We speculate that for slash pine, predictions were improved with the use of individual tree equations because possible site-to-site variability is better represented at the individual tree level for slash pine versus loblolly pine.

For individual diameter classes, the means of the Reynolds et al. (1988) error index for trees per acre for the four modified stand table

Table 3. Means (and standard deviations) of the Reynolds et al. (1988) error indices based on trees per acre and basal area per acre for slash pine for four models. Numbers in bold denote the smallest mean error among the four models.

Models	Projection length (years)					
	3	6	9	12	15	18
Error index based on number of trees/acre						
ETPPRP[a]	**78 (28)**	109 (32)	128 (54)	166 (54)	178 (45)	212 (63)
Nepal and Somers (1992)	80 (29)	**79 (26)**	85 (29)	**95 (19)**	**96 (21)**	**113 (24)**
Cao and Baldwin (1999)	82 (30)	**79 (27)**	**83 (29)**	**95 (19)**	103 (24)	114 (23)
Cao (2007)	92 (35)	115 (40)	144 (46)	149 (35)	129 (30)	154 (22)
Error index based on basal area/acre						
ETPPRP	**23 (10)**	37 (11)	46 (12)	69 (21)	82 (25)	109 (37)
Nepal and Somers (1992)	25 (13)	29 (14)	35 (17)	**42 (13)**	**47 (11)**	**59 (15)**
Cao and Baldwin (1999)	24 (13)	**28 (16)**	**34 (17)**	43 (12)	52 (14)	60 (12)
Cao (2007)	26 (14)	35 (16)	50 (18)	56 (17)	55 (15)	76 (24)
n (number of plots)	26	20	15	10	6	3

[a] ETPPRP, East Texas Pine Plantation Research Project.

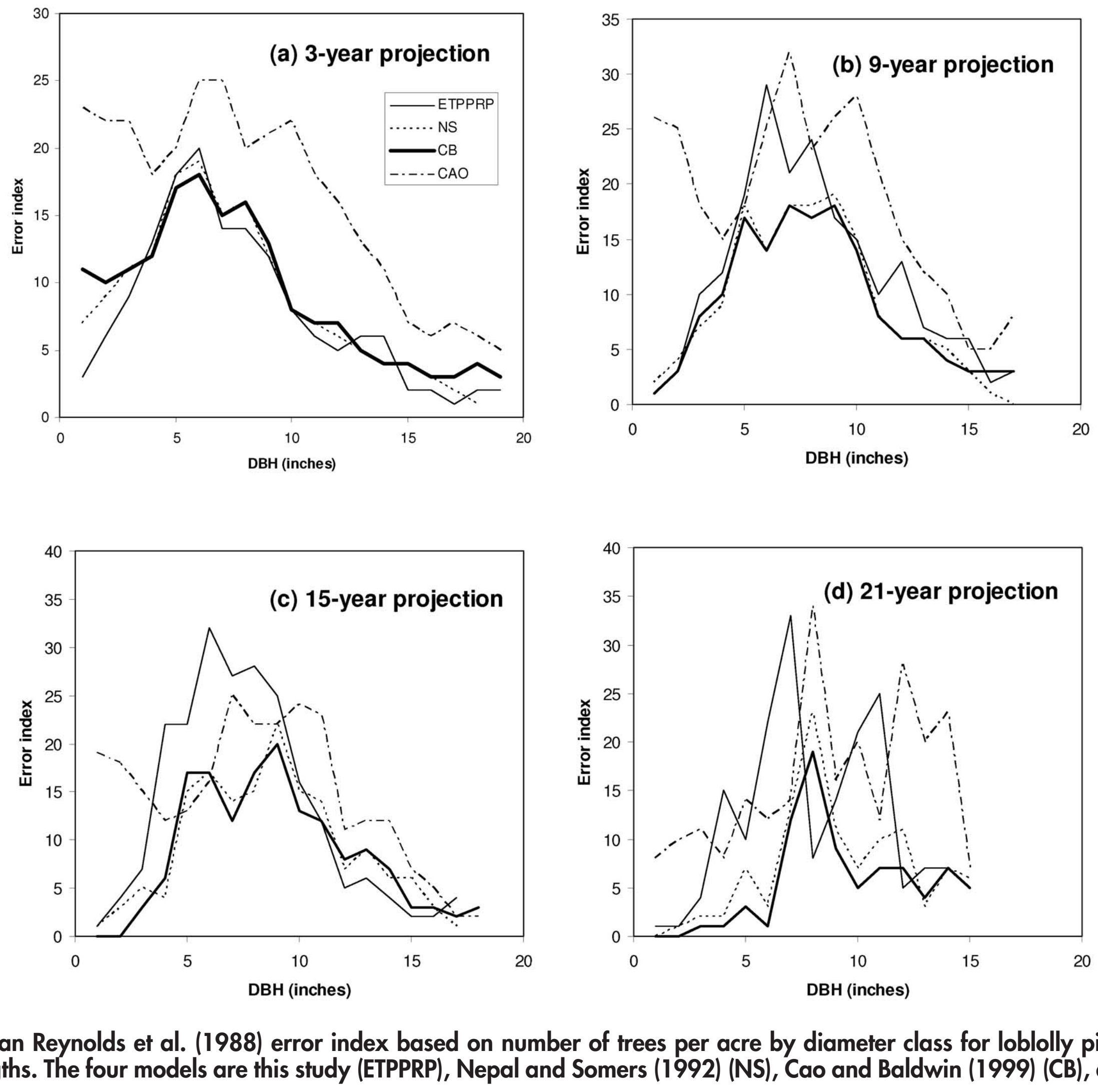

Figure 1. Mean Reynolds et al. (1988) error index based on number of trees per acre by diameter class for loblolly pine for different projection lengths. The four models are this study (ETPPRP), Nepal and Somers (1992) (NS), Cao and Baldwin (1999) (CB), and Cao (2007) (CAO).

projection models based on the 10% validation data set are shown in Figure 1 for loblolly pine. For 3-year projections, the ETPPRP model performed the best for most diameter classes (Figure 1a). The CB and NS models performed next best across the range of diameters, followed by the CAO model. The CB and NS models produced similar results for the shorter projection lengths, but the CB model outperformed the NS models as the projection length increased. At the 21-year projection length, the CB model produced the lowest error indexes for all diameter classes except the 8-in. and 12-in. classes (Figure 1d). For these two classes, the ETPPRP model produced the lowest error indexes, followed by the CB model and then the NS model.

For individual diameter classes of slash pine, the means of the Reynolds et al. (1988) error index for trees per acre for the four

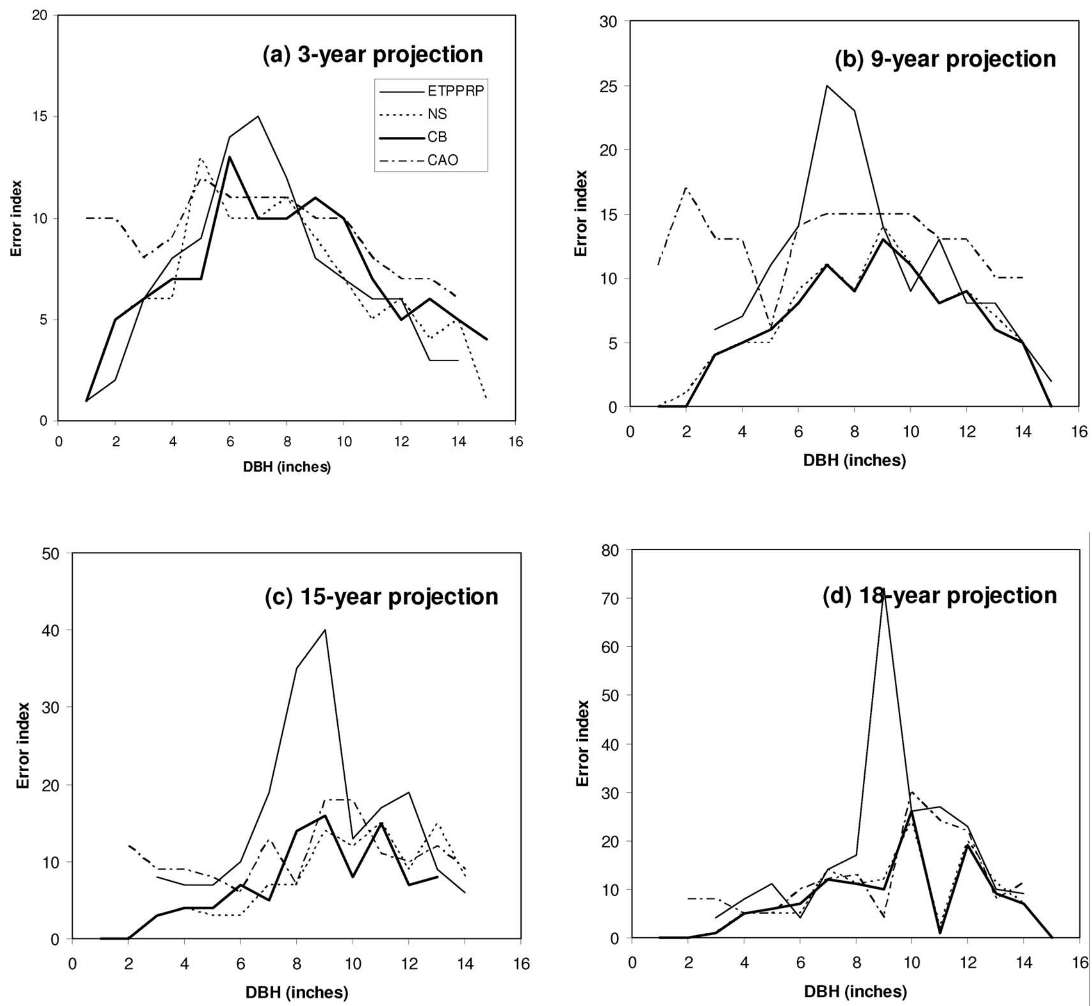

Figure 2. Mean Reynolds et al. (1988) error index based on number of trees per acre by diameter class for slash pine for different projection lengths. The four models are this study (ETPPRP), Nepal and Somers (1992) (NS), Cao and Baldwin (1999) (CB), and Cao (2007) (CAO).

modified stand table projection models based on the 10% validation data set are shown in Figure 2. For 3-year projections, the ETPPRP, CB, and NS models performed similarly (Figure 2a). The CB and NS models outperformed the ETPPRP model as projection length increased. Unlike the loblolly results, the CB and NS models produced similar results across all projection lengths. The CAO model again did poorly versus the other three models, though it outperformed the ETPPRP model for the 18-year projection length (Figure 2d).

On the basis of these results, we recommend that the CB model be used to project stand tables for unmanaged loblolly and slash pine plantations in East Texas. This model typically produced the lowest mean errors for individual diameter classes and all diameter classes combined compared with the other models considered, especially for loblolly as projection length increased. We considered recommending the NS model, because the results were similar to those of the CB model, but the CB model outperformed the NS model at the diameter-class level for loblolly pine at the longer projection lengths. We believe the CB and NS models outperformed the ETPPRP model because the γ coefficient in Equation 18 was estimated for the entire population. In contrast, the Lagrangian multipliers (Equations 13–15) calculated for the CB model and the proportions (Equation 4) calculated for the NS model used data from each individual plot. This plot-level estimation procedure probably contributed to less error in the final predictions of trees per acre and basal area per acre versus the ETPPRP model. Furthermore, we expected that the CAO model would outperform the other models, based on the findings of Cao (2007), but that was not the case in this study. A possible explanation could be differences between the data sets used in Cao (2007). For example, Cao (2007) used data from a loblolly South-wide seed source study that included 15 seed sources planted at 13 locations across 10 southern states. Our study used data from East Texas loblolly and slash pine plantations planted with woods-run seedlings.

A Numerical Example

This example presents the results of a stand table projection using the recommended CB model from age 27 (time 1) to age 33 (time 2) for plot 45 in the loblolly pine plantation model validation database (Table 4). The current and future observed stand attributes are given in columns 2 and 3, respectively. The first step is to project the whole-stand variables dominant height, arithmetic mean diameter, trees per acre, and basal area per acre.

Table 4. Numerical example for loblolly plot number 45, cycle 6, of the East Texas Pine Plantation Research Project data set.

	Observed trees/ac		Predicted trees/ac		
Diameter (in.) (1)	Age 27 (2)	Age 33 (3)	After mortality (4)	After D growth (5)	Final (6)
4	17.4	0.0	6.7	3.9	3.9
5	47.9	8.7	29.9	19.5	19.5
6	39.2	30.5	30.1	29.0	29.0
7	39.2	34.8	33.6	31.5	31.5
8	43.6	30.5	39.8	37.1	37.1
9	43.6	39.2	41.3	41.7	41.7
10	30.5	43.6	29.5	36.1	36.1
11	30.5	13.1	29.9	31.5	31.5
12	0.0	26.1	0.0	10.6	10.6
13	4.3	0.0	4.3	3.4	3.4
14	0.0	0.0	0.0	0.9	0.9
15	4.3	4.4	4.3	4.0	4.0
16	0.0	4.4	0.0	0.3	0.3
Hd	69.0	74.7			
Trees/ac	301.0	235.2	249.4	249.4	249.4
Dbar	7.7	8.9	8.1	8.6	8.6
Dq	8.1	9.2	8.4	8.9	8.9
Basal area/ac	106.71	108.91	96.71	106.61	106.64

1. Calculate Hd_2 from Equation 2 in Allen et al. (2010)

$$\mathrm{Hd}_2 = 69\left[\frac{1-e^{-0.073835*33}}{1-e^{-0.073835*27}}\right]^{1.445436} = 74.7 \text{ ft.}$$

2. Calculate N_2 from Equation 3 in Allen et al. (2010)

$$N_2 = 301 * e^{-0.00000797*67*(33^{1.994295}-27^{1.994295})} = 249.4 \text{ tpa.}$$

3. Calculate B_2 from Equation 1 in Allen et al. (2010) using projected values of Hd_2 and N_2

$$\begin{aligned}\ln B_2 = {} & -3.11553 \\ & + \frac{27}{33}\left(\begin{array}{r}\ln(106.71) + 3.11553 - 0.574834 * \ln(301) \\ -1.081493 * \ln(69) - 0.849356 * \dfrac{\ln(301)}{27} \\ - 5.384194 * \dfrac{\ln(69)}{27}\end{array}\right) \\ & + 0.574834 * \ln(249.4) + 1.081493 * \ln(74.7) \\ & + 0.849356 * \frac{\ln(249.4)}{33} + 5.384194 * \frac{\ln(74.7)}{33} \\ = {} & 4.66945,\end{aligned}$$

so $B_2 = e^{4.66945} = 106.64\ \text{ft}^2/\text{ac}$.

4. Calculate Dq_2 and then calculate $D\mathrm{bar}_2$ from Equation 4 in Allen et al. (2010)

$$\mathrm{Dq}_2 = \sqrt{\frac{B_2}{0.00545415 * N_2}} = \sqrt{\frac{106.64}{0.00545415 * 249.4}} = 8.9 \text{ in.}$$

and

$$\begin{aligned}\mathrm{Dbar}_2 = {} & \mathrm{Dq}_2 \\ & - e^{(-1.13318-3.28555)/27-0.04466*7.7+0.013421*69-0.00525*106.71} \\ = {} & 8.6 \text{ in.}\end{aligned}$$

Table 5. Weibull parameters.

Time	Average diameter (in.)	Basal area per acre (ft^2)	a	b	c
1	7.7	106.71	3.5	5.22095	2.15702
2	8.6	106.64	3.5	5.71124	2.39351

5. Apply the diameter-level survival Equation 7 in this study to the current stand table. The coefficient β_1 is calculated using the secant method to be -0.4885, so that the density of the stand is reduced from 301.0 to 249.41 trees per acre after mortality (column 4), which happens to equal N_2, calculated in step 2. Equations 10–12 indicate that there is no increase in minimum diameter, so no adjustment to Dmin is necessary.
6. The Weibull parameters recovered from the current and future stand attributes are shown in Table 5. The diameter growth function (Equation 2) implied from the current and future Weibull parameters can be rearranged to solve for diameter at time 1 (D_1):

$$D_1 = 3.5 + 5.22095\left(\frac{D_2 - 3.5}{5.71124}\right)^{2.39351/2.15702}.$$

The idea is to find the lower and upper endpoints of a future 1-in. midpoint diameter class in terms of current diameter. For example, consider the 6-in. diameter class at time 2: the lower limit is 5.5 in. and the upper limit is 6.5 in. Thus, the values for D_1 corresponding to the two future limits are

$$\text{Lower } D_1 = 3.5 + 5.22095\left(\frac{5.5 - 3.5}{5.71124}\right)^{2.39351/2.15702} = 5.13 \text{ in.}$$

and

$$\text{Upper } D_1 = 3.5 + 5.22095\left(\frac{6.5 - 3.5}{5.71124}\right)^{2.39351/2.15702} = 6.06 \text{ in.}$$

This means that the future 6-in. diameter class in current terms ranges from 5.13 to 6.06 in. Notice that this diameter range spans both the 5-in. (4.5–5.5 in.) and 6-in. (5.5–6.5 in.) current 1-in. diameter classes. To find the future trees per acre in the 6-in. diameter class, Equation 3 is applied to the two intervals, 5.13–5.5 in. and 5.5–6.06 in. This first requires that Equation 1 be used to calculate the cumulative Weibull density for five diameters:

$$\text{Upper } D_1 = 3.5 + 5.22095\left(\frac{6.5 - 3.5}{5.71124}\right)^{2.39351/2.15702} = 6.06 \text{ in.}$$

$$F_1(5.13) = 1 - e^{-((5.13-3.5)/5.22095)^{2.15702}} = 0.07798,$$

$$F_1(6.06) = 1 - e^{-((6.06-3.5)/5.22095)^{2.15702}} = 0.19344,$$

$$F_1(4.5) = 1 - e^{-((4.5-3.5)/5.22095)^{2.15702}} = 0.02790,$$

$$F_1(5.5) = 1 - e^{-((5.5-3.5)/5.22095)^{2.15702}} = 0.11858,$$

$$F_1(6.5) = 1 - e^{-((6.5-3.5)/5.22095)^{2.15702}} = 0.26115.$$

Now, Equation 3 can be used to calculate the future trees per acre for the two intervals, 5.13–5.5 in. and 5.5–6.06 in.:

$$n_{[5.13,5.5]} = n_{15}\left[\frac{F_1(5.5) - F_1(5.13)}{F_1(5.5) - F_1(4.5)}\right]$$

$$= 29.9 * \left[\frac{0.11858 - 0.07798}{0.11858 - 0.02790}\right] = 13.4 \text{ tpa},$$

$$n_{[5.5,6.06]} = n_{16}\left[\frac{F_1(6.06) - F_1(5.5)}{F_1(6.5) - F_1(5.5)}\right]$$

$$= 30.1 * \left[\frac{0.19344 - 0.11858}{0.26115 - 0.11858}\right] = 15.8 \text{ tpa}.$$

Finally, the unadjusted future trees per acre for the 6-in. diameter class are the sum of the trees per acre of the two intervals: 13.4 + 15.8 = 29.2 trees per acre. This process is repeated for all diameter classes in the stand table to produce unadjusted trees per acre by diameter class, which are found in column 5 of Table 4. Note: the value of 29.2 is slightly different from the tabulated value of 29.0 because of rounding error in this hand-calculated example.

7. The stand table can now be adjusted to match whole-stand values of basal area per acre as projected by the whole-stand equation. The Lagrangian multipliers for Equation 13 were calculated to be $\lambda_1 = -0.0092152$ (from Equation 14) and $\lambda_2 = 0.000080835$ (from Equation 15). Now, the final stand table (Table 4, column 6) can be computed using Equation 13, which will sum to the whole-stand estimates.

Literature Cited

ALLEN, M.G. II. 2010. *A modified stand table projection model for unthinned loblolly and slash pine plantations in east Texas.* Master's thesis, Arthur Temple College of Forestry and Agriculture, Stephen F. Austin State University, Nacogdoches, TX. 96 p.

ALLEN, M.G. II, D.W. COBLE, I. HUNG, AND J. YEISER. 2010. *A whole-stand growth and yield model for unmanaged east Texas loblolly and slash pine plantations.* East Texas Pine Plantation Research Project Report No. 67. Arthur Temple College of Forestry and Agriculture, Stephen F. Austin State University, Nacogdoches, TX. 12 p.

AVERY, T.E., AND H.E. BURKHART. 1983. *Forest measurements*, 3rd ed. McGraw-Hill, Inc., New York. 331 p.

BAILEY, R.L. 1980. Individual tree growth derived from diameter distribution models. *For. Sci.* 26(4):626–632.

BORDERS, B.E., W.M. HARRISON, F.Y. ZHANG, B.D. SHIVER, M. CLUTTER, C. CIESZEWSKI, AND R.F. DANIELS. 2004. Growth and yield models for second rotation loblolly pine plantations in the Piedmont/Upper Coastal Plain and Lower Coastal Plain of the southeastern U.S. PMRC Res. Pap. School of Forest Resources, Univ. of Georgia, Athens, GA. 64 p.

CAO, Q.V. 2007. Incorporating whole-stand and individual-tree models in a stand-table projection system. *For. Sci.* 53(1):45–49.

CAO, Q.V., AND V.C. BALDWIN, JR. 1999. A new algorithm for stand table projection models. *For. Sci.* 45(4):506–511.

CHAPMAN, H.H., AND W.H. MEYER. 1949. *Forest mensuration*. McGraw-Hill, Inc., New York. 522 p.

COBLE, D.W. 2009. A new whole-stand model for unmanaged loblolly and slash pine plantations in east Texas. *South. J. Appl. For.* 33(2):69–76.

LENHART, J.D., E.V. HUNT, JR., AND J.A. BLACKARD. 1985. Establishment of permanent growth and yield plots in loblolly and slash pine plantations in east Texas. P. 436–437 in *Proc. of the 3rd Biennial Southern Silvicultural Research Conf.,* Shoulders, E. (ed.). US For. Serv. Gen. Tech. Rep. SO-54.

MATNEY, T.G., K.L. BELLI, AND R.M. FARRAR, JR. 1990. Parameter-free diameter-distribution recovery. P. 95–107 in *Division 4 Proc. International Union of Forestry Research Organizations (IUFRO) World Congress Meet.* Pub. FWS-2-90. School of For. Wildl. Resources, Virginia Polytechnic Institute and State University, Blacksburg, VA.

NEPAL, S.K., AND G.L. SOMERS. 1992. A generalized approach to stand table projection. *For. Sci.* 38(1):120–133.

PIENAAR, L.V., AND W.M. HARRISON. 1988. A stand table projection approach to yield prediction in unthinned even-aged stands. *For. Sci.* 34(3):804–808.

PRESS, W.H., S.A. TEUKOLSKY, W.T. VETTERLING, AND B.P. FLANNERY. 1996. *Numerical recipes in Fortran 77: The art of scientific computing,* 2nd Ed. Cambridge University Press, New York. 933 p.

REYNOLDS, M.R., T.E. BURK, AND W.C. HUANG. 1988. Goodness-of-fit-tests and model selection procedures for diameter distribution models. *For. Sci.* 34(2): 373–399.

SAS. 2004. *SAS 9.1 user's guide.* SAS Institute Inc., Cary, NC. 2,456 p.

TANG, S., Y. WANG, L. ZHANG, AND C.H. MENG. 1997. A distribution-independent approach to predicting stand diameter distribution. *For. Sci.* 43(4):491–500.