An annual tree survival and diameter growth model for loblolly and slash pine plantations in east Texas

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ABSTRACT

Annual growth and survival of individual trees is routinely predicted with individual tree growth models (Stage 1973, Mitchell 1975, Hilt 1985, Larsen and Hann 1985, Burkhart et al. 1987, Wensel and Biging 1987, Hilt and Teck 1989, Marquis and Ernst 1992). Traditionally, annual growth is assumed to be constant during the projection period (Cao and Strub 2008). This can be an oversimplification of reality in some cases because growth and survival are nonlinear through time. McDill and Amateis (1993), Cao (2000), Cao et al. (2002), and Nord-Larsen (2006) subsequently introduced interpolation methods to improve on the traditional constant rate method. However, these improvements still required estimates of stand-level variables in the individual tree equations for the years between the beginning and end of the projection period. To address this problem, Cao (2002, 2004) incorporated a whole-stand growth model into his interpolation method to predict stand-level variables on an annual basis. Cao and Strub (2008) examined four methods to estimate the parameters of an annual individual tree survival and diameter system of equations that incorporates annual updates of stand-level variables. They found that the predicting attributes method, which incorporates a whole-stand model to predict stand-level variables on an annual basis, outperformed two other methods that used traditional linear interpolation or assumed that growth and survival was constant for the projection period.

The objective of this study was to develop individual-tree annual survival and diameter growth equations for unmanaged loblolly pine (Pinus taeda) and slash pine (Pinus elliottii) plantations in East Texas based on the methodologies of Cao and Strub (2008). The prediction equations were validated with observed data to examine the model’s performance in terms of bias and precision for projection lengths from 3 to 24 years.

Data Description

This study used 104,035 observations from 174 remeasured permanent plots located in East Texas loblolly pine plantations (Table 1). For slash pine, a total of 37,515 observations from 80 permanent plots were used (Table 1). These plots are part of the East Texas Pine Plantation Research Project (Lenhart et al. 1985), which covers 22 counties across East Texas. Generally, the counties are located within the rectangle from 30° to 35° north latitude and 93° to 96° west longitude. Each plot is approximately 0.25 ac in size (100 × 100 ft). Within a plot, dbh (in inches, measured at 4.5 ft above the groundline), total height (feet), and the survival status (live or dead) were measured three to nine times for each planted tree. Plots were remeasured at fixed 3-year intervals. Data were randomly selected from only one measurement cycle per plot to avoid serial autocorrelation problems among individual tree measurements through time. Dominant height (feet) was determined by averaging the total heights of the 10 tallest trees on a plot (which approximately represents the tallest 40 trees per acre) that were free of damage, forks, and stem fusiform rust.

Keywords: growth and yield, maximum likelihood, simultaneous equations, West Gulf Coastal Plain

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The loblolly and slash pine plot data sets were randomly divided in half to create separate model development and model validation data sets. All possible projection lengths (from 3 to 24 years) in the model validation data sets were used to examine the effects of projection length on the predictions. For example, if a plot was measured at 10, 13, 16, 19, 22, 25, 28, and 31 years, then there will be seven 3-year projection periods, six 6-year periods, five 9-year periods, four 12-year periods, three 15-year periods, two 18-year periods, and one 21-year period. After validation was completed, all observations were pooled to estimate the final regression parameters.

### Statistical Methods

This study examined two methodologies from Cao and Strub (2008) to predict annual diameter growth and survival of loblolly and slash pine trees in East Texas: (1) The predicting attributes method, which uses a whole-stand model to predict intermediate values of stand-level variables on an annual basis within the projection period, and (2) the constant rate method, which uses initial values to represent the traditional approach that assumes a constant rate of growth. Although the constant rate method is an oversimplification of reality, we included it in this study to consider the traditional approach. This enabled us to evaluate four combinations of the two methodologies for the East Texas data: (1) predicting attributes for both tree survival and diameter growth, (2) predicting attributes for tree survival and constant rate for tree diameter growth, (3) constant rate for tree survival and predicting attributes for tree diameter growth, and (4) constant rate for both tree survival and diameter growth. The possibility exists that a combination of the two methodologies will work best for the East Texas data.

For the predicting attributes method, we used a whole-stand growth model developed for Eastern Texas (Allen et al. 2010) to predict stand-level variables required in the individual tree equations, which consists of a tree survival equation and a diameter growth equation. In practice, any whole-stand growth model can be used, but the method of Allen et al. (2010) was developed for Eastern Texas pine plantations, so it should work better than other models for this study. The probability of an individual tree surviving to the next growing season was predicted with the equation,

\[
p_{i,t+1} = \left(1 + e^{(\beta_1 + \beta_2 A_i + \beta_3 H_i + \beta_4 B_i + \beta_5 d_i qt + \beta_6 R_{ist,t})}\right)^{-1} + e_i,
\]

and the annual diameter growth of an individual tree was predicted with the equation,

\[
d_{i,t+1} = \left(d_{i,t} + e^{(\beta_1 + \beta_2 A_i + \beta_3 H_i + \beta_4 B_i + \beta_5 d_i qt + \beta_6 R_{ist,t})}\right) + e_i,
\]

where \(p_{i,t+1}\) is the probability that the \(i\)th tree survives at time \(t + 1\), \(A_i\) is plantation age (years) at time \(t\), \(H_i\) is predicted stand dominant height (feet) at time \(t\), \(B_i\) is predicted stand basal area (ft²/ac) at time \(t\), \(d_{i,t}\) is predicted diameter breast height (dbh, inches) of the \(i\)th tree at time \(t\), \(d_{i,t+1}\) is dbh of the \(i\)th tree at time \(t + 1\), \(\Delta d_t = d_{i,t+1} - d_{i,t}\) is predicted growth (in.); \(R_{ist,t}\) is the probability of the individual tree surviving at time \(t\); \(\beta_6\) is parameters to be estimated; \(k = 1\) to 11; and \(e_i\) is random error.

For the constant rate method, Equations 1 and 2 were modified to use only the dominant height, basal area per acre, quadratic mean diameter, and individual tree diameter observed at the beginning of the projection period, rather than using intermediate values predicted annually by the whole-stand model for these independent variables:

\[
p_{i,t} = \left(1 + e^{(\beta_1 + \beta_2 A_i + \beta_3 H_i + \beta_4 B_i + \beta_5 d_i qt + \beta_6 R_{ist,t})}\right)^{-\frac{q}{g}} + e_i,
\]

\[
\Delta d_t = e^{(\beta_1 + \beta_2 A_i + \beta_3 H_i + \beta_4 B_i + \beta_5 d_i qt + \beta_6 R_{ist,t})} + e_i,
\]

where \(p_t\) is probability that the \(i\)th tree survives the growing period; \(g\) is the length of the growth period; \(\Delta d_t = (d_{i,t+q} - d_{i,t})/q\); \(\beta_6\) are parameters to be estimated (\(k = 12\) to 22); and all other variables are as defined above.

This study adapted the maximum likelihood estimation technique described in Cao and Strub (2008) to simultaneously estimate parameters for Equations 1 and 2. Cao and Strub (2008) also present maximum likelihood methods to estimate parameters for Equations 3 and 4 separately. For all four equations, the maximum likelihood parameter estimates were obtained with the NLIN procedure of SAS (SAS Institute 2004). Like Cao and Strub (2008), autocorrelation was not considered in the maximum likelihood methods used to estimate the parameters for Equations 1 and 2. Autocorrelation is not as important for Equations 3 and 4 because only the initial measurements were used.

### Model Validation

The systems of Equations 1 and 2 for predicting attributes and Equations 3 and 4 for constant rate were validated using the validation data sets. Mean difference (MD) was used to evaluate bias in the tree survival and tree diameter growth predictions, whereas mean absolute difference (MAD) was used to evaluate precision in tree survival and tree diameter growth predictions. Standard error of the estimate (SEE) was also included to evaluate precision. Low absolute values for MD, MAD, and SEE represented the best model. MD, MAD, and SEE were defined as

\[
MD = \frac{1}{n} \sum_{i=1}^{n} (Y_i - \hat{Y}_i),
\]

\[
MAD = \frac{1}{n} \sum_{i=1}^{n} |Y_i - \hat{Y}_i|,
\]

\[
SEE = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2},
\]
i, Yi Ŷi /n, 1n Yi Ŷi /n, k

where \(Y_i\) = observed tree survival or tree diameter value for observation \(i\); \(Ŷ_i\) = predicted tree survival or tree diameter value for observation \(i\); \(n\) = number of observations in the validation data sets; and \(k\) = number of estimated parameters in the equation (\(k = 6\) for Equations 1 and 3, whereas \(k = 5\) for Equations 2 and 4).

Results and Discussion

For loblolly pine, tree survival bias (MD) increased similarly for both methods as projection length increased. MD was similar at projection lengths of 3 and 6 years, as well as 24 years, but the predicting attributes method (PA) increased to a higher level of bias than the constant rate method (CR) at all other projection lengths (Figure 1a). Tree survival precision decreased (i.e., MAD and SEE increased) at a greater rate for PA versus CR across all projection lengths (Figure 1b and 1f). Tree diameter bias was lower and precision was higher for PA versus CR (Figure 1c to 1e). Furthermore, bias and precision for tree diameter estimated with PA were both relatively constant across the range of projection lengths compared with CR (Figure 1c to 1e). Tree diameter was overpredicted by less than 1 in. at all projection lengths (Figure 1c), whereas precision (MAD) was a minimum of 0.5 in. for the 3-year projection length; it then increased to just under 2 in. for the 6-year projection length and remained relatively constant through 24 years (Figure 1d).

Figure 1. Loblolly pine evaluation results for the two methods used in this study (predicting attributes and constant rate). a, Mean difference (MD) for tree survival versus projection length; b, mean absolute difference (MAD) for tree survival versus projection length; c, MD for tree diameter versus projection length; d, MAD for tree diameter versus projection length; e, standard error of the estimate (SEE) for tree diameter versus projection length; f, SEE for tree survival versus projection length.

The validation results for slash pine were similar to those for loblolly pine. Tree survival bias increased for both methods through the 21-year projection length and then dropped at 24 years (Figure 2a). MD was similar for both methods at projection lengths of 3 and 6 years, but PA began to diverge from CR at the 9-year projection length (Figure 2a). Tree survival precision decreased at a greater rate for PA versus CR across all projection lengths (Figure 2b and 2f). Tree diameter bias was lower and precision was higher for PA versus CR (Figure 2c to 2e). Like loblolly pine, slash pine bias and precision for tree diameter estimated with PA were both relatively constant across the range of projection lengths compared with CR (Figure 2c to 2e). Tree diameter was overpredicted by less than 1 in. at all projection lengths (Figure 2c), whereas precision (MAD) was a minimum of 0.5 in. for the 3-year projection length; it then increased to just under 2 in. for the 6-year projection length and remained relatively constant through 24 years (Figure 2d).

To summarize the validation results, PA and CR produced similar predictions for the 3-year projection length. For longer projection lengths, the most notable differences can be seen in the tree diameter predictions. For tree diameter bias, CR differed from PA by 0.2 in. for loblolly and slash pine (Figures 1c and 2c, respectively) at the 3-year projection length. By the 24-year projection length, the difference increased to 9.4 and 9.9 in. for loblolly and slash pine, respectively. Precision for tree diameter followed a similar pattern. At the 24-year projection length, the percentage differences reached 6.7 and 8.2 in. for loblolly and slash pine, respectively (Figures 1d and 2d). Clearly, the predicting attributes method should be preferred over the constant rate method for annual predictions of tree diameter. However, for tree survival predictions, the constant rate method was more precise than the predicting attributes method for both loblolly and slash pine (Figures 1b, 1f, 2b, and 2f), which
supports the assumption of constant annual mortality in the pine plantations represented in this study. On the basis of the validation results, we recommend the predicting attributes method for annual diameter growth predictions and the constant rate method for annual tree survival predictions in East Texas loblolly and slash pine plantations.

After validation was completed, the data were pooled to produce the final parameter estimates of Equations 2 and 3 for loblolly and slash pine (Table 2). For loblolly pine, the parameter estimates for all independent variables in Equations 2 and 3 were significantly different from zero ($P < 0.001$ for all $\beta$, except $P = 0.03$ for $\beta_{13}$) (Table 2). For Equation 2, the positive sign for $\beta_8$ indicates that diameter growth slows as the trees get older. This may seem counterintuitive, but recall that the variable is the reciprocal of age, so the sign for $\beta_8$ is positive rather than negative. The negative sign for $\beta_9$ indicates that diameter growth slows as dominant height increases. This may also seem counterintuitive, but it is related to how diameter growth slows as the trees get older as much as it is related to tree height and site quality. In a plantation, tree height is relatively uniform, because overtopped trees die from competition, which occurs relatively early in the life of the plantation (i.e., the stem exclusion stage; Oliver and Larson 1990). Thus, dominant height increases as the plantation gets older, which causes diameter growth to slow. This process occurs at a faster rate for higher quality sites than for lower quality sites. The negative sign for $\beta_{10}$ indicates that diameter growth slows as the stand density (basal area in ft$^2$/ac) increases. The positive sign for $\beta_{13}$ indicates that diameter growth is higher for larger trees versus smaller trees. Larger trees typically have larger crowns and root systems, so they are able to secure more site resources than smaller trees. For Equation 3, the expression containing the exponential function inside the brackets is raised to a negative power (i.e., $q$ is negative), so negative signs for the parameter estimates result in an increase of survival probability, whereas positive signs result in a decrease of survival probability. The positive sign for $\beta_{13}$ indicates that the probability of survival decreases as the trees get older. The positive sign for $\beta_{14}$ indicates that the probability of survival decreases as dominant height increases. This may seem

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Slash pine evaluation results for the two methods used in this study (predicting attributes and constant rate). a, Mean difference (MD) for tree survival versus projection length; b, mean absolute difference (MAD) for tree survival versus projection length; c, MD for tree diameter versus projection length; d, MAD for tree diameter versus projection length; e, standard error of the estimate (SEE) for tree diameter versus projection length; f, SEE for tree survival versus projection length.

<table>
<thead>
<tr>
<th>Species</th>
<th>Equation</th>
<th>Parameter</th>
<th>Parameter estimate</th>
<th>Standard error</th>
<th>$P$ value</th>
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</table>
counterintuitive, but the reason for this decrease is similar to the decrease of diameter growth as dominant height increases. Taller dominant height is related to older plantations, and survival probability decreases as the trees get older. As with diameter growth, this process occurs at a faster rate for higher versus lower quality sites. The positive sign for $\beta_{15}$ indicates that the probability of survival decreases as stand density (basal area in $ft^2/ac$) increases. The negative sign for $\beta_{14}$ indicates that the probability of survival increases as individual tree diameter approaches the average tree diameter of the plantation (i.e., as the ratio $d/Dq$ approaches or exceeds 1). A larger tree has a greater probability of survival because it can secure more site resources than smaller trees. The positive sign for $\beta_2$ indicates that the probability of survival decreases if a tree stem is infected by fusiform rust.

For slash pine, the final parameter estimates for all independent variables in Equations 2 and 3 were significantly different from zero ($P < 0.001$ for all $\beta_s$, except $P = 0.04$ for $\beta_{13}$) (Table 2). The only difference in signs from the loblolly pine parameter estimates was found with $\beta_{13}$. The negative sign for $\beta_{13}$ indicates that the probability of survival increases as stand density increases. This result is opposite that of loblolly pine and is difficult to explain. For this data set, the average basal area per acre for slash pine is 68 $ft^2/ac$ versus 92 $ft^2/ac$ for loblolly pine (Table 1). Slash pine plantations typically have fewer trees per acre because of higher stem fusiform rust infection, which results in greater mortality rates versus loblolly pine plantations (Coble and Lee 2004). Perhaps the slash pine plantations used in this study do not reach high enough basal areas to decrease the probability of survival simply because so much mortality has already occurred in these plantations, even though the basal areas can get high enough to slow diameter growth.

**Conclusion**

We recommend the predicting attributes method for annual growth predictions of tree diameter (Equation 2) and the constant rate method for annual growth predictions of tree survival (Equation 3) in East Texas loblolly and slash pine plantations. For the plantation data used in this study, most mortality occurred as a result of competition and, for slash pine, fusiform rust, which resulted in a uniform reduction in trees per acre over time. Diameter growth, however, was nonlinear rather than uniform over time. Comparison with other studies of these species in this region, such as that of Cao and Strub (2008), was problematic because different variables and equations were used in the different studies. The individual-tree and whole-stand estimates in this study are not compatible, which means that the individual-tree estimates when summed on a per-acre basis do not necessarily equal the whole-stand estimates. If users wish to reconcile the potential differences between the two levels of resolution, they can (1) assume that the whole-stand estimates of basal area per acre and survival are more reliable and use the disaggregation method to update the individual trees (Ritchie and Hann 1997, Qin and Cao 2006) or (2) compute the weighted average of individual-tree and whole-stand estimates (Yue et al. 2008). Any whole-stand growth model can be used with these individual-tree models. We recommend Allen et al. (2010) because their model is best suited for East Texas loblolly and slash pine plantations.

**A Numerical Example**

This example shows how to use Equations 2 and 3 to grow an individual loblolly pine tree from the $A_1 = 14$ years to future $A_2 = 25$ years (typical rotation age in East Texas). At the current age $= 14$ years, the following information is available from a standard inventory: $Hd_1 = 51$ ft, site index (25 year index age) = 72 ft, trees per acre $= N_1 = 366$, $B_1 = 94 ft^2/ac$, $Dq_1 = 6.9 in.$, $d_1 = 5.7 in.$, and the tree has no stem fusiform rust. The values for this projection can be found in Table 3. The steps below illustrate how to calculate the values for age $= 15$ years. Once a year is completed, the values are then used to make the next year’s calculations.

1. Calculate $Hd_2$ from Equation 2 in Allen et al. (2010):

$$Hd_2 = 51 \left[ 1 - e^{-0.073835 \times 15} \right]^{1.145436} = 53.9 \text{ feet.}$$

2. Calculate $N_2$ from Equation 3 in Allen et al. (2010):

$$N_2 = 366 \times e^{-0.000000797 \times 15^{0.849356 - 14 \times 0.0020874}} = 360.1 \text{ tpa.}$$

3. Calculate $B_2$ from Equation 1 in Allen et al. (2010) using projected values of $Hd_2$ and $N_2$:

$$\ln B_2 = -3.11553 + 14 \left( \ln(94) + 3.11553 
- 0.574834 \times \ln(366) - 1.081493 \times \ln(51) 
- 0.849356 \times \ln(366) - 5.384194 \times \ln(51) \right) 
+ 0.574834 \times \ln(360.1) + 1.081493 \times \ln(53.9) 
+ 0.849356 \times \ln(360.1) + 5.384194 \times \ln(53.9) \right) \right)$$

$$= 4.61180.$$ 

Thus, $B_2 = e^{4.61180} = 100.66 \text{ or } 100.7 \text{ ft}^2/\text{ac.}$
4. Calculate $Dq_2$:

$$Dq_2 = \sqrt{\frac{B_2}{0.0054541 * N_2^2}} = \sqrt{\frac{100.7}{0.0054541 * 360.1}} = 7.2 \text{ inches.}$$

5. Calculate the $d'_{age} = 15$ years using Equation 2 and the parameter estimates from Table 2:

$$d'_{age} = 5.7 + e^{-0.6174 + 2.9993/14 - 0.82444 * 51 - 0.0064 + 94 + 0.3983 * \ln(12.5)} = 5.9 \text{ inches.}$$

6. Calculate the probability of survival for age = 15 years using Equation 3 and the parameter estimates from Table 2:

$$p_2 = [1 + e^{-1.8421 + 0.00674 * 14 + 0.0255 * 51 + 0.08858 * 94 - 5.7783 * (5.7/6.9) + 0.9549}]^{-1} = 0.989.$$}

The value of 0.989 can be applied to the expansion factor of each individual tree, thereby reducing the stand density (i.e., trees per acre) as the plantation age increases. For example, if the plot size is 0.1 ac, each individual tree represents 1/0.1 or 10 trees per acre at the beginning of the growth period and 10 x (0.989) or 9.89 trees per acre at the end of the growth period.

7. The next step is to calculate steps 1 to 6 for age = 15 years using the values calculated for age = 15 as starting values in steps 1 to 6. This process is repeated for subsequent years. Each year’s values are presented in Table 3.

**Literature Cited**


