Turret Configurations and Particle Paths
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Abstract
The research originated from the question of how many rows of soldiers or turrets could be placed and allow for all turrets to have a projectile path. We find the relationship between the safe radius for each turret/soldier and the number of rows for which each soldier will have a clear projectile path. As shown, we place turrets in a grid configuration in the Cartesian plane:

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\end{array} \]

Figure 1:

Initial Radius Function
Assume a particle shot at angle \( \alpha \) from the origin passing through \((1, k)\). If the particle is tangent to the circle one unit above the origin, then the particle will tangent to the circle one unit below the circle centered at \((1, k)\).

Using trigonometry, we find:
\[
\begin{align*}
\tan \alpha &= \frac{k}{1} \\
\sin \alpha &= \frac{k}{\sqrt{k^2 + 1}} \\
\cos \alpha &= \frac{1}{\sqrt{k^2 + 1}} \\
\end{align*}
\]

Therefore using that \( \tan \alpha = \frac{\sin \alpha}{\cos \alpha} \) we have:
\[
\frac{1}{r^2} = 1 + k^2.
\]

Figure 2:

Scale Transformation
So when the gap between the turrets is 1 we previously found the relationships to satisfy:
\[
\frac{1}{r^2} = k^2 - 1
\]

Suppose we let the gap between turrets we variable, say \( d \). Then the problem scales nicely:

Comparing to the equations we deduced for Figure 2 we have:
\[
\begin{align*}
\tan \alpha &= \frac{k}{d} \\
\sin \alpha &= \frac{k}{\sqrt{k^2 + d^2}} \\
\cos \alpha &= \frac{d}{\sqrt{k^2 + d^2}} \\
\end{align*}
\]

Therefore using that \( \tan \alpha = \frac{\sin \alpha}{\cos \alpha} \) we have:
\[
\frac{d^2}{r^2} = d^2 + k^2.
\]

Figure 3:

Extending the Project
Further research will mainly focus on the generalization of each turret’s firing range and also study other turret configurations. Depending on the progress, we may build the 3-D model for grid and other configurations.

Configurations
Previously, we assumed that we were given the number of rows of turrets and found the maximum radius for the turret. We now study the reverse question: Given a radius, what is the maximum number of rows. Because the equation does not dictate that \( k \) be an integer we end up with an angular site or projectile range for each turret.

In Figure 2 we saw that the maximal angular range of the turret is \( \alpha \) that makes the path of the particle pass through \((1, k)\). When \( k \) is not an integer, we end up with a range of angles for which the projectile can pass without running into the other turrets. The minimum of this angular range is \( \beta \).

Figure 4:

References
Thank You to: Jeremy Becnel, Ph.D. SFASU
• Sage 4.7: sagenum.org
• \LaTeX