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## AN APPROACH FOR PROJECTING MIXED-SPECIES FOREST GROWTH AND MORTALITY

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### ABSTRACT

Mixed-species forests are common throughout the United States and growth and yield models are needed which reflect complex interrelationships. A stand-table projection approach with upgrowth and mortality components is presented for modeling growth and development in such forests. The method involves concepts previously used to model mixed-species diameter growth, in the upgrowth component of the projection system. Tentative equation forms are presented to help clarify the general approach.

Decisions regarding timber harvesting and stand density are of prime concern to forest managers. Harvesting directly influences both the profitability and the environmental impacts of forestry operations. Analyses of intermediate and final harvest alternatives require reliable means of projecting growth and yield, including responses to harvesting and other cultural practices. Models of growth and yield which recognize the mixed-species nature of most forest types are particularly needed for considering harvesting options. Only when such models are available can detailed management alternatives be evaluated for vast numbers of existing mixed-species stands.

Stand-table projection is a growth and yield technique which can reflect differential growth and responses to harvesting. A stand-table projection method is presented for modeling growth and structural changes in even-aged forests of mixed-species. The general approach to modeling mixed-species forest types is emphasized, rather than a specific, empirically estimated model. Following an outline of notation, a general discussion of the approach is presented, including tentative equation forms. Relative advantages and disadvantages of the proposed method are also discussed.

The following is a list of the notations used during the course of this article:

- $i$  = index for species group;
- $j$  = index for diameter class;
- $k$  = index for growth period;
- $N_{ijk}^{U,M,I,C,R}$  = number of trees by species, diameter class, and growth period, where superscripts U, M, I, C, and R denote upgrowth, mortality, initial, cut, and residual numbers, respectively;
- $P_{ijk}$  = potential proportion of upgrowth by species, diameter class, and growth period;
- $ADJ_{ijk}$  = adjustment term or proportion of potential growth realized by species, diameter class, and growth period;
- $b_{0,1,2,3}^{ij}$  = upgrowth ( $b_0, b_1$ ) and mortality ( $b_2, b_3$ ) parameter estimates by species and diameter class;
- $D_k$  = stand density measures by growth period; and
- $e$  = base of the natural logarithms.

### PROJECTING MIXED-SPECIES FORESTS

Stand-table projection has often been used to predict short term growth and structural changes for uneven-aged forests. Projections for such stands may be accomplished by predicting three components of forest development: the ingrowth of trees into the smallest diameter class recognized, the upgrowth or number of trees growing into larger diameter classes, and mortality. Each of the three components is predicted for each diameter class, for a growth period of fixed length. Stand growth and structure at the end of a growth period are estimated by aggregating the net results from predicting ingrowth, upgrowth, and mortality for each diameter class.

In many cases, mixed-species stands of even age may be projected without an ingrowth component. As discussed by Oliver, such stands may appear uneven-aged by diameter distribution and/or vertical stratification of the canopy, yet the appearance may be attributed to differential species growth and development, rather than to ingrowth of trees into the stand [1]. Upgrowth and mortality processes alone are considered in the following discussion for projecting mixed-species forest growth and development. As individual species groups are recognized, growth components are predicted for each species/diameter class combination.

#### Upgrowth

Upgrowth for each species/diameter class combination is the number of trees advancing into the next larger class during a growth period. This number is estimated by multiplying the number in each species/diameter class at the start of the growth period by the estimated proportion of upgrowth for the class.

Wahlenberg discussed three factors influencing the proportion of trees advancing from one diameter class to another during a fixed growth interval: diameter growth rate, diameter class size, and the distribution of the number of trees within the diameter class [2]. Stand-table projection models may consider each of these components separately, or may estimate upgrowth directly. Models presented by Hann [3] and Ek [4] respectively, are examples of the two approaches. Estimating upgrowth directly is considered in the present article, as the resulting growth model is simpler and more readily integrated with stand-level decision-making techniques.

Possibilities for incorporating species composition in stand simulators were discussed by Hann and Bare [5]. Adams and Ek, for example, concluded that a stand simulator at the individual tree level of resolution would be required to recognize individual species [6]. Hann's model for ponderosa pine, however, is a whole stand simulator which recognizes two vigor classes in a manner which could be used to recognize separate species [3]. To incorporate separate species in a stand-table projection model which predicts upgrowth directly, the upgrowth relation must explicitly reflect interspecific growth rates and possible responses to release.

The U. S. Forest Service modeled diameter growth on mixed-species plots by first establishing an upper limit or potential on diameter growth [7]. Using measures of stand density, the upper potential is adjusted downward to an actual estimate of diameter growth. In the present article, a related procedure is proposed for modeling the upgrowth component in mixed-species stand-table projections. The upgrowth process is presented symbolically in relation (1):

$$N_{ijk}^U = (N_{ijk-1}^I)(P_{ijk})(ADJ_{ijk}). \quad (1)$$

For a given species (i) and diameter class (j), after each growth period (k), the number of trees estimated as upgrowth ( $N_{ijk}^U$ ) is the corresponding number of trees at the beginning of the growth period ( $N_{ijk-1}^I$ ) multiplied by the estimated upgrowth proportion ( $P_{ijk}$ )( $ADJ_{ijk}$ ). The upgrowth proportion is the potential proportion for that species, diameter class, and growth period ( $P_{ijk}$ ), reduced to an actual estimate by the adjustment factor ( $ADJ_{ijk}$ ).

The upper limit on upgrowth ( $P_{ijk}$ ) reflects the potential ability of a tree to respond to release from competition. Three factors affecting the potential are fixed in the expression. That is, species (i) and diameter class (j) are constant, and age is reflected by (k) since the stand is even-aged and growth periods are of fixed length. Site quality is the remaining factor in determining a tree's ability to respond to release. Potential proportions of upgrowth may therefore be estimated as a function of species, diameter, age, and site quality. Values of ( $P_{ijk}$ ) used in relation (1) are constants with respect to density, however, since maximum diameter growth occurs when competition is minimized. Such values may therefore be predicted outside the stand-table projection system.

The adjustment factor ( $ADJ_{ijk}$ ), however, is a primary component of the mixed-species model. Potential upgrowth proportions may be reduced to actual estimates based on measures of stand competition such as density. The adjustment term therefore represents an estimate of the proportion of potential diameter growth which will be realized during a growth period. For constants  $b_0^{ij}$  and  $b_1^{ij}$ , and appropriate measures of stand density ( $D_{k-1}$ ), a possible form for the adjustment factor is:

$$ADJ_{ijk} = b_0^{ij} e^{-b_1^{ij} D_{k-1}} \quad (2)$$

Relation (2) is presented as an example specification of the adjustment relationships. The negative exponential form is widely used in growth and yield studies in forestry and reflects expected relationships in the present case. That is, relation (2) ensures that each estimated proportion of potential realized will be between 0 and 1, and represents an inverse relationship between diameter growth rate and stand density. Relation (2) also implies that changes in the diameter growth rate with respect to density are proportional to the growth rate. This result is demonstrated in the Appendix by deriving the negative exponential form from the proportionality statement.

The appropriateness of a more general form of relation (2), including alternative measures of stand density, was discussed by Hann in modeling basal area growth in ponderosa pine [3]. The choice of appropriate density variables may depend on such considerations as data requirements, relative ability to predict remeasurement data, and the ultimate use for the projection model.

For mixed-species stands, measures of total stand density as well as terms for each species may be incorporated in the upgrowth relation. Such terms may be used to reflect the relative position of a diameter class within the stand, and to model interspecific growth effects, i.e., the effects on growth of one species resulting from the presence of other species groups in the stand. During a fixed growth interval, the proportion of potential diameter growth realized by trees of a given species and diameter class may therefore be modeled as a function of total stand density, and the presence and relative size of other species groups within the stand. The general upgrowth relation presented therefore has potential for modeling a wide range of mixed-species projection problems.

### Mortality

The second component of the even-aged stand-table projection model is the mortality relation. This process represents the number of trees in each species/diameter class combination dying from resource competition over the growth period.

Overstorey tree mortality in northern hardwoods was predicted by Monserud using diameter and diameter increment, a competition index, and the length of the growth period as independent variables [8]. With stand-table projection,

diameter class and growth period length are fixed. Diameter growth, however, was modeled as a function of competition or density in the upgrowth relation. The important factors in modeling mortality are therefore incorporated in the stand-table approach.

As with the upgrowth relation, a tentative form is discussed for the stand-table mortality equation. Relation (3) is presented as an example specification of mortality over a fixed growth interval as a function of density.

$$N_{ijk}^M = (N_{ijk-1}^I)(1 - b_2^{ij} e^{-b_3^{ij} D_{k-1}}) \quad (3)$$

Estimated mortality for each species/diameter class group is the number of trees present at the beginning of the growth period, multiplied by the proportion of trees dying. The relation ensures that the estimated proportion dying is between 0 and 1, and is positively related to density. As discussed for the upgrowth relation, density terms may be used to reflect the relative position of each diameter class within the stand, and in relation to other species groups within the stand.

## DISCUSSION

Harvesting may be incorporated in the stand-table projection model by projecting residual numbers of trees at the beginning of each growth period ( $N_{ijk-1}^R$ ). Residuals before each period may be defined as the difference between the initial number of trees in each species/diameter class combination ( $N_{ijk-1}^I$ ), and the number of trees cut ( $N_{ijk-1}^C$ ). Management programs may thus be formulated for selecting numbers of trees to remove from each combination, prior to each growth period projected. After thinning, diameter growth accelerates for the residual stand, since density measures for the next growth period are reduced, resulting in higher upgrowth and lower mortality proportions.

A disadvantage of the proposed modeling approach is the large number of parameters to be estimated from remeasurement data. Data which reflect various thinning policies are needed for specifying and estimating the potential and adjustment portions of the upgrowth relation, and the mortality equation. Parameter estimates would be needed for each forest type or combination of species groups to be modeled. Given appropriate data, however, parameter estimation would not be difficult, as the nonlinear forms specified are log-linear.

The general method presented for stand-table projection is designed for short-term projections of even-aged stand growth and structure for mixed-species forest types. The method is an original synthesis of growth and yield approaches which have been successfully applied. That is, concepts which have been used in modeling mixed-species forest types are proposed for use in mixed-species stand-table projection.

Projecting growth components for stands of mixed-hardwood, pine-hardwood, alder-couifer, Douglas-fir-hemlock, etc., with relatively simple equations would



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facilitate analyzing management policies for many existing stands. Growth and yield information is provided by diameter class and species, allowing such evaluations to reflect the often pronounced interspecific growth rates and value by size class relationships frequently associated with mixed-species forest types.

#### APPENDIX

Where  $C$ 's represent constants, the negative exponential form of the adjustment process in relation (2) is derived from the premise (A1) that changes in the growth rate with respect to density are proportional to the growth rate.

$$\frac{dADJ_{ijk}}{dD_{k-1}} = C_1 ADJ_{ijk} \quad (A1)$$

Rewriting (A1) yields:

$$\frac{1}{ADJ_{ijk}} dADJ_{ijk} = C_1 dD_{k-1} \quad (A2)$$

Integrating both sides of (A2),

$$\int_{-\infty}^{\infty} \frac{1}{ADJ_{ijk}} dADJ_{ijk} = \int_{-\infty}^{\infty} C_1 dD_{k-1},$$

yields:

$$\ln(ADJ_{ijk}) + C_2 = C_1 D_{k-1} + C_3. \quad (A3)$$

(A3) may be rewritten as:

$$ADJ_{ijk} = e^{C_1 D_{k-1} + C_4} = C_5 e^{C_1 D_{k-1}}.$$

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