Consider the function $y = x$ on the interval [0, 1].

Suppose that this function is continuous from $[a, b]$.

The volume of the solid is given by

$$
\int_{a}^{b} \pi (x^2) dx
$$

Where $A(x)$ is the area at $x$.

Therefore the volume of the function on the interval [0, 1] is

$$
\int_{0}^{1} \pi x^2 dx
$$

The formula with a radius of

$$
1 - x^2
$$
on the interval [0, 1].

This graph shows the region that would be used to evaluate the volume of the function

$$
\int_{0}^{1} \pi (1 - x)^2 dx
$$

The value $\pi$ is the volume in terms of $\pi$ and the approximate value is

$$
\approx 1.67050892
$$

Notice we are only using 100 rectangles in the above problem and our answer agrees with the answer found using standard calculations to a large degree of accuracy.

Limitations

The limits for the methods that are used in Calculus II only allow the rotation of curves around the

- $x$-axis
- $y$-axis
- $y=C$ (where $C$ is some value in the integers)
- $x=C$ (where $C$ is some value in the integers)

The methods used in Calculus II are

- Shell Method
- Disk Method
- Washer Method

Each has its own purpose but none of them have the capability of revolving a curve around a line that has an equation of

$$
y = \arctan{x} + b
$$

Problem Description

This verifies our algorithm for finding the volume of a solid of revolution about the line $y = 0$. To verify our method works we make use of the symmetry of the circle. We rotate the chord from (0,0) to (1,1) on the unit circle $x^2 + y^2 = 1$ about the line $y = x$ (the line on which the chord sits) using the algorithm we developed.

We then use the conventional formula from Calculus II to rotate the chord from (0,0) to (1,1) on the unit circle. Notice both chords have the same length and are both on a circle of radius 1. Therefore the solids formed should be of the same volume.

$$
\frac{\pi}{4}
$$

about the line $y = x$ (the line on which the chord sits).

The variable $n$ represents the number of rectangles used in approximating the volume.

The quantity $\Delta x$ below computes the distance between elements of the partition of [0,1].

$\Delta x = \frac{1}{n}$.

The function $f(x)$ is used to traverse through the partition of the segment [0,1].

The usual notation is for the partition is $x_0, x_1, \ldots, x_n$.

$\Delta x_i = f(x_i) - f(x_{i-1})$.

The function $f$ is the $x$-coordinate of the intersection point between the line and the function $f(x)$. Moreover it is the specific intersection point located in the interval $[x_{i-1}, x_i]$.

$4\pi \cdot \text{length of chord} = \pi \cdot \text{length of chord}$.

The coordinates which cause the "bottom" of the rectangle are given by $(x_i, y_i)$ and $(x_{i-1}, y_{i-1})$. Using the distance formula we find the length of the bottom of the rectangle which we label to be $2$ for width. This can be considered to be the width of the rectangle or the width of the approximating disk.

$\frac{\pi}{4}$

We know the radius used in the Disk Method, recall that the radius of the disk is the height of the rectangle being used in the approximation. So we find the distance between the point on the line of rotation $(x_i, y_i)$ and our point on the line in the direction perpendicular to the line $(x_i, y_i)$.

The plane $x = \text{constant}$

Hence our answer is...

Our Answer: $0.127030427$

Notice we are only using 100 rectangles in the above problem and our answer agrees with the answer found using standard calculations to a large degree of accuracy.

Verification Circle

To verify the correctness of our method we now use the conventional formula from Calculus II to find the volume of the solid generated by the chord from $(0,0)$ to $(1,1)$ on the unit circle $x^2 + y^2 = 1$ about the line $y = x$ (the line on which the chord sits).

The function which yields the bottom half of the above circle is given by $y = x$.

$$
\int_{0}^{1} \pi x^2 dx
$$

Here we graph the solid formed by our rotation.

We now compute the volume of the above solid using the formula from Calculus II in order to verify our algorithm is correct.

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