Consider the function $y = \frac{1}{x}$ on the interval $[0, 1]$. Suppose that this function is continuous on $[a,b]$. The volume of the solid is given by
\[ \int_a^b \pi \left( \frac{1}{x^2} \right)^2 \, dx \]
Therefore the volume of the function on the interval $[0, 1]$ is
\[ \int_0^1 \pi \left( \frac{1}{x^2} \right)^2 \, dx \]
The formula with a radius of $r = \sqrt{1 - x^2}$ on the interval $[0, 1]$:
\[ \int_0^1 \pi \left( \sqrt{1 - x^2} \right)^2 \, dx \]

This graph shows the region that would be used to evaluate the volume of the function.

The value $\frac{1}{x}$ is the volume in terms of $\pi$ and the approximate value is $\int_0^1 \pi \left( \sqrt{1 - x^2} \right)^2 \, dx 
\approx 0.7854\]  

We note that we only use 100 rectangles in the above problem and our answer agrees with the answer found using standard calculations to a large degree of accuracy.

The volume is represented by the slope of the line we are rotating about.

Here we define the function for which we are about to rotate.

Next we compute the two intersection points to determine where the rotation starts and ends.

We begin by plotting the function $f(x)$ and the line $y = mx$.

The solid for which we are finding the volume is graphed below.

The limited methods used in Calculus II are:
- Shell Method
- Disk Method
- Washer Method

Each has its own purpose but none of them have the capability of resolving a curve around a line that has an equation $y = mx + b$.

This verifies our algorithm for finding the volume of a solid of revolution about the line $y = mx$. To verify our method we take a look at the symmetry of the circle. We rotate the chord from $(0,0)$ to $(1,1)$ on the unit circle $x^2 + y^2 = 1$ about the line $y = x$ (the line on which the chord sits) using our algorithm developed.

We then use the conventional formula from Calculus II to rotate the chord from $(0,0)$ to $(1,1)$ on the unit circle.

Notice both chords have the same length and are both on a circle of radius $1$. Therefore the solids formed should be of the same volume:
\[ \frac{\pi}{8} \]  
about the line $y = x$ (the line on which the chord sits)

The variable $n$ represents the number of rectangles used in approximating the volume.

The quantity $\Delta x$ is used to compute the difference between elements of the partition of $[a,b]$.

The function $y = mx$ is used to iterate through the partition of the segment $[a,b]$. The usually notation for the partition is $x_0, x_1, \ldots, x_n$.

The point of intersection is the $x$-coordinate of the intersection point between $y = mx$ and the line perpendicular to $y = mx$, which is given by $y = mx$.

The function $f(x)$ is the $x$-coordinate of the intersection point between this line and the function $y = mx$. Moreover it is the specific intersection point located in the interval $[a, b]$.

Here we compute the radius used in the Disk method. Recall that the radius of the disk is the height of the rectangle being used in the approximation. So we find the distance between the point on the line of rotation $(x, mx)$ on $y = mx$ and the point on the function in the direction perpendicular to the line $(x, f(x))$.

Our answer: $0.212703427$

Notice we are only using 100 rectangles in the above problem, and our answer agrees with the answer found using standard calculations to a large degree of accuracy.

To verify the correctness of our method we once again use the conventional formula from Calculus II to find the volume of the solid generated by the chord from $(0,0)$ to $(1,1)$ on the unit circle $x^2 + y^2 = 1$ about the line $y = x$ (the line on which the chord sits).

The function which yields the bottom half of the above circle is given by $y = \sqrt{1 - x^2}$.

Here we graph the solid formed by our rotation.

We now compute the volume of the above solid using the formula from Calculus II in order to verify our algorithm is correct.

Finally we compare our algorithm to the answer found using techniques from Calculus II, and notice that the answers are a difference of $\sqrt{\pi} = 3.14159265358979323846$.

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