Communication Across Random Landings

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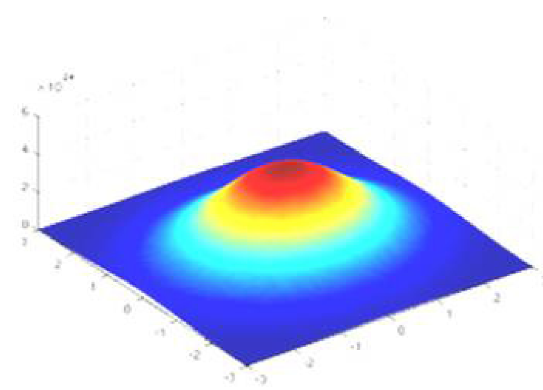
Abstract

The idea to this project began with a simple question: Suppose that people carrying communication radios parachute out a plane, if each device has a certain range, what is the probability that once everyone lands they will be able to communicate. To study this problem I assumed that the spot where each individual lands is normally distributed (see the figure below.)



We discuss the different ways the communication radios can work. In particular we examine the situation where all the radios have to be within a certain radius *r* to operate correctly and the situation where the radios work on a relay system. We discuss how the probabilities differ in each situation. For the sake of clarity here we only take into account horizontal distance between the jumpers.

Tables

Below, the tables show a comparison with results when the formulas and theorems were applied versus the results from a computer simulation. The first table corresponds to the non relay system and the second, the relay system, both with 3 jumpers.

RELAY

Distance	Simulation Results	Calculation Results
0.1	0.006	0.005497
0.5	0.122	0.127868
1	0.416	0.414697
2	0.866	0.863469
3	0.983	0.983863
4	0.999	0.998953

NON-RELAY

Distance	Simulation Results	Calculation Results
0.1	0.003	0.002752
0.5	0.066	0.066578
1	0.240	0.240713
2	0.661	0.666501
3	0.914	0.914457
4	0.986	0.987012

Method

In both cases, we notice that the jumpers landing spot specified by the standard normal random variables $X_1, X_2, X_3, \ldots, X_n$ can be ordered in *n!* ways. More importantly since the variables are i.i.d. standard normal, each ordering is equally likely.

In the case where the radios do not work on a relay system we need all the jumpers to be within a certain distance *r* of each other. We assume the ordering $X_1 < X_2 < X_3 < \ldots < X_n$. Thus, we compute

$P(X_1 < X_2 < X_3 < \ldots < X_n \text{ and } X_n - X_1 < r)$ or

$P(X_2 - X_1 > 0 \text{ and } \ldots \text{ and } X_n - X_{n-1} > 0 \text{ and } X_n - X_1 < r)$,

noting that if X_n and X_1 are within *r* of each other then all will be within *r* of each other.

In the case where the radios work on a relay system we need consecutive jumpers to be within a distance *r* of each other. Similarly, we assume that there is an ordering and compute

$P(X_1 < X_2 < X_3 < \ldots < X_n \text{ and } X_2 - X_1 < r \text{ and } X_3 - X_2 < r \text{ and} \ldots \text{and } X_n - X_{n-1} < r)$.

Equations and Theorems

Some of the main equations and theorems used to answer this question stated below with a brief explanation.

1.For simplicity, we only consider the horizontal distance on a line.

2. The normal distribution is

$$f(x) = e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \frac{1}{\sqrt{2\pi\sigma^2}}$$

3.If X~N(0,1), Y~N(0,1), where X and Y are independent, then X+Y and X-Y are both ~N(0,2)

4.The multivariate normal distribution is a generalization of the normal distribution to higher dimensions. The distribution function is

$$F_n(\vec{y}) = e^{-\frac{1}{2}(\vec{y}-\vec{\mu})^T \Sigma^{-1} (\vec{y}-\vec{\mu})} \frac{1}{(2\pi)^{n/2} \det(\Sigma)^{1/2}}$$

where Σ is the covariance matrix. It can sometimes be used when you have several normal random variables which are not independent.

Results

In general, both the formula for probability of communication amongst *n* jumpers is given by:

NON-RELAY

$$n! \int_0^r \int_0^{r-y_{n-1}} \int_0^{r-y_{n-1}-y_{n-2}} \Lambda \int_0^{r-y_2-y_3-\Lambda-y_{n-1}} F_{n-1}(\vec{y})\, dy_1 dy_2 \mathrm{K}\, dy_{n-1}$$

RELAY

$$n! \int_0^r \int_0^r \int_0^r \Lambda \int_0^r F_{n-1}(\vec{y})\, dy_1 dy_2 \mathrm{K}\, dy_{n-1}$$

where $Y_k = X_{k+1} - X_k$ for $k = 1,2,3, \ldots, n-1$ and

$$\Sigma = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & O & O \\ 0 & O & O & -1 \\ 0 & O & -1 & 2 \end{bmatrix}$$