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THINNING OPTIMIZATION FOR MIXED-SPECIES FORESTS

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ABSTRACT

An approach is summarized for estimating optimal thinning and final harvest age for existing, mixed-species stands. The method involves stand-table projection with upgrowth and mortality equations, formulated as an integer-nonlinear programming problem. Random search methods are proposed for estimating optimal cutting prescriptions. Such solution methods warrant further study in forestry, since their use enables broad application of stand-specific modeling results.

INTRODUCTION

Mixed-species forests in the United States range from mixed-hardwood and pine-hardwood forest types in the South, to alder-conifer and Douglas-fir-hemlock stands in the Pacific Northwest. Management decisions in such stands can be very complex, especially for species with different growth rates, values, and value-by-size-class relationships. Thinning optimization involves the timing, frequency, and intensity of partial harvests over time. We address the problem of optimizing harvests for existing, mixed-species stands of even age, estimating optimal thinning and the optimal age of final harvest.

Thinning evaluations are based on models of growth and yield. We propose a mixed-species growth model and formulate a thinning model for selecting and comparing cutting options. Solution methods which recognize the discrete nature of numbers of trees cut over time are discussed. The methods are demonstrated to yield near-optimal harvesting strategies with very little computer time or storage.

GROWTH MODEL

Three criteria were necessary for modeling mixed-species growth and yield. The approach had to project volume by species group and diameter class. The species/diameter level of resolution is necessary to reflect potential differences in growth rates and values by size classes. The growth model also had to reflect responses to thinning, and had to be compatible with optimization procedures.

Stand-table projection models provide growth and yield estimates at the diameter class level of resolution. These models also account for growth responses to thinning, and may be formulated to determine optimal harvesting plans, as demonstrated for all-aged northern hardwoods by Adams and Ek (1974). We propose a stand-table projection model for mixed-species stands of even age. Mixed-species modeling concepts used by the U.S. Forest Service (1979) are incorporated in the upgrowth and mortality components of a stand-table projection system.

Upgrowth

Upgrowth is the number of trees advancing from one diameter class to the next during a fixed-length growth period. Estimates are needed for each species/

diameter class, for each growth period projected, and are obtained by multiplying the number of trees in each class by the estimated upgrowth proportion for that species/diameter group:

$$\text{Upgrowth} = (\#\text{Trees})[(\text{Potential Proportion})(\text{Adjustment Factor})] \qquad (1)$$

The upgrowth proportion has two terms. The potential proportion is an upper limit on upgrowth, and should represent the proportion of trees advancing for stand densities near zero. The ability of trees to respond to release is related to past competition, and since upgrowth is projected for each species, diameter class, and growth period, site quality is sufficient for predicting potential upgrowth. That is, site quality, age, diameter, and species reflect past competition and potential for responding to release. Harvesting, however, does not affect the upper bound on upgrowth, and potential proportions are constant with regard to thinning strategies.

Upgrowth estimates are affected by harvesting through the adjustment factor. The adjustment term represents the estimated proportion of the potential which will be realized during a growth period, and is a decreasing function of stand density.[1/] Reducing density through cutting results in higher adjustment values, thereby resulting in greater upgrowth proportions for the residual stand.

Mortality

The mortality component of the stand-table projection model represents the number of trees dying by species, diameter class, and growth period. These numbers are estimated as:

$$\text{Mortality} = (\#\text{Trees})(\%\ \text{Mortality}) \qquad (2)$$

[1/] We tentatively specified the adjustment term as a negative exponential function of volume:

$$\text{Upgrowth} = (\#\text{Trees})[(\text{Potential})(B_0 e^{-B_1'V})].$$

Where B_0 is a constant and B_1 and V are vectors of constants and stand volume variables, respectively. Regression constants vary for each species/diameter class group. Potential proportions vary by growth period, thus reflecting stand age.

The mortality proportion is an increasing function of stand density[2/]. Partial harvests therefore reduce the rate of mortality estimated for post-harvest growth periods.

Mixed-species stand-table projection meets the criteria necessary for modeling harvesting options. Both upgrowth and mortality are affected by thinning, and the model provides information at the species/diameter level of resolution. The stand-table projection method can also be used to formulate a thinning model for estimating optimal harvesting plans.

THINNING OPTIMIZATION

In recent years, dynamic programming has become one of the most frequently used thinning optimization techniques (see Brodie and Kao 1979, Chen et al. 1980, and Riitters et al. 1982). Dynamic programming is a logical choice of methods, exploiting the sequential nature of harvesting problems in forestry. For thinning problems which recognize diameter classes, however, state-space dimensionality problems result (Hann and Brodie 1980). The problems expand for applications with separate diameter classes for each species. Thinning problems can result in billions of discrete nodes, each representing a management alternative to be evaluated in the dynamic programming network.

We formulated the problem of thinning and final harvest for even-aged, mixed-species stands using integer-nonlinear programming. The problem is integer since numbers of trees cut by species, diameter, and growth period are the decision variables. The formulation is nonlinear due to the nonlinear upgrowth and mortality equations. We discuss the formulation, including the constraints and objective function, as well as proposed solution methods.

Formulation

The primary set of constraints in the nonlinear formulation defines the residual

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[2/] The specification we used for mortality was also a function of stand volume:

$$\text{Mortality} = (\#\text{Trees})(1 - B_2 e^{-B_3'V}).$$

Where different regression constants are used for each species and diameter class.

stand after each growth period. The general form of these constraints is:

$$\begin{bmatrix}\text{Residual \#trees}\\ \text{after growth}\\ \text{period k}\end{bmatrix} = \begin{bmatrix}\text{Residual \# trees}\\ \text{after growth}\\ \text{period k-1}\end{bmatrix} - \begin{bmatrix}\text{\#trees advancing}\\ \text{as upgrowth}\end{bmatrix} - \begin{bmatrix}\text{\#trees lost through}\\ \text{mortality}\end{bmatrix} + \begin{bmatrix}\text{\#trees gained}\\ \text{through upgrowth}\end{bmatrix} - \begin{bmatrix}\text{\#trees harvested}\end{bmatrix} \quad (3)$$

Such constraints are needed for each species/diameter class, after each growth period projected. From (3), the residual number for each group is the number of trees at the beginning of the growth period, minus the number growing into the next larger diameter class, minus the ones dying, plus the trees growing into the diameter class, minus the number of trees cut. In this manner, the residual stand is estimated for each growth period, with residual numbers used in the density terms of the upgrowth and mortality functions for the next interval projected[3]. Numbers of trees harvested are the decision variables, and harvests are possible after each of the fixed-length growth periods.

Residuals defined in (3) are for intermediate diameters since some terms are not applicable to the smallest and largest classes. For the smallest diameter class, for example, no trees are gained through upgrowth since stands are even-aged. Numbers projected in the largest diameters for each species, after each period, are comprised entirely of upgrowth minus numbers harvested. Upgrowth and mortality terms from (1) and (2) are substituted into (3) to complete the nonlinear programming constraint set (see Bullard (1983) for details of the modeling procedure).

Other constraints may also be included in modeling thinning and final harvest age. Constraints setting minimum and maximum thinning levels, for example, are easily included as functions of numbers of trees cut by species and diameter.

[3] As noted, density terms were tentatively specified as functions of volume. Numbers of residual trees are converted to volume using average volumes per tree by species and diameter class.

The most appropriate economic objective for valuing existing stands is the present value of future income (see Clutter et al. 1983). Income is obtained from the present stand, and from the property after the present stand is removed. The objective in thinning and final harvest optimization is to estimate the policy which maximizes:

$$\begin{bmatrix}\text{Present}\\ \text{Value}\end{bmatrix} = \begin{bmatrix}\text{Harvest values}\\ \text{by species}\\ \text{and diameter}\end{bmatrix} \cdot \begin{matrix}\text{.discounted}\\ \text{from each}\\ \text{growth period}\end{matrix} + \begin{bmatrix}\text{Income after}\\ \text{stand removal}\end{bmatrix} \cdot \begin{matrix}\text{.discounted}\\ \text{from final}\\ \text{harvest}\end{matrix} - \begin{bmatrix}X_k\text{*Fixed costs}\end{bmatrix} \cdot \begin{matrix}\text{.discounted}\\ \text{from each}\\ \text{growth period}\end{matrix} \quad (4)$$

Harvest values from the present stand are discounted by growth period since cutting is possible after each interval. Reflecting different values and values by sizes is an important modeling attribute for many mixed-species forests. The second term in (4) represents future income from the property. This term may be a soil expectation value if forestry uses are expected, or a future market value for land if property sale or change in land use is considered. By discounting from final harvest age, the opportunity costs of holding an existing stand are reflected. Fixed costs are subtracted after growth periods where thinning is chosen ($X_k = 1$), but are not incurred without harvesting ($X_k = 0$).

Solution

The mixed-species thinning and final harvest age problem is combinatorial; i.e., it involves the selection of a finite number of discrete objects from a larger set. There are no efficient converging algorithms for large combinatorial problems (see Muller-Merbach 1981), and heuristics or inexact solution methods are often advocated. We considered simple and multistage random search for estimating optimal harvesting prescriptions for mixed-species stands.

Simple random search involves randomly generating and evaluating solutions to integer mathematical programming problems. The approach has also been termed Monte-Carlo Integer Programming (Conley 1980), and is often advocated based on the following simple probability argument. Given all solutions to an integer problem, the objective value relative frequency can be plotted and is bounded on the right by the optimum for maximization problems (fig. 1). The objective in generating random solutions is to obtain at least one with an objective value which is near-optimal. This goal is represented in figure 1 by obtaining at least one solution whose objective value is within a desired sub-region (a) of the extreme or optimum value.

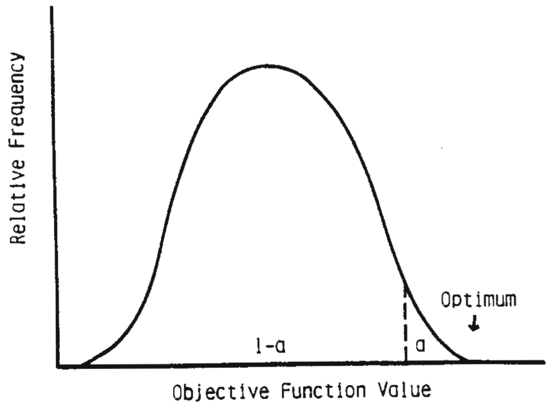

Figure 1. Relative frequency of objective function values for a finite, integer mathematical programming problem.

The probability that a random solution has a lower objective value is (1 - a), and the probability that all (n) solutions in a random sample have lower values is $(1 - a)^n$. Therefore, the probability that at least one solution from a sample of (n) will have an objective value within the desired sub-region of the optimum is:

$$\text{Probability} = 1 - (1 - a)^n \quad (5)$$

The major defense of simple random search is that for large values of n, the probability approaches 1. Two disadvantages, however, are that near-optimal solutions may not be adequate for problems with multi-million dollar objective values, and the degree to which estimated solutions approach optimal values is unknown.

Multistage random search, or Multistage Monte-Carlo Integer Programming (Conley 1981), refines the simpler approach by directing random searches to concentrated areas of the feasible region. Sets of random solutions are called stages, and after each set the ranges for generating decision variable values (numbers of trees cut by species and diameter) are narrowed, centered around the values in the best solution obtained thus far. Sampling proceeds until the number of stages specified has been completed.

In forestry applications of random search methods, the greatest disadvantage is not knowing how close an estimated solution is to the optimum objective value. Means are available, however, for gaining confidence in random search algorithms. Methods have been developed for statistically estimating the optimal objective value to a problem. The value is estimated from the random sampling results (see Zanakis and Evans 1981). The best solution obtained through random sampling can then be compared with the estimated optimal objective value, with continued sampling until the comparison is acceptable.

We applied both simple and multistage random search methods to mixed-species thinning problems using assigned growth model coefficients. In an example with a known optimal present value of $485.76, both methods produced thinning schedules with per acre values within 99 percent of the optimum, using only seconds of execution time on an IBM 3081. Both methods were also used to estimate solutions to problems with unknown optima. In all cases, the multistage approach produced solutions with higher objective values than simple random search. Further research in using random search in forestry applications should incorporate statistical techniques in estimating optimal objective values.

DISCUSSION

Mixed-species thinning decisions must account for interspecific growth rates, values, and value-by-size-class relationships. We propose an approach for modeling mixed-species cutting options which recognizes both the size and species of material harvested over time. The method involves mixed-species stand-table projection with two equations, and is formulated to estimate present value maximizing schedules for removing existing stands.

Random search methods are particularly appropriate for estimating cutting strategies because of the problems' integer nature, the lack of efficient converging algorithms, and the use of inexact data or growth projections. Multistage methods appear especially promising for thinning and other stand- and forest-level problems. Random search methods can be statistically evaluated and use very little computer memory or storage. Ideally, master programs could be used to generate harvesting prescriptions on microcomputers. Biometric and economic parameters, stored by forest type, would allow users to input stand-table data and generate near-optimal cutting policies with specified confidence levels. A possible disadvantage in using microcomputers, however, may be the time needed to examine thousands of solutions. Further work should examine the extent of this problem, since broad implementation of stand-specific modeling relies on wide user access. Random search methods, in general, however, have many applications and certainly warrant further study for complex decision-making problems in forestry.

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