1988

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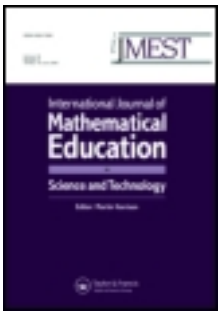

International Journal of Mathematical Education in Science and Technology

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmes20

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Published online: 09 Jul 2006.

To cite this article: Keith A. Blatner & Steven H. Bullard (1988) Present net worth and internal rate of return: a note on equivalence in use , International Journal of Mathematical Education in Science and Technology, 19:6, 851-855, DOI: 10.1080/0020739880190609

To link to this article: http://dx.doi.org/10.1080/0020739880190609

INT. J. MATH. EDUC. SCI. TECHNOL., 1988, VOL. 19, NO. 6, 851–855

Present net worth and internal rate of return: a note on equivalence in use†

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(*Received 7 April 1986*)

Present net worth and internal rate of return are frequently used for ranking investment alternatives. The relative merits and limitations of the two criteria are discussed at length in numerous journal articles and texts. For analysts wishing to present consistent project comparisons, the issue is not which criterion is superior, but when are they equivalent. In this article, conditions for equivalence in use are outlined under alternative assumptions. Although the conditions appear complex, they are not unduly restrictive.

1. Introduction

In many diverse disciplines, the increasing importance of a sound educational background in the evaluation of capital investments is without question. Two of the most popular financial investment criteria are present net worth (PNW), the present value of a project's revenues or benefits minus the present value of the costs, and internal rate of return (IRR), the interest rate at which the present value of the benefits equals the present value of the costs.

PNW is often preferred by economists and academicians, yet recent surveys show businees managers often prefer IRR due to its uniquely descriptive nature [1]. Most textbooks emphasize the theoretical superiority of PNW, yet in many cases, the critical issue is not which criterion is superor, but that students and analysts fully understand the mathematical equivalence of the two criteria for different uses and comparisons of projects.

2. Equivalence in use

IRR and PNW are often used to describe alternative investment opportunities or to answer the accept–reject question for individual projects. Where alternatives are not mutually exclusive and capital is not rationed, they provide equivalent results [1]. For particular projects, if the present value of benefits (PVB) exceeds the present value of costs, IRR is greater than the alternative rate or return and vice versa. Decision makers must often choose between mutually exclusive investments, however, or must budget limited capital between investment opportunities. Under such conditions, project rankings by PNW and IRR are not necessarily equivalent. A sufficient condition for investment ranking by IRR to be equivalent to ranking by

† This paper was approved by the Director, Washington State University Agricultural Research Center, as Scientific Paper 7381, Project 0671.

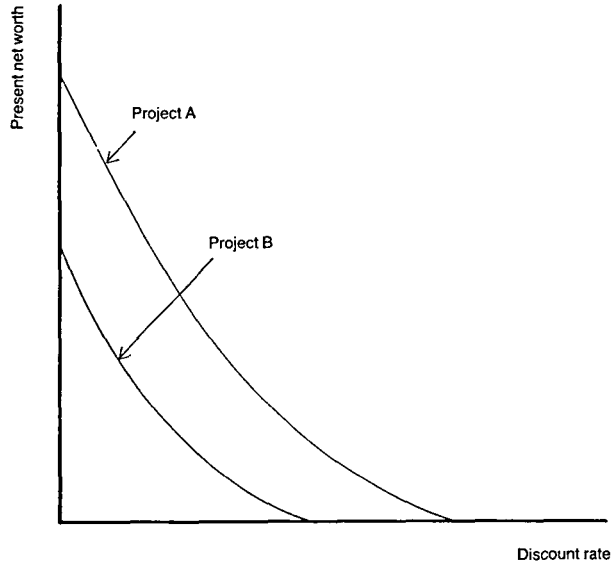

Figure 1. Non-intersecting PNW relations.

PNW is that PNW relations for competing projects be non-intersecting within the range of positive values (figure 1) [2]. If the PNW relations intersect at one point (discount rate r^* in figure 2), however, project ranking by IRR is equivalent to ranking by PNW only in those cases where the alternative rate of return (ARR) is greater than r^* (figure 2).

Factors contributing to the intersection of PNW relations within the positive range include scale of the investment, timing of the cash flows, and cash flows yielding no single IRR solution [1].

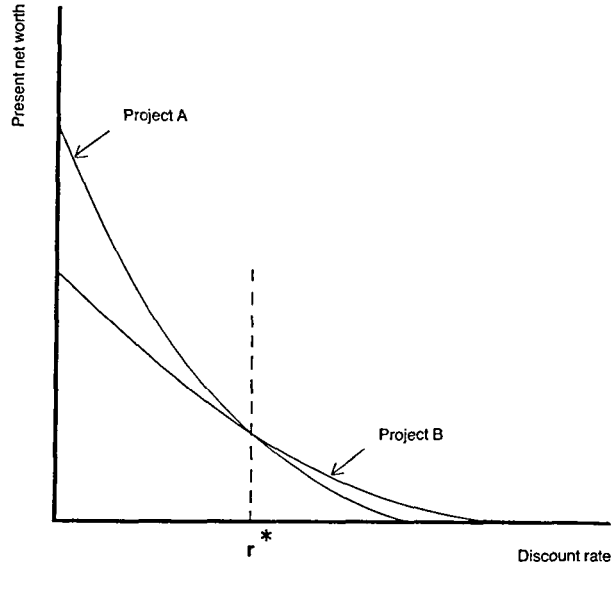

Figure 2. Intersecting PNW relations.

3. Application of IRR on an incremental basis

In many cases, inconsistent project rankings between PNW and IRR may be overcome by applying IRR on an incremental basis. Under this approach, the analyst chooses between alternatives by deciding whether or not the differences in project costs are justified by the differences in project benefits [3]. Consider the following example in table 1, where investment alternatives differ in scale.

Table 1. Cash flows for investment alternatives 'A' and 'B' in U.S.$.

Investment alternative	Year			
	0	1	2	3
A	−1000	500	500	500
B	−11000	5000	5000	5000

With a 10 per cent cost of capital, the two criteria rank the projects differently (table 2).

Table 2. PNW and IRR values for investment alternatives 'A' and 'B'.

Investment alternative	PNW (U.S.$)	IRR (per cent)
A	243·42	23·37
B	1434·25	17·27

PNW relations for the two projects are shown in figure 3. The two functions intersect at a discount rate (r^*) of 16·65 per cent. As previously noted, for any alternative rate of return greater than 16·65 per cent, PNW and IRR yield equivalent project rankings. With discount rates less than 16·65 per cent, project rankings disagree.

To resolve the ranking inconsistency, assume the firm uses IRR exclusively, and therefore plans to undertake investment alternative 'A', yet is faced with an investment capital limitation of U.S.$11 000. A logical question is 'Would it be worthwhile to add an additional or incremental investment alternative "B-A"?'. If the answer is yes, the total investment would equal A+(B–A). However, since A+(B–A)=B, by accepting 'B–A' one has actually decided investment alternative 'B' is superior to 'A'.

In terms of our example, the incremental investment alternative would be as shown in table 3. The IRR of this alternative is 16·65 per cent or r^*. Since the cost of capital is 10 per cent, the firm would choose to accept 'B–A', thereby changing its investment decision to alternative 'B'. This is the same decision the firm would have made using PNW for project comparisons.

Table 3. Cash flows for investment alternative 'B–A' in U.S.$.

Investment alternative	Year			
	0	1	2	3
B–A	−10000	4500	4500	4500

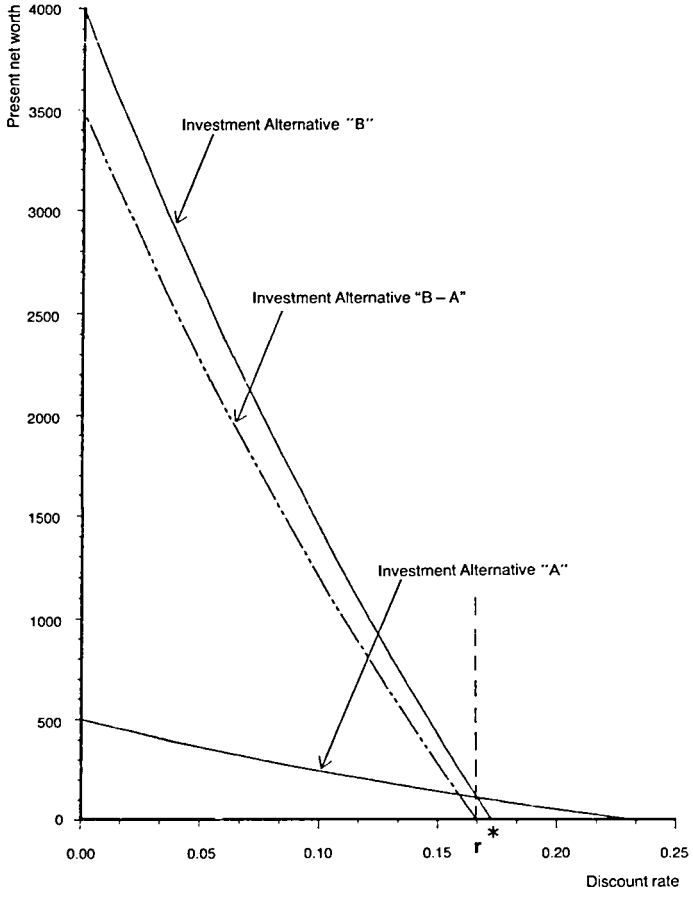

Figure 3. Comparison of investment alternatives using incremental analysis.

Applying IRR on an incremental basis yields project rankings equivalent to PNW if (1) both investment alternatives are viable, and (2) the PNW relations only intersect once in the first quadrant. Mathematically this implies the following relationships.

$$\text{If IRR 'A'} \geqslant \text{ARR and IRR 'B'} \geqslant \text{ARR} \tag{1}$$

Then

$$\text{PNW 'B'} \geqslant \text{PNW 'A'} \rightarrow \text{IRR 'B–A'} \geqslant \text{ARR} \tag{2}$$

or

$$\text{PNW 'A'} > \text{PNW 'B'} \rightarrow \text{IRR 'B–A'} < \text{ARR} \tag{3}$$

The validity of relations (2) and (3), follows directly from figure 3 and the definitions of the investment criteria. Consider relationship (2) and figure 3, for example: for discount rates between ARR and r^*, PNW 'B' > PNW 'A' and the IRR of 'B–A' will exceed ARR (since IRR 'B–A' equals r^*). Similarly, if ARR were greater than r^*, PNW 'A' > PNW 'B' and IRR 'B–A' < ARR.

IRR can also be applied on an incremental basis where more than two projects are involved. Newman [3], among others, outlines the procedure for multiple investment alternatives.

4. Conclusions

PNW and IRR yield equivalent investment evaluations in most cases. Both criteria may be used to answer the fundamental accept/reject question with equivalent results. Investment rankings by PNW and IRR are not equivalent, however, unless the PNW relations are non-intersecting or unless they have a unique intersection in the positive quadrant and IRR was applied on an incremental basis. Obtaining non-contradictory results also depends on a unique rate of return for each investment alternative.

Although the aforementioned conditions appear complex and restrictive, the number of investment alternatives failing to meet these conditions are relatively limited. The criterion used should therefore reflect the analyst's or firm's preferences, except in cases where investment alternatives are often characterized by nonconventional cash flows. For such problems, PNW and/or the incremental application of IRR should be preferred over project rankings by simple internal rates of return.

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