MULTIPLICATICVE "CONIC"

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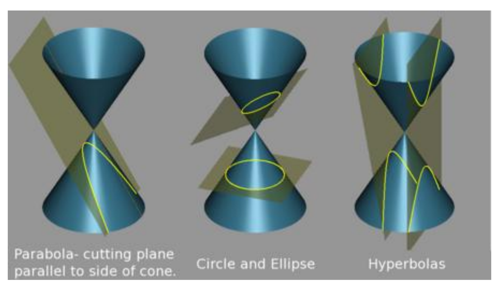
SFA MATH STATS

BACKGROUND

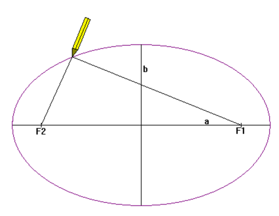
Conic Sections

- A curve obtained as the intersection of a cone and a plane



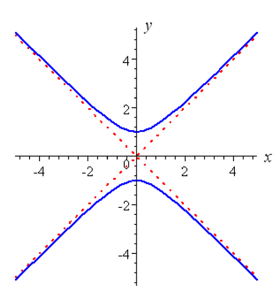
Ellipses and Hyperbolas

- An **Ellipse** is the collection of points in the plane whose distance to two points, foci, add to a fixed constant.



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

- A **Hyperbola** is the collection of points in the plane whose difference in distances from two points, foci, is a fixed constant.



$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

RESEARCH IDEA

What happens when, instead of the sum or difference, we examine the product of the distances from a set of points to two foci?

METHODS

As is customary, we use the points (c,0) and (-c,0) as our default foci. We are trying to find the set of points (x,y) that have the distance from (c,0) multiplied by the distance from (x,y) to (-c,0) is a fixed constant k. Using the distance formula this leads to the equation.

$((x-c)^2+y^2)^{1/2}((x+c)^2+y^2)^{1/2} = k$.

I squared both sides and obtained

$((x-c)^2+y^2)((x+c)^2+y^2) = k^2$.

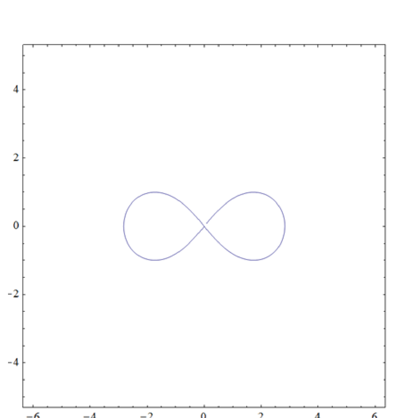
After simplification we end up with the equation,

$x^4 + y^4 + c^4 + 2x^2y^2 - 2x^2c^2 + 2y^2c^2 = k^2$.

RESULTS

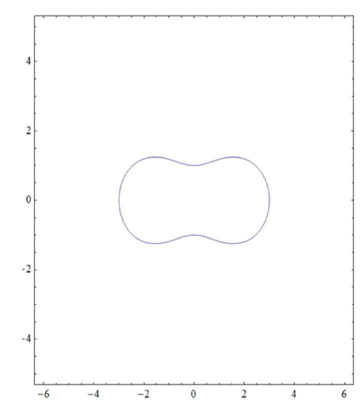
We then studied properties of the graph created by these 4th degree polynomials. These properties are with the assumption that we are centered about the origin.

- The x-intercepts are located at $x=\pm\sqrt{\frac{2c^2\pm\sqrt{(-2c^2)^2-4(c^4-k^2)}}{2}}$
- Simplified the equation becomes $x=\pm\sqrt{c^2\pm k}$
- The y-intercepts are located at $y=\pm\sqrt{\frac{-2c^2\pm\sqrt{(2c^2)^2-4(c^4-k^2)}}{2}}$
- Simplified the equation becomes $y=\pm\sqrt{-(c^2)\pm k}$
- When $c^2 = k$, we have x-intercepts of $(c\sqrt{2},0)$, $(-c\sqrt{2},0)$, and (0,0) which is also the only y-intercept.

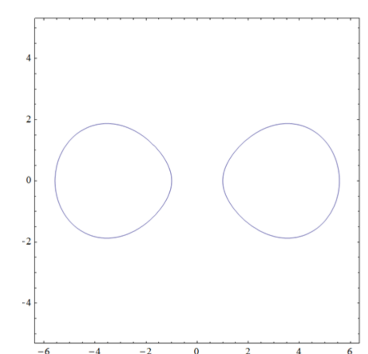


RESULTS CONTINUED

- When $c^2 < k$,
- These two x-intercepts are the same as two of the previous intercepts. They are $(-c\sqrt{2},0)$ and $(c\sqrt{2},0)$
- We take $y=\pm\sqrt{-(c^2)+k}$ for the location of the y-intercepts, because $y=\pm\sqrt{-(c^2)-k}$ does not yield real roots
- . The two y-intercepts are $\left(0,\sqrt{-(c^2)+k}\right)$ and $\left(0,-\sqrt{-(c^2)+k}\right)$

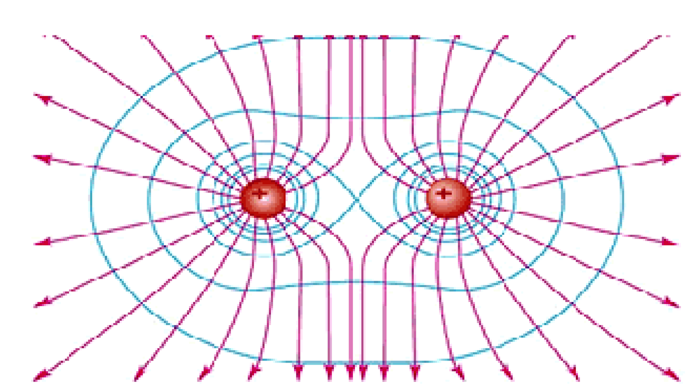


- When $c^2 > k$,
- The x-intercepts are $\left(\sqrt{c^2+k},0\right)$, $\left(-\sqrt{c^2+k},0\right)$, $\left(\sqrt{c^2-k},0\right)$, and $\left(-\sqrt{c^2-k},0\right)$.
- There are no real y-intercepts.



Applications and Future Research

We looked at equipotential field lines created by two particles of the same charge. The reason is because those field lines, represented by the blue curves in the picture below, form similar graphs as our equation for the product of the distances from a point to two foci.



We need to apply the equation for potential. The equation for potential for two body system is

$(KQ/r_1) + (KQ/r_2) = V$.

Here Q is the point charge, r is the distance from that charge, and K is a given constant.

For simplicity we begin with the equation
$V = (1/r_1) + (1/r_2)$
where r_1 and r_2 are the distances from a point to the foci.

After simplification we arrive at
$(r_1 + r_2)/(r_1 * r_2) = V^2$. Where V^2 is the potential.

Using the points (c,0) and (-c,0) as the location of our particle we can replace r_1 with $((x-c)^2+y^2)$ and r_2 with $((x + c)^2+y^2)$.
This translates into:
$(2x^2 + 2y^2 + 2c^2) / (x^4 + y^4 + c^4 + 2x^2y^2 - 2x^2c^2 + 2y^2c^2) = V^2 = 1 / k^2$, using our constant k in place of V.

Thus the equipotential field lines are an instance of the general equation.
$x^4 + y^4 + 2x^2y^2 + 2y^2A - 2x^2B + C = 0$.
where,

- $A = c^2 - k^2$,
- $B = k^2 + c^2$, and
- $C = c^4 - 2c^2k^2$

Future Research:

- I will take a more in-depth look into the general form obtained in this research.
- I will also take a look at the **quotient** of the distances from a point, (x,y) to two foci, (c,0) and (-c,0).