**BACKGROUND**

**Conic Sections**
- A curve obtained as the intersection of a cone and a plane

**Ellipses and Hyperbolas**
- An Ellipse is the collection of points in the plane whose distance to two points, foci, are add to a fixed constant.
- A Hyperbola is the collection of points in the plane whose difference in distances from two points, foci, is a fixed constant.

**RESULTS CONTINUED**

- When \( c^2 < k \),
  - These two x-intercepts are the same as two of the previous intercepts. They are (-c\(\sqrt{2}\),0) and (c\(\sqrt{2}\),0).
- We take \( y = \pm \sqrt{(c^2 + k)} \) for the location of the y-intercepts, because \( y = \pm \sqrt{(c^2 - k)} \) does not yield real roots
  - The two y-intercepts are \((0,\sqrt{(c^2 + k)})\) and \((0,-\sqrt{(c^2 + k)})\).
- When \( c^2 > k \),
  - The x-intercepts are \((\sqrt{c^2 + k},0)\), \((-\sqrt{c^2 + k},0)\), \((\sqrt{c^2 - k},0)\), and \((-\sqrt{c^2 - k},0)\).
  - There are no real y-intercepts.

**Applications and Future Research**

We looked at equipotential field lines created by two particles of the same charge. The reason is because these field lines, represented by the blue curves in the picture below, form similar graphs as our equation for the product of the distances from a point to two foci.

We need to apply the equation for potential. The equation for potential for two body system is:

\[(KQ/r_1) + (KQ/r_2) = V.\]

Here \(Q\) is the point charge, \(r\) is the distance from that charge, and \(K\) is a given constant.

For simplicity we begin with the equation

\[V = (1/r_1) + (1/r_2),\]

where \(r_1\) and \(r_2\) are the distances from a point to the foci.

After simplification we arrive at

\[(r_1 + r_2)(r_1r_2) = V^2.\]

Using the points \((-c,0)\) and \((c,0)\) as the location of our particle we can replace \(r_1\) with \((c x^2 + y^2)^{1/2}\) and \(r_2\) with \((x^2 + y^2)^{1/2}\). This translates into:

\[2x^2 + 2y^2 + 2x^2y^2 + x^2 + y^2 - 2xy^2 - 2x^2 + 2y^2 = V^2 = 1/k^2,\]

using our constant \(k\) in place of \(V\).

Thus the equipotential field lines are an instance of the general equation:

\[x^2 + y^2 + 2x^2y^2 + 2x^2 - 2xy^2 + C = 0,\]

where:
  - \(A = c^2 - k^2\),
  - \(B = k^2 + c^2\), and
  - \(C = c^2 - 2ck^2\).

**Future Research:**

- I will take a more in-depth look into the general form obtained in this research.
- I will also take a look at the quotient of the distances from a point, \((x,y)\) to two foci, \((c,0)\) and \((-c,0)\).