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Physics in a Glitter Ball

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Maui Toys' Water Bouncer (Fig. 1) is a water-filled ball containing glitter. Buy one and put it on your desk and students can't keep their hands off of it. Pitch the ball in the air giving it a quick spin. When you catch it you will see a sparkling vortex. Twist the ball around different ways and the angular momentum of the fluid keeps the axis of the glitter vortex fixed in one direction.

Let the glitter settle and place the ball under a lamp. Look at the light reflecting from the glitter on the bottom of the ball. Around the perimeter of the sphere you see the colors produced by chromatic aberration. Some of the light entering our sphere can be reflected from the water/air interface at the back side where no glitter is present. This light undergoes dispersion both entering and exiting the sphere. This dispersion combined with the internal reflection gives rise to rainbows and dew bows.[1] If this occurs in the sky we "ooh and aah!" but if it occurs in a lens it is a nuisance and we call it an aberration, like some beautiful flowers are weeds if we don't want them in our garden.

After all, the ball is a spherical lens, so let's calculate the focal length. Taking into account that the sphere is a thick lens, the focal length is

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n-1)}{n}\frac{d}{R_1R_2}\right]. \tag{1}$$

For a sphere of radius R, $R_1 = R$, $R_2 = -R$, and the thickness of the lens $d = 2R$, so our thick lens equation gives us the focal length of a sphere as

$$f=\frac{n}{2(n-1)}R. \tag{2}$$

Put the index of refraction for water $n=\frac{4}{3}$ into Eq. (2) and run the numbers and you get $f = 2R$.

Equation (1) and the equations for the location of the principal planes are derived in *Introduction to Geometrical and Physical Optics* by Joseph Morgan.[2] The derivation for the focal length of a thick spherical lens is rather lengthy since, in general, the two faces of a thick lens have different radii and centers of curvature. You would think that the derivation of the focal length of a sphere, Eq. (2), should be easier since the front and back surfaces have the same center of curvature and radii. Let's give it a try.

In Fig. 2 we have a sphere of index n_2 in a medium of index n_1. A ray parallel to the principal axis $\overline{VV'}$ strikes the sphere at point P at an angle of incidence θ_1 measured from the normal, and refracts at an angle θ_2 measured from the same normal, passes through the sphere, and exits at point Q, crossing the principal axis a distance q beyond the back vertex V'. Applying a little elementary geometry and trigonometry, we see

$$\alpha+\theta_1=2\theta_2 \tag{3}$$

$$\theta_1=\alpha+\gamma \tag{4}$$

$$\tan\alpha=\frac{y}{R-\delta} \tag{5}$$

$$\tan\gamma=\frac{y}{q+\delta} \tag{6}$$

Now consider a paraxial ray parallel to the principal axis by pulling point P and the incident ray down close to V, keeping the ray parallel to the principal axis. This brings the point Q down close to V' and forces δ to become negligibly small. Additionally, the angles θ_1, θ_2, α, and γ become so small that we can use the small-angle approximation: $\tan\theta \simeq \sin\theta \simeq \theta$. The results of the small-angle approximation on Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, is $n_1\theta_1 = n_2\theta_2$ so that $\theta_1 = n\theta_2$. Here $n=\frac{n_2}{n_1}$ is the index of the sphere relative to the surrounding medium. Using Eq. (4) and replacing θ_2 with $\frac{\theta_1}{n}$ in Eq. (3), we get, after a little rearranging,

$$(2-2n)\alpha=(2-n)\gamma. \tag{7}$$

Since δ is essentially zero, the small-angle approximations to Eqs. (5) and (6) are $\alpha=\frac{y}{R}$ and $\gamma=\frac{y}{q}$. Substitute these into Eq. (7), rearrange, and obtain

$$q=\frac{2-n}{2n-2}R. \tag{8}$$

But the initial ray was parallel to the principal axis, which makes this image distance q the focal length of our sphere, but changing q to f won't make Eq. (8) look like Eq. (2). For the lens equation

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

to work for thick lenses, the focal length f, the object distance p, and the image distance q for a thick lens must all be measured from their respective principal planes. For the special case of a sphere, the two principal planes coincide at the center of the sphere, and we can treat the sphere as if it were a thin lens by taking all measurements from the center of the sphere. In Eq. (8) we were measuring from the back vertex of our sphere, so to measure the focal length from the center of the sphere we need to add R to the right side of Eq. (8). Doing so we get

$$f=\frac{n}{2(n-1)}R.$$

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Fig. 1. Circle of light, the caustic.

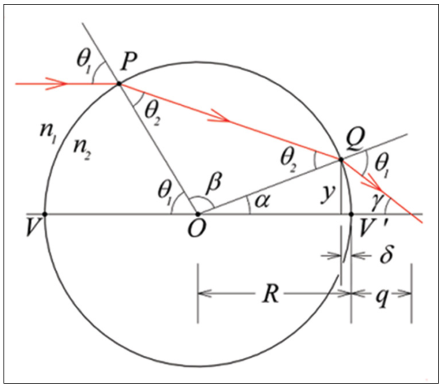

Fig. 2. Ray tracing geometry.

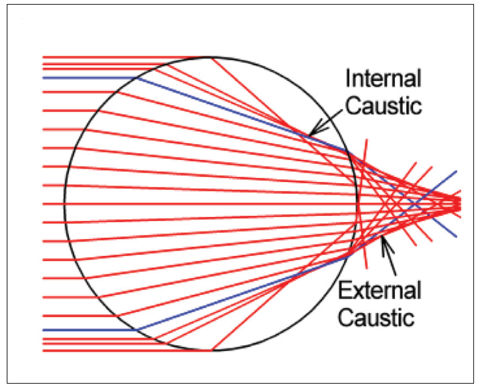

Fig. 3. Multiple rays forming the caustic.

The focal length of our water sphere is $f = 2R$. Hold the "lens" back under a desk lamp and form an image. The image of the light bulb is more than R beyond the ball because the object distance isn't infinite. Take the ball outside into the sunshine and focus the Sun on your hand. The image of the Sun is warm, not hot, and a distance R behind the ball. Water is a good infrared absorber. Measure the image distance. We didn't take into account the index of refraction of the thick vinyl shell of the ball, so our calculation may be off slightly.

Hold the ball up toward the Sun and look at the bright circle of light formed on the inside concave surface of the ball as in Fig. 1. Notice the circumference of the circle of light is brighter than the rest of the circle. If you trace the rays through the sphere with a good CAD program like AutoSketch, you will obtain a drawing like Fig. 3. The bright ring is the intersection of the internal caustic with the surface of the sphere. All rays parallel to the principal axis outside this blue ray are refracted to the inside the circle of light.

In Chapter 8 of *Fundamentals of Optics*, Jenkins and White[3] show an elegant graphical construction that uses Snell's law and allows paraxial *and* oblique rays to be traced through a single lens or an entire optical system, exactly. A CAD program such as AutoSketch makes this process easy and accurate and will provide a student interested in optics many enjoyable hours of ray tracing.

To calculate the internal caustic, refer again to the geometry in Fig. 2. From Eq. (3), $\alpha = 2\theta_1 - \theta_2$. The blue ray in Fig. 3 corresponds to α being a maximum. So use Snell's law, $\sin\theta_1 = n \sin\theta_2$ with $n = \frac{n_2}{n_1}$, set $\frac{d\alpha}{d\theta_1} = 0$, and get

$$\sin\theta_{\rm cir} = \sqrt{\frac{4-n^2}{3}}\,. \qquad (9)$$

The angle $\theta_{\rm cir}$ is the angle of incidence for the ray parallel to the principal axis that refracts through the *circumference* of the bright circle. Again using $n = \frac{4}{3}$, Eq. (9) gives

$$\sin\theta_{\rm cir} = \frac{2}{3}\sqrt{\frac{5}{3}},\ \sin\theta_2 = \frac{1}{2}\sqrt{\frac{5}{3}},$$

and $\alpha = 0.3668$ rad. This means that the bright circle has an arc diameter of $0.734R$.

Let the glitter settle to the bottom of the ball. Now drop the ball without spinning it. This may take a little practice. The way I do it is to rest the ball on the palm of my hand and accelerate my hand downward with acceleration greater than g. When the ball hits the floor, the glitter produces a mushroom cloud. After the glitter settles, rotate the ball 90° about a horizontal axis in about one second and watch the turbidity current. Oops! That's geology.

The glitter ball costs about the same as a good enchilada dinner, but lasts a whole lot longer.

References

1. Jearl Walker, *The Flying Circus of Physics With Answers* (Wiley, 1977), p. 122.
2. Joseph Morgan, *Introduction to Geometrical and Physical Optics* (McGraw-Hill, 1953), p. 57.
3. Francis A. Jenkins and Garvey E. White, *Fundamentals of Optics*, 3rd ed. (McGraw-Hill, 1957), p. 119.

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