Project Report No. 66, Metric Volume and Biomass Prediction Equations for Loblolly and Slash Pine Trees Planted in Unmanaged Pine Plantations in East Texas

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METRIC VOLUME AND BIOMASS PREDICTION EQUATIONS FOR LOBLOLLY AND SLASH PINE TREES PLANTED IN UNMANAGED PINE PLANTATIONS IN EAST TEXAS

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Abstract. - Metric equations are presented to predict the volume of wood and/or biomass contained in individual loblolly (Pinus taeda, L.) and slash (Pinus elliottii, Englem.) pine trees growing in unmanaged pine plantations in east Texas. Taper equations are also presented for both species that describe tree form in metric units.

Mathematical equations are used extensively in forest management to predict the amount of wood and/or biomass contained in individual trees. Lenhart et al. (1987) and Lapongan et al. (1993) published tree content (volume and green-weight biomass) and taper equations for individual loblolly and slash pine trees growing in unmanaged east Texas plantations. Their equations provided estimates of tree form (i.e., taper) as well as volume and green-weight biomass for stem wood, bark, branches, and needles in Imperial (British) units, which are useful to forest managers. However, forest scientists need equations that predict tree content and taper in metric units. In this study, we present metric unit tree content and taper equations for loblolly and slash pine trees growing in unmanaged east Texas plantations. The intent was to provide content and taper equations similar in form to those of Lenhart et al. (1987) and Lapongan et al. (1993), except that the new equations use metric units. Thus, this study utilized the same data and equation forms as Lenhart et al. (1987) and Lapongan et al. (1993).

MATERIALS AND METHODS

This study used the 101 loblolly pine trees and 86 slash pine trees felled and measured by Lenhart et al. (1987) and Lapongan et al. (1993). This section will provide a brief description of how they collected the tree samples. Summary statistics of the tree data in metric units are provided in Table 1.

All sample trees were located near permanent research plots located in loblolly and slash pine plantations across east Texas, which are part of the East Texas Pine Plantation Research Project (ETPPRP; Lenhart et al. 1985). The ETPPRP study area covers 22 counties across east Texas. Generally, the counties are located within the rectangle from 30° - 35° north latitude and 93° - 96° west longitude.

Prior to felling a sample tree, the diameter at breast height (dbh, 1.3 meters above the groundline) was measured. After felling, the total height from ground to tip of stem was determined. All branches were removed and weighed: wood, bark, and needles. A representative branch was selected and weighed with and without needles.

Eight branch segments about 30 cm in length were randomly selected and weighed with and without bark. Using these sub-sample values, appropriate ratios were calculated to convert the total weight of the branches to the weight of its three basic components: wood, bark, and needles. The stem was cut into 1-meter long bolts, and each bolt was weighed. A 2 – 10 cm thick disk was sawn from the bottom of each bolt and weighed with and without bark. Diameter with and without bark was measured at each cut point, and cubic-meter volume of wood and bark and wood only was calculated using Smalian's formula (Avery and Burkhart 2002). For each sample tree, appropriate ratios of stem volume to stem weight were multiplied by total branch weight to obtain an estimate of the volume of wood and bark and wood only in the branches. For these relatively small and young trees, stem ratios should be reasonably representative of branch ratios.

COMPLETE TREE CONTENT PREDICTIONS EQUATIONS

Scatter plots of observed complete tree content, excluding stump, in stem and branches (C) over dbh in centimeters (D) and total height above ground in meters (H) indicate that the most suitable model to predict complete tree content (excludes stump and roots but includes branches) is:

$$C = \beta_0 D^{\beta_1} H^{\beta_2} \qquad (1)$$

Unweighted non-linear least squares regression (NLS) produced the following equations for complete tree content:

Loblolly

$$CCMWB = 0.000060D^{2.2186}H^{0.6261} \qquad (2)$$

$$CCMW = 0.000025D^{2.2440}H^{0.8423} \qquad (3)$$

$$CGWWBN = 0.0712D^{2.0557}H^{0.7415} \qquad (4)$$

$$CGWWB = 0.0432D^{2.0702}H^{0.8828} \qquad (5)$$

$$CGWW = 0.0294D^{2.1313}H^{0.9208} \qquad (6)$$

Slash

$$CCMWB = 0.000068D^{1.9757}H^{0.8841} \qquad (7)$$

$$CCMW = 0.000023D^{2.0938}H^{1.0330} \qquad (8)$$

$$CGWWBN = 0.0755D^{2.0616}H^{0.7490} \qquad (9)$$

$$CGWWB = 0.0519D^{1.9881}H^{0.9274} \qquad (10)$$

$$C = \beta_0 D^{\beta_1} H^{\beta_2} - 0.00007854\left(1-\left(\frac{2}{\beta_3}\right)\right)\left(\frac{udob^{\beta_3}}{D^{\beta_3 - 2}}\right)(H-1.3) \qquad (12)$$

Where: udob = upper-stem diameter (cm) outside bark.

Equation (12) was fit to the volume data using NLS to produce stem volume prediction equations. These fitted equations were algebraically rearranged and simplified to produce the following cubic-meter stem volume equations:

Loblolly

$$SCMWB = 0.000035 D^{1.9575} H^{1.0486} - 0.000029\left(\frac{udob^{3.1828}}{D^{1.1828}}\right)(H-1.3) \qquad 13)$$

$$SCMW = 0.000018 D^{1.9727} H^{1.2053} - 0.000025\left(\frac{udob^{2.9408}}{D^{0.9408}}\right)(H-1.3) \qquad (14)$$

Slash

$$SCMWB = 0.000054 D^{1.7747} H^{1.1274} - 0.000035\left(\frac{udob^{3.5959}}{D^{1.5959}}\right)(H-1.3) \qquad (15)$$

$$SCMW = 0.000023 D^{1.8520} H^{1.2684} - 0.000028\left(\frac{udob^{3.0806}}{D^{1.0806}}\right)(H-1.3) \qquad (16)$$

Where:

SCMWB= total stem cubic-meters of wood and bark, and

SCMW= total stem cubic-meters of wood only.

As mentioned above, Equation (12) can be simplified to:

$$CGWW = 0.0319D^{2.0556}H^{0.9766} \quad (11)$$

Where:

CCMWB= complete tree cubic-meters of wood and bark,

CCMW= complete tree cubic-meters of wood only,

CGWWBN=complete tree green weight in kilograms of wood, bark and needles,

CGWWB=complete tree green weight in kilograms of wood, and bark, and

CGWW=complete tree green weight in kilograms of wood.

All regression coefficients were significant at the $\alpha = 0.05$ level. All Pseudo-R^2's ($\text{Pseudo} - R^2 = 1 - \sum(y-\hat{y})^2 / \sum(y-\bar{y})^2$) were greater than 0.98. No bias was evident in the residual plots (results not shown). SAS (2001) PROC NLIN was used for NLS fitting. Bark and needle content can be determined by differencing the values from the appropriate equations listed above.

STEM CONTENT PREDICTION EQUATIONS

The same equation used by Lenhart et al. (1987) and Lapongan et al. (1993), which was developed by McTague (1985) and modified by Pienaar et al. (1985), was used in this study to predict partial stem content (excludes both stump and branches) to an upper-stem diameter:

$$C = \alpha_0 D^{\alpha_1} H^{\alpha_2} - \alpha_3 \left(\frac{\text{udob}^{\alpha_4}}{D^{\alpha_4 - 2}} \right) (H - 1.3) \tag{17}$$

Equation (17) was fit to the weight data using NLS to produce the following green-weight prediction equations:

Loblolly

$$SGWWB = 0.0248 D^{1.8578} H^{1.2645} - 0.0283 \left(\frac{\text{udob}^{2.7755}}{D^{0.7755}} \right) (H - 1.3) \tag{18}$$

$$SGWW = 0.0201 D^{1.9027} H^{1.2643} - 0.0265 \left(\frac{\text{udob}^{2.7777}}{D^{0.7777}} \right) (H - 1.3) \tag{19}$$

Slash

$$SGWWB = 0.0391 D^{1.7983} H^{1.1788} - 0.0321 \left(\frac{\text{udob}^{3.3435}}{D^{1.3435}} \right) (H - 1.3) \tag{20}$$

$$SGWW = 0.0294 D^{1.8399} H^{1.1923} - 0.0285 \left(\frac{\text{udob}^{3.3581}}{D^{1.3581}} \right) (H - 1.3) \tag{21}$$

Where:

SGWWB= total stem green weight in kilograms of wood, and bark, and

SGWW= total stem green weight in kilograms of wood only.

All regression coefficients were significant at the $\alpha = 0.05$ level. All Pseudo-R^2's were greater than 0.98. No bias was evident in the residual plots (results not shown). SAS (2001) PROC NLIN was used for NLS fitting.

TAPER EQUATIONS

The same procedure used by Lenhart et al. (1987), Lapongan et al. (1993), and Pienaar et al. (1985), was used to develop taper equations in this study. Ormerod (1973) first presented the mathematical forms of these taper equations. The following taper equations were derived (see Appendix for derivation) from Equations (13) and (14) for loblolly pine, and Equations (15) and (16) for slash pine:

Loblolly

$$\text{udob} = D\left(\frac{H-h}{H-1.3}\right)^{0.8455} \tag{22}$$

$$\text{udib} = 0.8896D\left(\frac{H-h}{H-1.3}\right)^{0.7432} \tag{23}$$

Slash

$$\text{udob} = D\left(\frac{H-h}{H-1.3}\right)^{0.6266} \tag{24}$$

$$\text{udib} = 0.8296D\left(\frac{H-h}{H-1.3}\right)^{0.4652} \tag{25}$$

Where:

udob = upper-stem outside bark diameter (cm),

udib = upper-stem inside bark diameter (cm), and

h = upper-stem height (m) where udob or udib occurs.

These taper equations can be algebraically rearranged to solve for h when udob or udib is known:

Loblolly

$$h = H - (H - 1.3)\left(\frac{udob}{D}\right)^{1.1828} \tag{26}$$

$$h = H - 1.1705(H - 1.3)\left(\frac{udib}{D}\right)^{1.3455} \tag{27}$$

Slash

$$h = H - (H - 1.3)\left(\frac{udob}{D}\right)^{1.5959} \tag{28}$$

$$h = H - 1.4942(H - 1.3)\left(\frac{udib}{D}\right)^{2.1496} \tag{29}$$

APPLICATIONS

We present two examples to illustrate how the prediction equations can be used. In the first example, we estimate the branch and needle biomass of a slash pine tree with D = 15 cm and H = 12 m. Use Equation (9) to calculate that the complete tree green-weight including wood, bark, branches, and needles (CGWWBN) = 129 kg. Use Equation (10) to calculate that the complete tree green-weight including wood, bark, and branches (CGWWB) = 113 kg. Subtract 113 kg from 129 kg to estimate 16 kg in needles for this tree. Use Equation (20) to calculate that the green-weight of wood and bark in the total stem (SGWWB, udob = 0) = 95 kg. Subtract this value from 113.3 kg (from above) to estimate that the amount of wood and bark in the branches = 18 kg.

In the second example, we estimate the cubic-meter volume of stem wood only in a loblolly pine tree with D = 20 cm, H = 15 m, and an upper-stem dob (udob) = 6 cm. Use Equation (14) to calculate that the volume of stem wood (SCMW)= 0.17 cubic-meters.

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Table 1. Descriptive statistics for the loblolly and slash pine sample trees.

Species	Variable	N	Mean	Standard deviation	Standard error	Minimum	Maximum
Loblolly	Diameter (cm)	101	13.3	6.6	0.7	2.0	31.2
	Height (m)		9.9	4.3	0.4	2.4	18.9
	Age (years)		10.9	4.1	0.4	4.0	19.0
Slash	Diameter (cm)	86	12.3	6.0	0.6	1.8	28.2
	Height (m)		9.0	4.2	0.5	1.8	17.9
	Age (years)		9.7	3.7	0.4	3.0	19.0

Table 2. Fit statistics for performance evaluation of the loblolly and slash pine equations.

Equation	Loblolly pine		Equation	Slash pine	
No.	R^2	RMSE	No.	R^2	RMSE
(2)	0.9951	0.0106	(7)	0.9921	0.0126
(3)	0.9912	0.0118	(8)	0.9912	0.0103
(4)	0.9861	17.2134	(9)	0.9889	14.8896
(5)	0.9908	13.3304	(10)	0.9898	12.8763
(6)	0.9903	12.7318	(11)	0.9903	11.0589
(13)	0.9828	0.0156	(15)	0.9838	0.0144
(14)	0.9829	0.0130	(16)	0.9799	0.0126
(18)	0.9790	15.6397	(20)	0.9806	14.0250
(19)	0.9792	14.6697	(21)	0.9794	13.0038

Note: $R^2 = 1 - \sum(y-\hat{y})^2 / \sum(y-\bar{y})^2$; $RMSE = \sqrt{\sum_{i=1}^{n}(y-\hat{y})^2 / n}$.

APPENDIX

Pienaar et al. (1985) showed that outside bark stem volume to an upper-stem diameter limit ($TVOB_m$) can be represented by:

$$TVOB_m = TVOB - TOPVOB$$

$$= \beta_0 D^{\beta_1} H^{\beta_2} - \beta_3 \left(\frac{udob^{\beta_4}}{D^{\beta_4 - 2}} \right) (H - 1.3),$$

Where: udob $\leq$ D,

TVOB = Total stem volume, outside bark,

TOPVOB = volume in top of tree above udob,

D = dbh (cm),

H = total height (meters),

udob = upper-stem diameter limit (cm), and

β_i's = regression parameters.

Clutter (1980) outlined a procedure to derive an implied taper equation from this relation using the portion for the top volume. The top volume (TOPVOB) can be written as:

$$k \int_0^T f(t)dt = \beta_3 \frac{H - 1.3}{D^{\beta_4 - 2}} [f(T)]^{\frac{\beta_4}{2}},$$

Where:

k = 0.00007854,

T = distance (m) from the top of the stem to udob = H - h,

h = distance from the groundline to T,

f(t) = udob^2, since diameter outside bark squared is a function of T.

This equation for top volume can be differentiated with respect to T:

$$kf(T) = \frac{\beta_3(H-1.3)}{D^{\beta_4-2}} * \frac{\beta_4}{2}[f(T)]^{\frac{\beta_4-2}{2}} * \frac{d[f(T)]}{dT}.$$

This separable differential equation can be integrated to give:

$$k(H-h) = \frac{\beta_3\beta_4(H-1.3)}{\beta_4-2}\left(\frac{udob}{D}\right)^{\beta_4-2} \qquad (A.1)$$

In this equation, udob must equal D when h = 1.3, so then:

$$k = \frac{\beta_3\beta_4}{\beta_4-2}.$$

Rearranging to solve for β_3 gives:

$$\beta_3 = k\left(1-\frac{2}{\beta_4}\right).$$

Now, the coefficients from Equation 12 can be substituted in Equation (A.1) to obtain the following equation:

$$k(H-h) = k\left(1-\frac{2}{\beta_4}\right) * \frac{\beta_4(H-1.3)}{\beta_4-2}\left(\frac{udob}{D}\right)^{\beta_4-2}.$$

This equation can be simplified to:

$$udob = D\left(\frac{H-h}{H-1.3}\right)^{\frac{1}{\beta_4-2}} \qquad (A.2)$$

Equation (A.2) was used to create Equations (22) and (24).

An inside-bark taper equation was derived in a similar fashion from the following inside-bark volume equation:

$$TVIB_m = TVIB - TOPVIB$$

$$= \alpha_0 D^{\alpha_1} H^{\alpha_2} - \alpha_3 \left(\frac{udob^{\alpha_4}}{D^{\alpha_4 - 2}} \right)(H - 1.3),$$

Where:

TVIBm = inside bark stem volume to an upper-stem diameter limit (udib),

TVIB = Total stem volume, inside bark, and

TOPVIB = volume in top of tree above udib.

The top inside-bark volume for wood only (TOPVIB) can be written as:

$$k\int_0^T f'(t)dt = \alpha_3 \left(\frac{udob^{\alpha_4}}{D^{\alpha_4 - 2}} \right)(H - 1.3),$$

Where:

$f'(t) = udib^2$ = inside-bark diameter at h where outside-bark diameter = udob.

This equation for top volume can be differentiated with respect to T:

$$kf'(T) = \frac{\alpha_3 \alpha_4 (H - 1.3)}{D^{\alpha_4 - 2}} * udob^{\alpha_4 - 1} * \frac{d(udob)}{dT}. \qquad (A.3)$$

From Equation (A.2) with T = H – h, diameter outside-bark is:

$$udob = D \left(\frac{T}{H - 1.3} \right)^{\frac{1}{\beta_4 - 2}}$$

So

$$\frac{d(udob)}{dT} = \frac{D(H-1.3)^{\frac{-1}{\beta_4-2}}}{\beta_4-2} * T^{\frac{3-\beta_4}{\beta_4-2}}.$$

Substituting the expressions for udob and d(udob)/dT above into Equation (A.3)

gives:

$$kf'(T) = k * udib^2 = \frac{\alpha_3\alpha_4 D^{\alpha_4}}{(\beta_4-2)D^{\alpha_4-2}}(H-1.3)^{\frac{\beta_4-\alpha_4-2}{\beta_4-2}}(H-h)^{\frac{\alpha_4-\beta_4+2}{\beta_4-2}}.$$

Simplifying this equation gives:

$$udib = \left[\frac{\alpha_3\alpha_4}{k(\beta_4-2)} D^2\left(\frac{H-h}{H-1.3}\right)^{\frac{\alpha_4-\beta_4+2}{\beta_4-2}}\right]^{\frac{1}{2}}$$

Or,

$$udib = \left[\frac{\alpha_3\alpha_4}{k(\beta_4-2)}\right]^{\frac{1}{2}} D\left(\frac{H-h}{H-1.3}\right)^{\frac{\alpha_4-\beta_4+2}{2(\beta_4-2)}} . \qquad (A.4)$$

Equation (A.4) was used to create Equations (23) and (25).