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SIMULTANEOUS LIGHT AND RADIAL VELOCITY CURVE SOLUTIONS FOR U CEPHEI

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ABSTRACT

The light-curve synthesis approach of Wilson & Devinney has been used to solve simultaneously light and radial velocity curves of the Algol-type eclipsing binary star U Cephei. We have performed eight new differential corrections solutions using the photometric data of Markworth and the radial velocity data of Batten to obtain a consistent set of orbital and astrophysical parameters for the light and velocity curves of this famous system. We find U Cephei to be best modeled using the semidetached (mode 5) system geometry of the Wilson & Devinney program, with a primary rotating at about 5.2 times its synchronous rate, and have found absolute system parameters to be $M_1 = 4.93\ M_\odot$, $M_2 = 3.27\ M_\odot$, $R_1 = 2.77\ R_\odot$, and $R_2 = 5.22\ R_\odot$.
Subject headings: stars: eclipsing binaries — stars: individual (U Cephei)

1. INTRODUCTION

Since its discovery as an eclipsing binary star over a century ago by Ceraski (1880), U Cephei has drawn the attention of many investigators. Detailed accounts of these studies have been presented by Batten (1974), Markworth (1977), Sahade & Wood (1978), and Olson (1984), while spectroscopic solutions and problems have been summarized by Kopal (1944), Struve (1944), Tomkin (1981), Plavec (1983) and McCluskey, Kondo, & Olson (1988). The system spectra are dominated by broad lines due to the hotter primary star, with deeper absorption cores which migrate from the violet to the red side of the line center throughout primary eclipse. Occasionally, hydrogen emission lines are observed (Batten, Baldwin, & Scarfe 1974) which coincide in time with major photometric anomalies (Olson 1976). A classic Rossiter profile near the primary eclipse phases suggests a rapidly rotating primary star, with values of the ratio of rotational to orbital angular velocities varying from 5 (Struve 1963; Plavec 1983) to 8 (Twigg 1980). Solutions for the radial velocity curves alone vary greatly in their values for the eccentricity and longitude of periastron, as shown in Table 1, although eccentricities much different from zero are forbidden by the light curves. Struve (1944) was the first to suggest that the radial velocity curves are contaminated by the presence of a substantial and highly time-dependent gas stream between the components. Hardie (1950) made corrections for this contamination by taking the line centers to be the bisection of the full width as measured at 75% of the nearby continuum level. These "corrected" radial velocity curves did offer lower eccentricities, but not with consistent results.

The light curves of U Cephei have been observed from the infrared (Khozov & Mineav 1969) through the visible and ultraviolet (Kondo, McCluskey, & Wu 1978; Kondo, McCluskey & Stencel 1979; Plavec 1983; Olson 1980a, b; Kondo, McCluskey, & Harvel 1981). The photometric anomalies are numerous and time-dependent. It is quite likely that a completely undisturbed epochal light curve for U Cep has never been observed, although the distortions are nowhere as severe as those encountered for more bizarre Algols such as the W Serpentis stars (Plavec 1980; Wilson, Rafert, & Markworth 1984). The light curve shows a depression of the light level of the ingress branch of the primary eclipse, and the depth of the primary eclipse appears to be variable with a periodic "slant" of the total phase Hall & Walter 1974). During episodes of photometric and spectroscopic activity, the shallow secondary eclipse becomes even shallower (Markworth 1979), and the top of the light-curve display dips, especially near phases of $0.6P$ and $0.2P$ (Olson 1978; Markworth 1977).

The well-observed outburst of 1974 has stimulated a concerted observational effort which has spanned the last 16 years, especially by Olson (1976, 1978, 1980a, b, 1984). A model for U Cep has emerged involving a gas stream from the G5–G8 IV–III component toward the B7 V component, an impact hot spot on the B component, and a disk about the B component (at least during times of high activity). The observed dips in the light curve may also require substantial dark areas on the B component (Olson 1978) or the masking of the light of the primary star by a cooler, thick disk (Crawford 1979). While many of the features cited above are not required to model all light curves, they served to illustrate the character of U Cep over the past decade.

Substantial progress has been made in understanding the general character of U Cep. Most models use parameters from one of the light curve solutions (e.g., Batten 1974; Hall & Walter 1974; Markworth 1979), as well as selected results from spectroscopic solutions (Batten 1974 or Tomkin 1981) as a starting point. Progress toward refining details of the U Cep model has been complicated, since the photometric and spectroscopic solutions suggest different values for the parameters of the system.

We present a solution of U Cep which is based on a simultaneous solution of both light and radial velocity data, so that a consistent set of orbital and astrophysical parameters may be recovered which apply to both sets. We have included all of the known geometrical, bolometric, and kinematic properties of the component star which can be included in the Wilson-Divinney model (Wilson & Divinney 1971) as modified to date. Two major advantages exist in a simultaneous solution: (1) the light and radial velocity curves contain complementary infor-

TABLE 1

RADIAL VELOCITY CURVE SOLUTIONS FOR U CEPHEI

e	w	K_1 (km s^{-1})	K_2 (km s^{-1})	v_γ (km s^{-1})	$a \sin i$ ($R_\odot$)	$a_1 \sin i$ ($R_\odot$)	$f(m)$ ($M_\odot$)	M_1 ($M_\odot$)	M_2 ($M_\odot$)	q	Source
0.47	25	109.9	...	−6.0	...	4.78	0.24	...	...	0.42[a]	Carpenter 1930
0.0[b]	...	62[c]	200	7.3	12.9	3.05	0.06	3.56	1.1	0.31	Kopal 1944
0.20	40	120	146	−5.0	12.8	5.79	0.42	2.52	2.07	0.82[d]	Struve 1944
0.30	30	122	...	13.0	...	5.73	0.41	...	...	0.63[a]	Hardie 1950, uncorrected 1943 spectra
0.0	...	85	...	0	...	4.19	0.16	...	...	0.32[a]	Hardie 1950, corrected 1943 spectra
0.15	10	85	...	22	...	4.14	0.15	...	...	0.31[a]	Hardie 1950, corrected 1949–1950 spectra
0.25	60	120	180	9	14.3	5.72	0.41	3.81	2.54	0.67[d]	Batten 1974, uncorrected
0.0	...	120	180	9	14.8	5.91	0.45	4.20	2.80	0.67[d]	Batten 1974, corrected
0.0	0.0	116	181	16.8	15.4	6.19	0.011	4.66	3.46	0.742	This paper, solution 5
0.0	0.0	110	159	16.8	14.06	6.83	0.010	3.40	3.09	0.906	This paper, solution 8

[a] Assumed $M_2 = 2.8\ M_\odot$, $i = 83$ (Batten 1974).
[b] Assumed.
[c] Synchronous rate for primary star.
[d] Computed on the basis of published e, K_1, K_2, P.

mation, i.e., information which also exists in the other set and which must logically have the same value for the same system; and (2) the agreement between the common parameters of separate light and velocity curves has been poor to date. It would be possible in principle to iterate between photometric and spectroscopic solutions until a consistent set of common parameters emerged, but to our knowledge this has never been done.

2. PREVIOUS SOLUTIONS

We have examined all previous spectroscopic and photometric solutions for U Cep, and we present several of these solutions in Tables 1 and 2. Table 1 shows several of the more frequently cited spectroscopic solutions, while Table 2 shows the results of previous graphical solutions, mostly of the Russell-Merrill type (Russell & Merrill 1950). Those marked with an asterisk are based on a visual inspection of the light curve. Although the Russell-Merrill approach is not widely practiced today, the rectification procedure (in which the assumed triaxial ellipsoid shape is "rectified" to the equivalent spherical case) offers the interesting advantage that successful rectification removes from the light curve those effects which cannot be properly accommodated by the model being used ("complications"). That is, rectification can be done in the absence of theoretical justification. Unfortunately, Hall & Walter (1974) and Markworth (1977, 1979) both present difficulties on successfully rectifying the light curve of U Cep, and these results must therefore be viewed as preliminary solutions.

Tables 3–6 contain previous solutions by Markworth (1979) and Twigg (1980) which used the differential corrections program of Wilson & Devinney (1971, as modified to date, hereafter WD), as well as the results of this study. The WD program is particularly applicable to U Cep, since it uses the Roche geometry, and allows either (or both) of the stars to rotate asynchronously. The F_1 parameter gives the ratio of rotational to orbital angular velocity. Markworth (1979) adopted a value of F_1 of 5.0 based on the measured line widths of Struve (1963), while Twigg (1980) used a fixed value of 8.0 based on his analysis of the Rositer effect in the radial velocity curves. We make special note that neither of these two solutions includes F_1 as an adjustable parameter, so that any inconsistency between the value adopted and the actual value would be absorbed, at least partially, by other adjustable geometrical or astrophysical parameters used in the solution. We

TABLE 2

GRAPHICAL SOLUTIONS FOR U CEPHEI

a_c	b_c	a_h	b_h	k	i	L_c	L_h	X	λ (Å)	Source
0.3225	0.3082	0.2000	0.1911	0.62	86.4	0.1615	0.8385	0.67	5500	Dugan 1920
0.325[a]		0.203[a]		0.63	87.8	0.0725	0.9275	0.6	5550	Broglia (solved by Batten 1974)
0.325[a]		0.203[a]		0.63	87.8	0.0325	0.9675		4350	Broglia (solved by Batten 1974)
0.325[a]		0.203[a]		0.63	87.8	0.0123	0.9877	0.4	3500	Broglia (solved by Batten 1974)
0.317[a]		0.207[a]		0.653	90	0.021	0.979		4300	Walter 1948
0.317[a]		0.207[a]		0.653	90	0.102	0.898	0.7	6100	Walter 1948
0.3408	0.3340	0.1699	0.1665	0.499	83.14	0.139	0.861	0.6	4600	Tschudovitchev 1950 (solved by Hall & Walter 1974)
0.3457	0.3340	0.1723	0.1665	0.498	82.91	0.340	0.660	0.2	8100	Khozov & Mineav 1969 (solved by Hall & Walter 1974)
0.3418	0.3340	0.1704	0.1665	0.499	82.98	0.184	0.816	0.4	5500	Catalano & Rodono 1974 (solved by Hall & Walter 1974)
0.3278	0.3243	0.1891	0.1871	0.577	83.43	0.1137	0.8863	0.6	5500	Markworth 1979
0.3278	0.3243	0.1891	0.1871	0.577	83.43	0.0513	0.9487	0.6	4300	Markworth 1979
0.3391	0.3358	0.1884	0.1866	0.556	82.84	0.0267	0.9733	0.2	3500	Markworth 1979

[a] Values determined by visual inspection of light curves.

TABLE 3

Wilson-Devinney Solutions for U Cephei (Mode 5): Fixed and Adjusted Parameters

Parameter	Solution 1 (simultaneous)	Solution 2 (T_1 = 11,250 K)	Solution 3 (radial velocity)	Solution 4 (light curve only)	Markworth 1979
a	15.5 ± 0.6 (p.e.)	15.5 ± 0.5	15.1 ± 0.8	15.5	...
F_1	5.2	5.2 ± 0.2	5.6 ± 0.4	5.2 ± 0.6	5.0
i	82.05 ± 0.28	82.24 ± 0.29	81.0 ± 0.4	82.6 ± 0.3	82.22 ± 0.15
g_1	1.000	1.000	1.000	1.000	0.46 ± 0.05
g_2	0.320	0.320	0.320	0.320	0.99 ± 0.02
T_1	13600	11250	13600	11250	13600
T_2	4919 ± 32	4533 ± 27	4817 ± 51	4566 ± 33	5454 ± 13
A_1	1.00	1.00	1.00	1.00	1.00
A_2	0.50	0.50	0.50	0.50	0.41 ± 0.02
Ω_1	7.02 ± 0.11	6.98 ± 0.11	7.41 ± 0.19	6.85 ± 0.14	6.911 ± 0.034
$\Omega_2 = \Omega_c$	3.1928	3.1779	3.2886	3.1779	3.143
q	0.672 ± 0.015	0.663 ± 0.015	0.725 ± 0.028	0.663 ± 0.017	0.644 ± 0.008
$L_1(5500)$	0.870 ± 0.001	0.870 ± 0.009	0.867 ± 0.012	0.871 ± 0.026	0.8661
$L_1(4300)$	0.939 ± 0.009	0.939 ± 0.012	0.938 ± 0.017	0.940 ± 0.035	0.935
$L_1(3500)$	0.974 ± 0.013	0.975 ± 0.013	0.975 ± 0.018	0.975 ± 0.037	0.971
$L_2(5500)$	0.1295	0.1304	0.1331	0.1289	0.1339
$L_2(4300)$	0.0615	0.0605	0.0616	0.0603	0.0652
$L_2(3500)$	0.0263	0.0253	0.0255	0.0255	0.0287
$x_1(5500)$	0.348	0.348	0.348	0.348	0.350 ± 0.034
$x_1(4300)$	0.441	0.441	0.441	0.441	0.621 ± 0.023
$x_1(3500)$	0.387	0.387	0.387	0.387	0.535 ± 0.041
$x_2(5500)$	0.806	0.806	0.806	0.806	0.0
$x_2(4300)$	0.980	0.980	0.980	0.980	0.0
$x_2(3500)$	0.987	0.987	0.987	0.987	0.0

have noticed (the WD model supplies the user with parameter correlation matrices) that the value of F_1 is strongly correlated with the values of i (inclination), Ω_1 (surface potential for component 1, modified by asynchronous rotation), and q (mass ratio). Undoubtedly, the assumed values of F_1 have biased the values of i, Ω_1, and q found in Tables 3–6. We also note that the values of the linear limb-darkening coefficients, the bolometric albedos, and the gravity darkening exponents used by Markworth are not consistent with the observationally determined results of Rafert & Twigg (1980) or Twigg & Rafert (1980) for semidetached binaries, and lie outside the range indicated by theory. The main point here is that any particular assumption which is made with respect to one particular (fixed) parameter will influence the final values of the other parameters (adjusted) in a solution, so an effort should logically be made to allow all relevant parameters to be adjustable. As noted later, however (§ 5), solutions which incorporate large numbers of free parameters are prone to several unique types of solution problems.

TABLE 4

Wilson-Devinney Solutions for U Cephei (Mode 6): Fixed and Adjusted Parameters

Parameter	Solution 6 (simultaneous)	Solution 7 (light curve only)	Twigg 1980
a	12.0 ± 0.4	15.531	
F_1	7.10 ± 0.11	6.27 ± 0.10	8.0
i	80.78 ± 0.33	86.55 ± 0.75	83.4 ± 0.5
g_1	1.000	1.000	0.99 ± 0.30
g_2	0.320	0.320	0.44 ± 0.22
T_1	11250	11250	13600
T_2	4433 ± 26	4641 ± 25	4820 ± 120
A_1	1.00	1.00	1.00
A_2	0.50	0.50	0.52 ± 0.04
$\Omega_1 = \Omega_c$	7.2533	6.4969	7.77 ± 0.12
$\Omega_2 = \Omega_c$	3.1650	2.9680	2.998
q	0.656 ± 0.023	0.549 ± 0.017	0.565 ± 0.030
$L_1(5500)$	0.8931 ± 0.0043	0.8967 ± 0.0038	0.879 ± 0.028
$L_1(4300)$	0.9530 ± 0.0041	0.9515 ± 0.0040	0.946 ± 0.036
$L_1(3500)$	0.9815 ± 0.0034	0.9794 ± 0.0034	
$L_2(5500)$	0.1068	0.1033	0.121
$L_2(4300)$	0.0470	0.0485	0.054
$L_2(3500)$	0.0185	0.0206	
$x_1(5500)$	0.348	0.348	0.35 ± 0.35
$x_1(4300)$	0.441	0.441	0.62 ± 0.27
$x_1(3500)$	0.387	0.387	
$x_2(5500)$	0.806	0.806	0.77
$x_2(4300)$	0.980	0.980	0.93
$x_2(3500)$	0.987	0.987	

TABLE 5

WILSON-DEVINNEY SOLUTIONS FOR U CEPHEI (MODE 5): AUXILIARY PARAMETERS

Parameter	Solution 1	Solution 2	Solution 3	Solution 4	Markworth 1979
r_1(pole)	0.157	0.158	0.149	0.161	0.159
r_1(side)	0.178	0.179	0.169	0.184	0.178
r_1(point)	0.179	0.180	0.170	0.186	0.179
r_1(back)	0.179	0.179	0.170	0.185	0.179
r_2(pole)	0.323	0.322	0.330	0.322	0.320
r_2(side)	0.338	0.337	0.345	0.337	0.335
r_2(point)	0.459	0.458	0.467	0.458	0.443
r_2(back)	0.370	0.369	0.377	0.369	0.367
M_1	4.86	4.93	4.35	4.93	
M_2	3.26	3.27	3.17	3.27	
$\sum wr^2$	0.0789	0.0848	0.265	0.0576	

The light curve synthesis program of Wood (1972), called WINK, has also been applied to U Cep by Olson (1984) and Markworth (1977), although a key parameter—the mass ratio—is practically indeterminate.

The previously mentioned problem regarding the eccentricities clearly affects the validity of early solutions in Table 1, and serves to illustrate the types of problems which are encountered when solving radial velocity curves alone. In order to form an easily comparable set of parameter values, we have computed (wherever possible) other parameters, such as $a \sin i$, $a_1 \sin i$, mass function, individual masses, and mass ratio, which can be inferred from the original solutions. For those solutions where the spectrum of the secondary star was measured, q can be computed from e, K_1, K_2 (velocity semiamplitudes), and P (orbital period). In several cases (marked with a plus sign) we have estimated the mass ratio using the commonly quoted mass of the secondary star and the inclination given by Batten (1974). We are aware that this procedure mixes two different data sets and analysis techniques, and that M_2 as measured by Batten is greater than other estimates previous to this study. The estimated mass ratios are therefore probably accurate to about 25%.

3. SELECTION OF PHOTOMETRIC AND SPECTROSCOPIC DATA SETS

The selection of the data sets and the weights assigned are of particular importance in a simultaneous solution. The true system parameters are expressed in all light and velocity curves, but only in those sets which are epochal and relatively free of stream, disk, and spot parameters can we unmask these parameters with some precision. The photoelectric data for this study are those of Markworth (1977). Although these data were obtained in 1974–1976 during a period of high average system activity (Scarfe, Delaney, & Gagne 1986), the data forming the normal points do not include nights where activity was present, as determined by the time-of-minima criterion of Crawford & Olson (1979). Fortunately, U Cep varies rapidly in the degree of photometric disturbance, so that an undisturbed light curve can be observed only a few cycles from a highly disturbed one (Olson 1976).

The radial velocity curve data are those of Batten (1974). The hydrogen line data of his Table 4 describe the velocity curve of the primary and secondary components. In addition, lines of the secondary spectrum measured during primary eclipse (Batten's Table 9) have been used. All of these spectra were obtained in 1967–1969, during a relatively quiescent period for U Cep (Hall & Walter 1974).

TABLE 6

WILSON-DEVINNEY SOLUTIONS FOR U CEPHEI (MODE 6): AUXILIARY PARAMETERS

Parameter	Solution 6	Solution 7	Twigg 1980
r_1(pole)	0.151	0.168	0.139
r_1(side)	0.208	0.229	0.177
r_1(point)	0.227	0.252	0.179
r_1(back)	0.217	0.239	0.179
r_2(pole)	0.321	0.307	0.309
r_2(side)	0.336	0.320	0.323
r_2(point)	0.457	0.439	0.429
r_2(back)	0.368	0.353	0.355
M_1	2.26	5.29	
M_2	1.48	2.90	
$\sum wr^2$	2.71	0.0999	

4. WEIGHTS

One of the early and most difficult problems of the simultaneous solution is the assignment of weights to the light and radial velocity data sets. Two problems exist: (1) assignment of weights internal to the three-color photometric data and (2) assignment of the relative weights of the spectroscopic and photometric data sets. The advantages derived from a simultaneous solution will be largely negated if weights are selected which do not adequately balance the information contained in both sets. It is unlikely that both sets contribute exactly equal amounts of information towards the solution. A strategy of weighting that maximizes the useful information content of each set should, however, be devised for each case. For example, the radial velocity curves for U Cep are plagued by gross distortions caused by gas streaming to the extent that they are not a reliable indicator of the orbital eccentricity. The solution must be forced to utilize the placement and width of the light minima to measure eccentricity. On the other hand, our knowledge of F_1 will be driven by the Rossiter profiles near the primary minimum.

The WD technique requires three different types of weighting. Each data point carries a weight. Since we wish to solve for F_1 in the simultaneous solution, the radial velocities near primary minimum were given extra weight. Beyond that, however, we were governed by the comments of Batten (1974) concerning the individual spectra. The individual weights of the radial velocities were then adjusted uniformly downward, except in the region of the Rossiter effect, so that distortions would not hamper the solution attempts. The light measures were normal points, where the weight used was the number of individual observations per normal point. The program next allows the user to input a "noise" factor. We used a weighting proportional to the inverse square of the light level and independent of the radial velocity. Last, a standard error for each data set can be assigned. In the case of the photometric data, this value is normally chosen to be the standard error of that particular light curve, as measured in normalized intensity, while for radial velocity curve data it is measured in kilometers per second. The sum of squared residuals uses residuals computed only up to the error bar given by this standard error. The user can occasionally force the program to examine one set of data more carefully by lowering the standard error for that set. We have used 15 km s^{-1} as the standard error for the radial velocity sets and 0.02, 0.02, and 0.03 as the errors in V, B, and U light curves, respectively.

5. THE SOLUTIONS

Eight new solutions are presented in Tables 1–6. Solutions 1–5 are the solutions which use a semidetached (mode 5) configuration. Solution 1 used both *UBV* and radial velocity curves for primary and secondary; solutions 2 and 3 were modified parallel solutions from solution 1 as discussed below. Solution 4 used light curve data only (Table 3), while solution 5 used radial velocity data only (Table 1). Solution 1 was initiated with a detached (mode 2) geometry, with a conversion to mode 5 only when several parameter subsets suggested corrections for Ω_2 which would have resulted in the secondary component exceeding its critical Roche lobe had those corrections been applied. Solutions 2–5 were subsequently initiated in mode 5. Solutions 6–8 used the so-called "double contact" configuration described by Wilson & Twigg (1980), in which case the secondary fills its Roche lobe in the usual way, i.e., synchronous rotation, and the primary component fills its critical equipotential adjusted for rotational effects (mode 6). Thus both components fill critical lobes, although they are not in contact. Solutions 6, 7, and 8 are parallel mode 6 solutions corresponding to solutions 2, 4, and 5.

Each of our solutions employed the method of parameter subsets (Wilson & Bierman 1976) and was terminated only when the parameter corrections for the base were all exceeded by their probable errors. The initial parameter values for each solution were those of Markworth (1979) for the photometric elements and those of Batten (1974) for the spectroscopic elements. The less sensitive parameters (bolometric albedos, limb-darkening coefficients, gravity darkening exponents) have been fixed at their theoretical values. The primary star was assumed to have a radiative atmosphere and the secondary star a convective atmosphere. The model atmosphere grid of Carbon & Gingerich (1969) was used to obtain limb-darkening coefficients.

Figure 1 and 2 show the simultaneous solution (*solid line*) plotted along with the observed light and radial velocity curves. There are several noteworthy results which the simultaneous solutions supply:

1. The value of $F_1 = 5.25$ found in solution 1 underestimates the Rossiter effect near the primary eclipse. Twigg (1980) and Wilson & Twigg (1980) found $F_1 = 8.0$ by fitting the "corrected" radial velocity curve data of Hardie (1950). A value of F_1 closer to 6, however, is suggested from the frequency of the small-amplitude light variations seen in the out-of-eclipse data present in the light curve. Such variations might be caused by a hot or cool spot on the surface of the primary stars. Our "light curve only" solution 5 obtains F_1 through the polar flattening of the primary star (viz., Wilson & Mukherjee 1988; Wilson 1988a, b) and yields an intermediate value of 7. In an effort to obtain a closer fit to the Rossiter profiles, we initiated an additional parallel solution (solution 3) in which all parameters were initially set at the values of solution 1, but the weights of the primary radial velocity curve within the phases 0.95–0.05 were manually adjusted upward by a factor of 18. As can be seen, only marginal improvement to the Rossiter profile was achieved, although the value of F_1 rose somewhat to $F_1 = 5.61$.

2. The values of the mass ratios found for solutions 1 and 2 (0.672 and 0.663) differ only slightly, and fall within the range of all previous solutions as shown in Tables 1–3. We have noticed that the values of F_1 and q are tightly correlated (by inspection of WD correlation matrix), particularly for "light

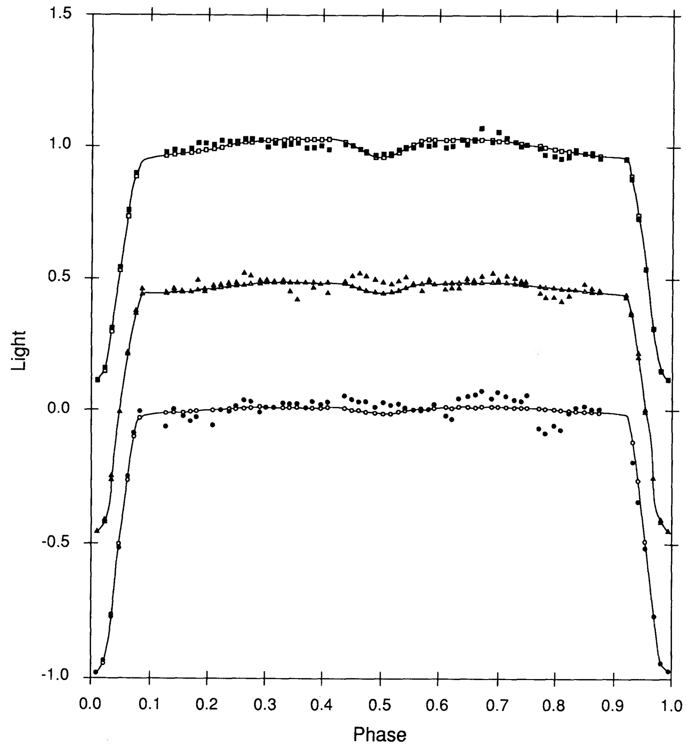

FIG. 1.—Our fit (solution 2, mode 5) to the *UBV* photometric data of Markworth (1979). Filled squares, triangles, and circles are the visual, blue, and ultraviolet observation normals, respectively, while open symbols are the theoretical values at that phase. The *B* curve is displaced downward by 0.5 relative to *V*, and the *U* curve by an additional 0.5.

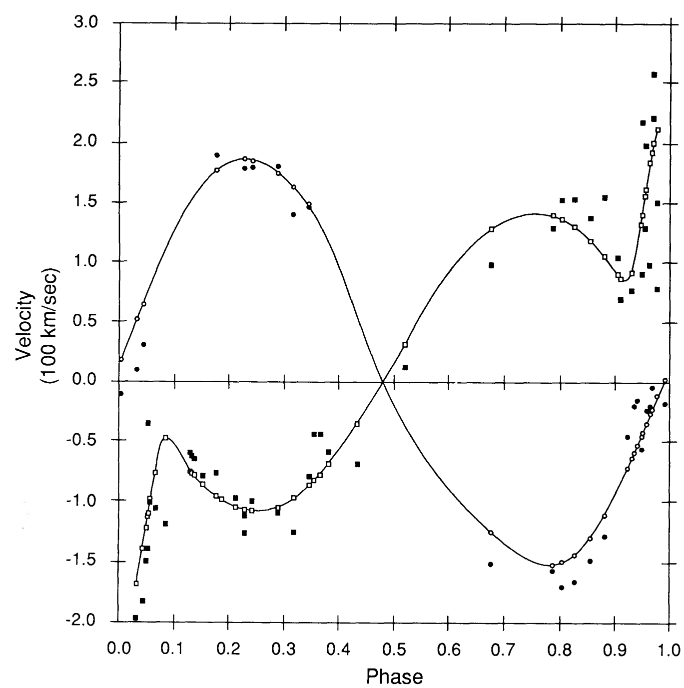

FIG. 2.—Our simultaneous fit (solution 2, mode 5) to the radial velocity data of Batten (1974). Filled squares and circles are the data for primary and secondary, respectively, while open symbols are the theoretical values at that phase.

curve only" solutions. This point is well made by comparing the values of F_1 and q given by Markworth (1979; 5.0 and 0.644), Twigg (1980; 8.0 and 0.565), and solution 4 (this study; 5.2 and 0.663). We note that of the three previous solutions cited, only solution 4 was performed with F_1 as an *adjustable* parameter—in the other two cases an *assumed* value was used. To a lesser degree, this correlation is preserved in the simultaneous solution (solution 1, 5.25 and 0.672; solution 2, 5.22 and 0.663), although these are impersonal fits in which both parameters are adjustable. Only the method of differential corrections is applicable in this case, where there are several nonlinear, model-dependent, correlated parameters (Wilson 1988a).

3. The orbital elements of the simultaneous solution can be used in conjunction with the orbital period to compute absolute elements (note that the semimajor axis of the orbit is one of the adjustable parameters). Values of the semimajor axis are presented in Tables 3 and 4, while masses and relative radii are tabulated in Tables 5 and 6.

A correlation exists in the parameter set which is forced by the system geometry, namely, between the inclination and the size of the primary star. This is expressed as in i-Ω_1 correlation in the WD solution. An i-Ω_2 correlation is also enforced for the same reason, but as Ω_2 is fixed by q for mode 5, this will appear as an i-q correlation. Clearly, other parameters which affect the sizes of the stars will also be correlated with inclination. We must therefore expect a correlation between F_1 and i. The multiple subset method (MSM) of Wilson & Biermann (1976) has been effective in reducing indeterminacy in the solutions due to such correlations.

A related point which influences the precision of derived parameters lies in the area of how many adjustable parameters should be employed. Rafert & Markworth (1986) have shown how failure to include essential parameters will inflict *guaranteed* correlations. Then the model (whatever one it might happen to be) is forced to consider the substantial polar flattening and change in emergent flux to be due to some other effect.

4. It is interesting to note that mode 6 ("double contact") is not the best representation for U Cep for our data. We note that our derived values (mode 5) of $F_1 = 5.25$–5.61 and (mode 6) $F_1 = 6.27$–7.10 are substantially less than those used by Wilson & Twigg (1980) for their double contact solution.

The value that we and Twigg (1980) chose for the polar temperature of the primary (13,600 K) follows from the B7 V classification of Batten (1974), and supplies a good fit to the optical spectral region. Plavec (1983) has obtained a best-fitting Kurucz model atmosphere with an effective temperature of 11,250 K which provides a good fit downward to about 100 Å. In order to determine whether a lower sum of the residuals squared could be obtained with this temperature, we initiated solution 2 with a value of $T_1 = 11{,}250$ K. As can be seen from Tables 3–6, the results are essentially the same as for solution 1. We draw no particular conclusion here, other than to recognize the inability of the WD program to differentiate between slightly different values of T_1. Nonetheless, the value of $T_1 = 11{,}250$ K was utilized for solutions 2, 4, 6, and 7.

6. THE GRID SEARCH

The parameter correlation problems must be addressed during the solution procedure if a global minimum is to be reached. To test the results of our solutions, we performed a final grid search test. The grid was constructed for values of $i = 90°$, $85°$, $80°$, and $75°$, for values of $q = 0.5$, 0.6, 0.7, and 0.8. All solutions were performed using both light and velocity curves as for the other solutions presented in this work. In order to reduce the number of adjustable parameters to a minimal level, we performed the grid search in mode 6. Although we feel that U Cep is best modeled in mode 5, the choice of operating the grid search in mode 6 offers the following advantages:

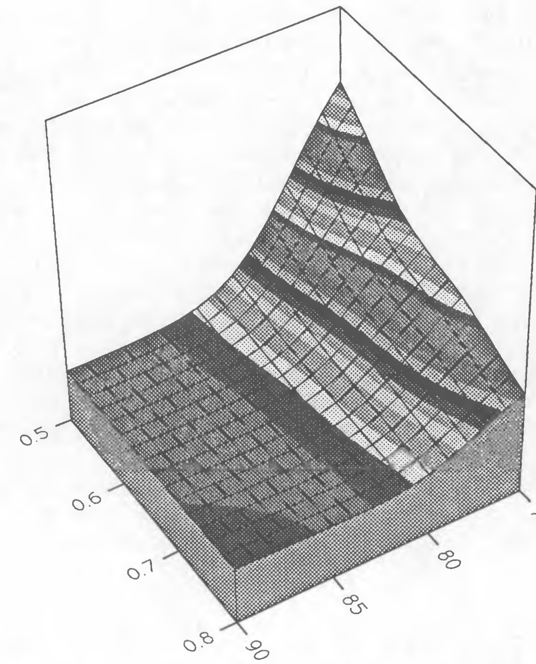

FIG. 3.—Mode 6 i-q hypersurface for the simultaneous solution. Note the broad, shallow area of low residual.

1. Normally, a grid-point solution would need to include a, F_1, i, T_2, Ω_1, Ω_2, q, and L_1 as adjustable parameters (eight parameters). By choosing i and q as grid values, we reduce the number of adjustable parameters to just six. Further reduction of adjustable parameters is possible via the use of mode 6, since Ω_1 and Ω_2 are eliminated as well, as they are computed by the program in this mode. Thus, only four parameters remain for differential corrections adjustment per grid point.

2. The system is very close to double contact, so mode 6 is a good approximation. Our goal is to obtain a global look at the i-q hypersurface to investigate possibilities of multiple local minima using a reasonable amount of computational effort. That is, we are willing to allow some trade-off in detail to obtain a global view.

The results of this analysis are shown in Figure 3. As can be seen, there is an extraordinarily broad and wide "valley" of approximately constant residual, in which our solutions are centrally located. We offer no special comment here, given the crude comparison between our model 5 results and the mode 6 grid, other than noting that there is no reason to expect our solution 2 to *not* be at a global minimum.

7. DISCUSSION

The simultaneous solutions fit the light and radial velocity curves with the same orbital, astrophysical, and geometrical elements, as shown in Figures 1 and 2. The results of our modeling efforts are summarized below:

1. Our value of F_1 results in a radial velocity curve which slightly underestimates the Rossiter profiles in the uncorrected spectroscopy. Wilson & Twigg's (1980) value of $F_1 = 8.0$, on the other hand, results in a good estimate for the corrected spectroscopy of Hardie (1950). The differences in our results depend upon a fundamental difference in solution technique, i.e., whether one wishes to extract spectroscopic complications before or after solving the radial velocity curve. The failure of the model of match the full amplitudes of the (uncorrected) profiles is reasonably strong evidence that the residuals from our present model represent as yet unmodeled physical effects. The solution might be improved using an iterative technique whereby the spectroscopic complications become included in the WD model. One line of attack which we plan to initiate includes the application of the WD model with a thick disk, since U Cep has been identified as a weak W Serpentis star (Plavec 1983).

2. There are severe correlation problems between a, i, Ω_1, q, and F_1. We included every possible subset of these parameters while using MSM, and preferentially selected corrections for subsequent iterations which also existed with the same approximate correction in other subsets. We have reached the bottom of a very broad and shallow global minimum in multiparameter space in which even a slight change in one parameter causes a substantial change in others (of essentially equivalent residual).

3. Our derived values of $M_1 = 4.93\ M_\odot$, $M_2 = 3.27\ M_\odot$, $R_1 = 2.77\ R_\odot$, and $R_2 = 5.22\ R_\odot$ (solution 2, Table 3) are near those of Tomkin (1981) and Olson (1984).

4. In conducting grid searches, we point out the value of eliminating adjustable parameters via appropriate choices of grid tabular values and WD program modes.

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REFERENCES

Batten, A. H. 1974, Pub. Dom. Ap. Obs., Vol. 16, No. 10
Batten, A. H., Baldwin, B. W., & Scarfe, C. D. 1974, IAU Circ., No. 2701
Carbon, D., & Gingerich, O. 1969, in Theory and Observations of Normal Stellar Atmospheres, ed. O. Gingerich (Cambridge: MIT Press), 377
Carpenter, E. F. 1930, ApJ, 72, 209
Catalano, S., & Rodonò, M. 1974, PASP, 86, 390
Ceraski, W. 1880, Astr. Nach., 97, 319
Crawford, R. C. 1979, PASP, 91, 111
Crawford, R. C., & Olsen, E. C. 1979, PASP, 91, 413
Dugan, R. S. 1920, ApJ, 52, 154
Hall, D. S., & Walter, K. 1974, A&A, 37, 263
Hardie, R. H. 1950, ApJ, 112, 542
Khozov, G. V., & Mineav, N. A. 1969, Trudy Astr. Obs. Leningrad, 26, 55
Kondo, Y., McCluskey, G. E., & Harvel, C. A. 1981, ApJ, 247, 202
Kondo, Y., McCluskey, G. E., & Stencel, R. E. 1979, ApJ, 233, 906
Kondo, Y., McCluskey, G. E., & Wu, C. C. 1978, ApJ, 222, 735
Kopal, Z. 1944, ApJ, 99, 239
Markworth, N. L. 1977, Ph.D. thesis, Univ. of Florida
———. 1979, MNRAS, 187, 699
McCluskey, G. E., Kondo, Y., & Olson, E. C. 1988, ApJ, 332, 1019
Olson, E. C. 1976, ApJS, 31, 1
———. 1978, ApJ, 220, 251
———. 1980a, ApJ, 237, 496
———. 1980b, ApJ, 241, 257
———. 1984, PASP, 96, 162
Plavec, M. J. 1980, in IAU Symposium 88, Close Binary Stars: Observations and Interpretation, ed. M. J. Plavec, D. M. Popper, & R. K. Ulrich (Dordrecht: Reidel), 251
———. 1983, ApJ, 275, 251
Rafert, J. B., & Markworth, N. L. 1986, AJ, 92, 678
Rafert, J. B., & Twigg, L. W. 1980, MNRAS, 193, 79
Russell, H. N., & Merrill, J. E. 1950, Princeton Obs. Contr., No. 23
Sahade, J., & Wood, F. B. 1978, in Interacting Binary Stars, ed. D. Ter Haar (Oxford: Pergamon).
Scarfe, C. D., Delaney, P. A., & Gagne, J. M. V. 1986, PASP, 95, 1165
Struve, O. 1944, ApJ, 99, 222
———. 1963, PASP, 75, 207
Tomkin, J. 1981, ApJ, 244, 546
Tschudovitchev, N. 1950, Astr. Circ., 100, 14
Twigg, L. W. 1980, Ph.D. thesis, Univ. of Florida
Twigg, L. W., & Rafert, J. B. 1980, MNRAS, 193, 775
Walter, K. 1948, Astr. Nach., 276, 225
Wilson, R. E. 1988a, in Critical Observations versus Physical Models for Close Binary Systems, ed. K. C. Leung (New York: Gordon & Breach), 193
———. 1988b, in Critical Observations versus Physical Models for Close Binary Systems, ed. K. C. Leung (Montreux: Gordon & Breach), 455
Wilson, R. E., & Biermann, P. 1976, A&A, 48, 349
Wilson, R. E., & Devinney, E. J. 1971, ApJ, 166, 605
Wilson, R. E., & Mukherjee, J. 1988, AJ, 96, 747
Wilson, R. E., Rafert, J. B., & Markworth, N. L. 1984, Comm. IAPPP, No. 16, 1
Wilson, R. E., & Twigg, L. W. 1980, in IAU Symposium 88, Close Binary Stars: Observation and Interpretation, ed. M. J. Plavec, D. M. Popper, & R. K. Ulrich (Dordrecht: Reidel), 263
Wood, D. B. 1972, A Computer Program for Modeling Nonspherical Eclipsing Binary Star Systems (Goddard Space Flight Center, X-110-72-473)