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SIMULTANEOUS LIGHT AND RADIAL VELOCITY CURVE SOLUTIONS FOR U CEPHEI

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ABSTRACT

The light-curve synthesis approach of Wilson & Devinney has been used to solve simultaneously light and radial velocity curves of the Algol-type eclipsing binary star U Cephei. We have performed eight new differential corrections solutions using the photometric data of Markworth and the radial velocity data of Batten to obtain a consistent set of orbital and astrophysical parameters for the light and velocity curves of this famous system. We find U Cephei to be best modeled using the semidetached (mode 5) system geometry of the Wilson & Devinney program, with a primary rotating at about 5.2 times its synchronous rate, and have found absolute system parameters to be $M_1 = 4.93 M_\odot$, $M_2 = 3.27 M_\odot$, $R_1 = 2.77 R_\odot$, and $R_2 = 5.22 R_\odot$.

Subject headings: stars: eclipsing binaries — stars: individual (U Cephei)

1. INTRODUCTION

Since its discovery as an eclipsing binary star over a century ago by Ceraski (1880), U Cephei has drawn the attention of many investigators. Detailed accounts of these studies have been presented by Batten (1974), Markworth (1977), Sahade & Wood (1978), and Olson (1984), while spectroscopic solutions and problems have been summarized by Kopal (1944), Struve (1944), Tomkin (1981), Plavec (1983) and McCluskey, Kondo, & Olson (1988). The system spectra are dominated by broad lines due to the hotter primary star, with deeper absorption cores which migrate from the violet to the red side of the line center throughout primary eclipse. Occasionally, hydrogen emission lines are observed (Batten, Baldwin, & Scarfe 1974) which coincide in time with major photometric anomalies (Olson 1976). A classic Rossiter profile near the primary eclipse phases suggests a rapidly rotating primary star, with values of the ratio of rotational to orbital angular velocities varying from 5 (Struve 1963; Plavec 1983) to 8 (Twigg 1980). Solutions for the radial velocity curves alone vary greatly in their values for the eccentricity and longitude of periastron, as shown in Table 1, although eccentricities much different from zero are recovered which apply to both sets. We have included all of the model has been complicated, since the photometric and spectroscopic activity, the shallow secondary eclipse becomes even shallower (Markworth 1979), and the top of the light-curve display dips, especially near phases of 0.6P and 0.2P (Olson 1978; Markworth 1977).

The well-observed outburst of 1974 has stimulated a concerted observational effort which has spanned the last 16 years, especially by Olson (1976, 1978, 1980a, b, 1984). A model for U Cep has emerged involving a gas stream from the G5-G8 IV–III component toward the B7 V component, an impact hot spot on the B component, and a disk about the B component (at least during times of high activity). The observed dips in the light curve may also require substantial dark areas on the B component (Olson 1978) or the masking of the light of the primary star by a cooler, thick disk (Crawford 1979). While many of the features cited above are not required to model all light curves, they served to illustrate the character of U Cep over the past decade.

Substantial progress has been made in understanding the general character of U Cep. Most models use parameters from one of the light curve solutions (e.g., Batten 1974; Hall & Walter 1974; Markworth 1979), as well as selected results from spectroscopic solutions (Batten 1974 or Tomkin 1981) as a starting point. Progress toward refining details of the U Cep model has been complicated, since the photometric and spectroscopic solutions suggest different values for the parameters of the system.

We present a solution of U Cep which is based on a simultaneous solution of both light and radial velocity data, so that a consistent set of orbital and astrophysical parameters may be recovered which apply to both sets. We have included all of the known geometrical, bolometric, and kinematic properties of the component star which can be included in the Wilson-Devinney model (Wilson & Devinney 1971) as modified to date. Two major advantages exist in a simultaneous solution: (1) the light and radial velocity curves contain complementary infor-
wag it would be possible in principle to iterate between photometric
formation, i.e., information which also exists in the other set and
which must logically have the same value for the same system;
(2) the agreement between the common parameters of
separate light and velocity curves has been poor to date. It
done.
parameters emerged, but to our knowledge this has never been
frequently cited spectroscopic solutions, while Table 2 shows
the results of previous graphical solutions, mostly of the
Russell-Merrill type (Russell & Merrill 1950). Those marked
with an asterisk are based on a visual inspection of the light
curves. Although the Russell-Merrill approach is not widely
assumed triaxial ellipsoid shape is "rectified" to the equivalent
rectification removes from the light curve those effects which
cannot be properly accommodated by the model being used
(“complications”). That is, rectification can be done in the
absence of theoretical justification. Unfortunately, Hall &
Walter (1974) and Markworth (1977, 1979) both present diffi-
culties on successfully rectifying the light curve of U Cep, and
these results must therefore be viewed as preliminary solutions.
Tables 3–6 contain previous solutions by Markworth (1979)
and Twigg (1980) which used the differential corrections
program of Wilson & Devinney (1971, as modified to date,
hereafter WD), as well as the results of this study. The WD
program is particularly applicable to U Cep, since it uses the
Roche geometry, and allows either (or both) of the stars to
rotate asynchronously. The \( F \) parameter gives the ratio of
rotational to orbital angular velocity. Markworth (1979)
adopted a value of \( F \) of 5.0 based on the measured line widths
of Struve (1963), while Twigg (1980) used a fixed value of 8.0
based on his analysis of the Rositer effect in the radial velocity
curves. We make special note that neither of these two solu-
tions includes \( F \) as an adjustable parameter, so that any
inconsistency between the value adopted and the actual value
would be absorbed, at least partially, by other adjustable geo-
metrical or astrophysical parameters used in the solution. We

\[ \begin{align*}
\text{TABLE 1} \\
\text{RADIAL VELOCITY CURVE SOLUTIONS FOR U CEPHEI} \\
\hline
\text{Source} & \text{Solution} & \text{F} & \text{X} & \text{Å} \\
Carpenter 1930 & ... & ... & ... & ... \\
Kopal 1944 & ... & ... & ... & ... \\
Struve 1944 & ... & ... & ... & ... \\
Hardie 1950, uncorrected & ... & ... & ... & ... \\
Hardie 1950, corrected & ... & ... & ... & ... \\
Batten 1974, corrected & ... & ... & ... & ... \\
This paper, solution 5 & ... & ... & ... & ... \\
This paper, solution 8 & ... & ... & ... & ... \\
\end{align*} \]

\[ \begin{align*}
\text{TABLE 2} \\
\text{GRAPHICAL SOLUTIONS FOR U CEPHEI} \\
\hline \text{Source} & \text{Solution} & \text{F} & \text{X} & \text{Å} \\
Dugan 1920 & ... & ... & ... & ... \\
Broglia (solved by Batten 1974) & ... & ... & ... & ... \\
Broglia (solved by Batten 1974) & ... & ... & ... & ... \\
Broglia (solved by Batten 1974) & ... & ... & ... & ... \\
Broglia (solved by Batten 1974) & ... & ... & ... & ... \\
Walter 1948 & ... & ... & ... & ... \\
Walter 1948 & ... & ... & ... & ... \\
Tschudovitchev 1950 (solved by Hall & Walter 1974) & ... & ... & ... & ... \\
Khovoz & Mineav 1969 (solved by Hall & Walter 1974) & ... & ... & ... & ... \\
Catalano & Rodoto 1974 (solved by Hall & Walter 1974) & ... & ... & ... & ... \\
Markworth 1979 & ... & ... & ... & ... \\
Markworth 1979 & ... & ... & ... & ... \\
Markworth 1979 & ... & ... & ... & ... \\
\end{align*} \]

* Values determined by visual inspection of light curves.
TABLE 3
WILSON-DEVINNEY SOLUTIONS FOR U CEPHEI (MODE 5): FIXED AND ADJUSTED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution 1</th>
<th>Solution 2</th>
<th>Solution 3</th>
<th>Solution 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(simultaneous)</td>
<td>(T = 11,250 K)</td>
<td>(radial velocity)</td>
<td>(light curve only)</td>
</tr>
<tr>
<td>a</td>
<td>15.5 ± 0.6 (p.e.)</td>
<td>15.5 ± 0.5</td>
<td>15.1 ± 0.8</td>
<td>15.5</td>
</tr>
<tr>
<td>F_{1}</td>
<td>5.2</td>
<td>5.2 ± 0.2</td>
<td>5.6 ± 0.4</td>
<td>5.2 ± 0.6</td>
</tr>
<tr>
<td>i</td>
<td>82.05 ± 0.28</td>
<td>82.24 ± 0.29</td>
<td>81.0 ± 0.4</td>
<td>82.6 ± 0.3</td>
</tr>
<tr>
<td>g_{1}</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>g_{2}</td>
<td>0.320</td>
<td>0.320</td>
<td>0.320</td>
<td>0.320</td>
</tr>
<tr>
<td>T_{1}</td>
<td>13600</td>
<td>11250</td>
<td>13600</td>
<td>11250</td>
</tr>
<tr>
<td>T_{2}</td>
<td>4191 ± 32</td>
<td>4533 ± 27</td>
<td>4817 ± 51</td>
<td>4566 ± 33</td>
</tr>
<tr>
<td>A_{1}</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>A_{2}</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>\Omega_{1}</td>
<td>7.02 ± 0.11</td>
<td>6.98 ± 0.11</td>
<td>7.41 ± 0.19</td>
<td>6.85 ± 0.14</td>
</tr>
<tr>
<td>\Omega_{2}</td>
<td>3.1928</td>
<td>3.1779</td>
<td>3.2868</td>
<td>3.1779</td>
</tr>
</tbody>
</table>

have noticed (the WD model supplies the user with parameter correlation matrices) that the value of \( F_{1} \) is strongly correlated with the values of \( i \) (inclination), \( \Omega_{1} \) (surface potential for component 1, modified by asynchronous rotation), and \( q \) (mass ratio). Undoubtedly, the assumed values of \( F_{1} \) have biased the values of \( i, \Omega_{1} \), and \( q \) found in Tables 3-6. We also note that the values of the linear limb-darkening coefficients, the bolometric albedos, and the gravity darkening exponents used by Markworth are not consistent with the observationally determined results of Rafert & Twigg (1980) or Twigg & Rafert (1980) for semidetached binaries, and lie outside the range indicated by theory. The main point here is that any particular assumption which is made with respect to one particular (fixed) parameter will influence the final values of the other parameters (adjusted) in a solution, so an effort should logically be made to allow all relevant parameters to be adjustable. As noted later, however (§ 5), solutions which incorporate large numbers of free parameters are prone to several unique types of solution problems.

TABLE 4
WILSON-DEVINNEY SOLUTIONS FOR U CEPHEI (MODE 6): FIXED AND ADJUSTED PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution 6</th>
<th>Solution 7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(simultaneous)</td>
<td>(light curve only)</td>
</tr>
<tr>
<td>a</td>
<td>12.0 ± 0.4</td>
<td>15.531</td>
</tr>
<tr>
<td>F_{1}</td>
<td>7.10 ± 0.11</td>
<td>6.27 ± 0.10</td>
</tr>
<tr>
<td>i</td>
<td>80.78 ± 0.33</td>
<td>86.55 ± 0.75</td>
</tr>
<tr>
<td>g_{1}</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>g_{2}</td>
<td>0.320</td>
<td>0.320</td>
</tr>
<tr>
<td>T_{1}</td>
<td>11250</td>
<td>11250</td>
</tr>
<tr>
<td>T_{2}</td>
<td>4433 ± 26</td>
<td>4641 ± 25</td>
</tr>
<tr>
<td>A_{1}</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>A_{2}</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>\Omega_{1}</td>
<td>3.1650</td>
<td>2.9680</td>
</tr>
<tr>
<td>\Omega_{2}</td>
<td>2.7533</td>
<td>6.4969</td>
</tr>
<tr>
<td>q</td>
<td>6.56 ± 0.023</td>
<td>0.549 ± 0.017</td>
</tr>
<tr>
<td>L_{1}(5500)</td>
<td>0.8933 ± 0.0043</td>
<td>0.8967 ± 0.0038</td>
</tr>
<tr>
<td>L_{2}(4300)</td>
<td>0.9530 ± 0.0041</td>
<td>0.9515 ± 0.0040</td>
</tr>
<tr>
<td>L_{2}(3500)</td>
<td>0.9815 ± 0.0034</td>
<td>0.9824 ± 0.0034</td>
</tr>
<tr>
<td>L_{2}(5500)</td>
<td>1.0168</td>
<td>1.033</td>
</tr>
<tr>
<td>L_{2}(4300)</td>
<td>0.0470</td>
<td>0.0485</td>
</tr>
<tr>
<td>L_{2}(3500)</td>
<td>0.0185</td>
<td>0.0206</td>
</tr>
<tr>
<td>x_{1}(5500)</td>
<td>0.348</td>
<td>0.348</td>
</tr>
<tr>
<td>x_{1}(4300)</td>
<td>0.441</td>
<td>0.441</td>
</tr>
<tr>
<td>x_{1}(3500)</td>
<td>0.806</td>
<td>0.806</td>
</tr>
<tr>
<td>x_{1}(5500)</td>
<td>0.980</td>
<td>0.980</td>
</tr>
<tr>
<td>x_{1}(3500)</td>
<td>0.987</td>
<td>0.987</td>
</tr>
</tbody>
</table>

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form an easily comparable set of parameter values, we have
computed (wherever possible) other parameters, such as $a \sin i$, $\sin i$, mass function, individual masses, and mass ratio,
which can be inferred from the original solutions. For those
solutions where the spectrum of the secondary star was
measured, $q$ can be computed from $e$, $K_1$, $K_2$ (velocity
semiamplitudes), and $P$ (orbital period). In several cases
marked with a plus sign) we have estimated the mass ratio
using the commonly quoted mass of the secondary star and the
primary minimum. On the other hand, our knowledge of $F_1$ will be driven by the Rossiter profiles near the
primary minimum.

The radial velocity curve data are those of Batten (1974).
The hydrogen line data of his Table 4 describe the velocity
curve of the primary and secondary components. In addition,
the radial velocity curves for U Cep are plagued by
gross distortions caused by gas streaming to the extent that
they are not a reliable indicator of the orbital eccentricity. The
solution must be forced to utilize the placement and width of
the light minima to measure eccentricity. On the other hand,
our knowledge of $F_1$ will be driven by the Rossiter profiles near the
primary minimum.

The WD technique requires three different types of weighting.
Each data point carries a weight. Since we wish to solve for $F_1$ in the simultaneous solution, the radial velocities near
primary minimum were given extra weight. Beyond that,
however, we were governed by the comments of Batten (1974)
concerning the individual spectra. The individual weights of
the radial velocities were then adjusted uniformly downward,
except in the region of the Rossiter effect, so that distortions
would not hamper the solution attempts. The light measures
were normal points, where the weight used was the number of
individual observations per normal point. The program next
allows the user to input a "noise" factor. We used a weighting
proportional to the inverse square of the light level and inde-
pendent of the radial velocity. Last, a standard error for each
data set can be assigned. In the case of the photometric data,
this value is normally chosen to be the standard error of that
particular light curve, as measured in normalized intensity,
while for radial velocity curve data it is measured in kilometers
per second. The sum of squared residuals uses residuals com-
puted only up to the error bar given by this standard error.
The user can occasionally force the program to examine one
set of data more carefully by lowering the standard error for
that set. We have used 15 km s$^{-1}$ as the standard error for the
radial velocity sets and 0.02, 0.02, and 0.03 as the errors in $V$, $B$, and $U$ light curves, respectively.

### 4. WEIGHTS

One of the early and most difficult problems of the simulta-
neous solution is the assignment of weights to the light and
radial velocity data sets. Two problems exist: (1) assignment
of weights internal to the three-color photometric data and (2)
assignment of the relative weights of the spectroscopic and
photometric data sets. The advantages derived from a simulta-
neous solution will be largely negated if weights are selected
which do not adequately balance the information contained in
both sets. It is unlikely that both sets contribute exactly equal
amounts of information towards the solution. A strategy of
weighting that maximizes the useful information content of
each set should, however, be devised for each case. For
example, the radial velocity curves for U Cep are plagued by
gross distortions caused by gas streaming to the extent that
they are not a reliable indicator of the orbital eccentricity. The
solution must be forced to utilize the placement and width of
the light minima to measure eccentricity. On the other hand,
our knowledge of $F_1$ will be driven by the Rossiter profiles near the
primary minimum.
5. THE SOLUTIONS

Eight new solutions are presented in Tables 1–6. Solutions 1–5 are the solutions which use a semidetached (mode 5) configuration. Solution 1 used both UBV and radial velocity curves for primary and secondary; solutions 2 and 3 were modified parallel solutions from solution 1 as discussed below. Solution 4 used light curve data only (Table 3), while solution 5 used radial velocity data only (Table 1). Solution 1 was initiated with a detached (mode 2) geometry, with a conversion to mode 5 only when several parameter subsets suggested corrections for \( \Omega_2 \), which would have resulted in the secondary component exceeding its critical Roche lobe had those corrections been applied. Solutions 2–5 were subsequently initiated in mode 5. Solutions 6–8 used the so-called “double contact” configuration described by Wilson & Twigg (1980), in which case the secondary fills its Roche lobe in the usual way, i.e., synchronous rotation, and the primary component fills its critical equipotential adjusted for rotational effects (mode 6). Thus both components fill critical lobes, although they are not in contact. Solutions 6, 7, and 8 are parallel mode 6 solutions corresponding to solutions 2, 4, and 5.

Each of our solutions employed the method of parameter subsets (Wilson & Bierman 1976) and was terminated only when the parameter corrections for the base were all exceeded by their probable errors. The initial parameter values for each solution were those of Markworth (1979) for the photometric elements and those of Batten (1974) for the spectroscopic elements. The less sensitive parameters (bolometric albedos, limb-darkening coefficients, gravity darkening exponents) have been fixed at their theoretical values. The primary star was assumed to have a radiative atmosphere and the secondary star a convective atmosphere. The model atmosphere grid of Carbon & Gingerich (1969) was used to obtain limb-darkening coefficients.

Figure 1 and 2 show the simultaneous solution (solid line) plotted along with the observed light and radial velocity curves. There are several noteworthy results which the simultaneous solutions supply:

1. The value of \( F_1 = 5.25 \) found in solution 1 underestimates the Rossiter effect near the primary eclipse. Twigg (1980) and Wilson & Twigg (1980) found \( F_1 = 8.0 \) by fitting the “corrected” radial velocity curve data of Hardie (1950). A value of \( F_1 \) closer to 6, however, is suggested from the frequency of the small-amplitude light variations seen in the out-of-eclipse data present in the light curve. Such variations might be caused by a hot or cool spot on the surface of the primary stars. Our “light curve only” solution 5 obtains \( F_1 \) through the polar flattening of the primary star (viz., Wilson & Mukherjee 1988; Wilson 1988a, b) and yields an intermediate value of 7. In an effort to obtain a closer fit to the Rossiter profiles, we initiated an additional parallel solution (solution 3) in which all parameters were initially set at the values of solution 1, but the weights of the primary radial velocity curve within the phases 0.95–0.05 were manually adjusted upward by a factor of 18. As can be seen, only marginal improvement to the Rossiter profile was achieved, although the value of \( F_1 \) rose somewhat to \( F_1 = 5.61 \).

2. The values of the mass ratios found for solutions 1 and 2 (0.672 and 0.663) differ only slightly, and fall within the range of all previous solutions as shown in Tables 1–3. We have noticed that the values of \( F_1 \) and \( q \) are tightly correlated (by inspection of WD correlation matrix), particularly for “light
curve only” solutions. This point is well made by comparing the values of } \( F_1 \) and } \( q \) given by Markworth (1979; 5.0 and 0.644), Twigg (1980; 8.0 and 0.565), and solution 4 (this study; 5.2 and 0.663). We note that of the three previous solutions cited, only solution 4 was performed with } \( F_1 \) as an adjustable parameter—in the other two cases an assumed value was used. To a lesser degree, this correlation is preserved in the simultaneous solution (solution 1, 5.25 and 0.672; solution 2, 5.22 and 0.663), although these are impersonal fits in which both parameters are adjustable. Only the method of differential corrections is applicable in this case, where there are several nonlinear, model-dependent, correlated parameters (Wilson 1988a).

3. The orbital elements of the simultaneous solution can be used in conjunction with the orbital period to compute absolute elements (note that the semimajor axis of the orbit is one of the adjustable parameters). Values of the semimajor axis are tabulated in Tables 3 and 4, while masses and relative radii are tabulated in Tables 5 and 6.

A correlation exists in the parameter set which is forced by the system geometry, namely, between the inclination and the size of the primary star. This is expressed as in } \( i-\Omega_1 \) correlation in the WD solution. An } \( i-\Omega_2 \) correlation is also enforced for the same reason, but as } \( \Omega_2 \) is fixed by } \( q \) for mode 5, this will appear as an } \( i-q \) correlation. Clearly, other parameters which affect the sizes of the stars will also be correlated with inclination. We must therefore expect a correlation between } \( F_1 \) and } \( i \). The multiple subset method (MSM) of Wilson & Biermann (1976) has been effective in reducing indeterminacy in the solutions due to such correlations.

A related point which influences the precision of derived parameters lies in the area of how many adjustable parameters should be employed. Rafert & Markworth (1986) have shown how failure to include essential parameters will inflict guaranteed correlations. Then the model (whatever one it might happen to be) is forced to consider the substantial polar flattening and change in emergent flux to be due to some other effect.

4. It is interesting to note that mode 6 (“double contact”) is not the best representation for U Cep for our data. We note that our derived values (mode 5) of } \( F_1 \) = 5.25-5.61 and (mode 6) } \( F_1 \) = 6.27-7.10 are substantially less than those used by Wilson & Twigg (1980) for their double contact solution.

The value that we and Twigg (1980) chose for the polar temperature of the primary (13,600 K) follows from the B7 V classification of Batten (1974), and supplies a good fit to the optical spectral region. Plavec (1983) has obtained a best-fitting Kurucz model atmosphere with an effective temperature of 11,250 K which provides a good fit downward to about 100 Å. In order to determine whether a lower sum of the residuals squared could be obtained with this temperature, we initiated solution 2 with a value of } \( T_1 \) = 11,250 K. As can be seen from Tables 3-6, the results are essentially the same as for solution 1. We draw no particular conclusion here, other than to recognize the inability of the WD program to differentiate between slightly different values of } \( T_1 \). Nonetheless, the value of } \( T_1 \) = 11,250 K was utilized for solutions 2, 4, 6, and 7.

6. THE GRID SEARCH

The parameter correlation problems must be addressed during the solution procedure if a global minimum is to be reached. To test the results of our solutions, we performed a final grid search test. The grid was constructed for values of } \( q \) = 0.5, 0.6, 0.7, and 0.8. All solutions were performed using both light and velocity curves as for the other solutions presented in this work. In order to reduce the number of adjustable parameters to a minimal level, we performed the grid search in mode 6. Although we feel that U Cep is best modeled in mode 5, the choice of operating the grid search in mode 6 offers the following advantages:

1. Normally, a grid-point solution would need to include } \( a \), } \( F_1 \), } \( i \), } \( T_1 \), } \( \Omega_1 \), } \( \Omega_2 \), } \( q \), and } \( L_1 \) as adjustable parameters (eight parameters). By choosing } \( i \) and } \( q \) as grid values, we reduce the number of adjustable parameters to just six. Further reduction of adjustable parameters is possible via the use of mode 6, since } \( \Omega_1 \) and } \( \Omega_2 \) are eliminated as well, as they are computed by the program in this mode. Thus, only four parameters remain for differential corrections adjustment per grid point.

2. The system is very close to double contact, so mode 6 is a good approximation. Our goal is to obtain a global look at the } \( i-q \) hypersurface to investigate possibilities of multiple local minima using a reasonable amount of computational effort. That is, we are willing to allow some trade-off in detail to obtain a global view.

The results of this analysis are shown in Figure 3. As can be seen, there is an extraordinarily broad and wide “valley” of approximately constant residual, in which our solutions are centrally located. We offer no special comment here, given the crude comparison between our model 5 results and the mode 6 grid, other than noting that there is no reason to expect our solution 2 to not be at a global minimum.

7. DISCUSSION

The simultaneous solutions fit the light and radial velocity curves with the same orbital, astrophysical, and geometrical elements, as shown in Figures 1 and 2. The results of our modeling efforts are summarized below:
1. Our value of $F_r$ results in a radial velocity curve which slightly underestimates the Rossiter profiles in the uncorrected spectroscopy. Wilson & Twigg's (1980) value of $F_r = 8.0$, on the other hand, results in a good estimate for the corrected spectroscopy of Hadr[135]e (1950). The differences in our results depend upon a fundamental difference in solution technique, i.e., whether one wishes to extract spectroscopic complications before or after solving the radial velocity curve. The failure of the model of match the full amplitudes of the (uncorrected) profiles is reasonably strong evidence that the residuals from our present model represent as yet unmodeled physical effects. The solution might be improved using an iterative technique whereby the spectroscopic complications become included in the WD model. One line of attack which we plan to initiate includes the application of the WD model with a thick disk, since U Cep has been identified as a weak W Serpentis star (Plavec 1983).

2. There are severe correlation problems between $a$, $i$, $\Omega_1$, $q$, and $F_r$. We included every possible subset of these parameters while using MSM, and preferentially selected corrections for subsequent iterations which also existed with the same approximate correction in other subsets. We have reached the bottom of a very broad and shallow global minimum in multiparameter space in which even a slight change in one parameter causes a substantial change in others (of essentially equivalent residual).

3. Our derived values of $M_1 = 4.93 M_\odot$, $M_2 = 3.27 M_\odot$, $R_1 = 2.77 R_\odot$, and $R_2 = 5.22 R_\odot$ (solution 2, Table 3) are near those of Tomkin (1981) and Olson (1984).

4. In conducting grid searches, we point out the value of eliminating adjustable parameters via appropriate choices of grid tabular values and WD program modes.

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